A

COMPLETE SYSTEM

OF

PRACTICAL ARITHMETIC;

WITH

VARIOUS BRANCHES

IN THE

MATHEMATICS:

ADAPTED FOR THE USE OF THE

GENTLEMAN AND SCHOLAR,

AS WELL AS FOR THE MAN OF BUSINESS.

THE WHOLE INTERSPERSED WITH

A great Variety of useful QUESTIONS worked at full Length.

ADORNED WITH

PROPER PLATES, CUTS, AND TABLES,

In order to excite the Curiosity, and form the Minds of Youth.

TO WHICH IS ADDED,

BY WAY OF APPENDIX,

BOOK-KEEPING, by SINGLE and DOUBLE ENTRY,

WITH DIFFERENT

Forms of Acquittances, Bills of Exchange, &c. &c.

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THE

PREFACE.

SCIENCE may be compared to a highly finifhed pile of building, all the parts of which being difpofed in the moft exact fymmetry, they muft affect our perception, and gratify our internal fenfation with a more exquifite pleafure, than if viewed in a feparate ftate: For, in fuch a ftate, to all but the learned, they would appear broken and unconnected materials of a mighty ftructure, which the mind, wanting power to conceive, could enjoy no fatisfaction in the contemplation of fuch a train of imperfect and confufed ideas. But, when thus exhibited in their true proportion, it will be eafy, even for the youngeft fcholar, to gain a perfect notion of each; and, as he advances, a gradual comprehenfion of the beauty refulting from their connexion, and how they mutually affift and ornament each other.

When we confider the utility of ARITHMETIC, on which fcience almoft all others do abfolutely depend, we need not be furprifed that fo many efforts have been made to bring it to the utmoft degree of perfection, fince the real value of its ufe, certainly merits all the ftudy and pains that can be beftowed upon it.

It muft be owned, that the progrefs of mathematical fciences is but flow, owing to the difficulty

of the ſeveral branches that come under conſideration; but then, it is ſure and certain: the acquiſition here gained is real knowledge. For this reaſon, it is the work of ages to bring even a ſingle branch to perfection; therefore, it is no wonder if the ancients have, in many caſes, made uſe of round-about methods to encompaſs their ends, and given us long and tedious demonſtrations, laying down many propoſitions, either of no uſe, or too ſimple and trifling to be taken notice of; whence moſt of their inventions may be demonſtrated ſhorter, propounded eaſier, diſpoſed in a better method, and taught in a more compendious way.

There are two things abſolutely neceſſary to make the acquiſition of any ſcience as eaſy as its nature will admit. Firſt, the diſpoſition of the work, ſo that the rules may be clear and diſtinct; ſecondly, the illuſtration of theſe rules, by a ſufficient number of proper and uſeful examples; and, as the great difficulty in this ſcience is acquiring the knowledge of ſtating and ſolving queſtions, I have given a great variety of theſe in all the different parts of this Treatiſe, in the moſt particular, diſtinct, and plain manner I poſſibly could, with their anſwers at full length, and explicit directions, where the leaſt difficulty ſeemed to occur.

The ſeveral rules follow in the ſame order, as ſpecified in the table of contents: thus, Part I. Book i. contains the four primary rules; i. e. Addition, Subtraction, Multiplication, and Diviſion, in integers, and Reduction aſcending and deſcending, with the

the tables of money, weights, meaſures, &c. which the learner ſhould be well acquainted with before he proceeds to the uſe of thoſe rules in compound numbers.

In Book II. the rules follow in the ſame order in which they are generally taught in ſchools; but they are all placed in ſuch a manner as to have little or no dependence on each other; therefore, they may be taught in what order every maſter chuſes.

In the ſecond and third parts, which treat of vulgar and decimal fractions, the rules and examples are laid down in ſo plain and intelligent a manner, as to be underſtood by the meaneſt capacities.— The fourth part treats of Geometry, Menſuration, Gauging, Land Surveying, and the Specific Gravity of Metals, &c. in which I have given every thing that is uſeful, taking all the care I poſſibly could to make them plain and eaſy to be underſtood: and that the learner might not be at a loſs in the firſt rudiments of Geometry, &c. I have given him the draught of every operation on a large Copper-plate, in order that he may the more eaſily comprehend the Problems, having every where purpoſely omitted the ſpeculative part, or things that are uſeleſs to beginners, and would prove ſtumbling blocks, rather than any way to improve the mind.

As to thoſe parts which treat of Chronology, Aſtronomy, Geography, and Algebra, I have taken all the care poſſible (within the compaſs of ſuch a limitation)

limitation) to make them plain, and eaſily underſtood by young beginners.

And in order to make this Book as uſeful as poſſible, I have added, by way of Appendix, firſt, a courſe of Book-keeping, by ſingle entry, with a deſcription of the books, and directions for uſing them.—Secondly, Book-keeping, by double entry, according to the Italian method; with various Forms of Acquittances, Bills of Exchange, &c. &c.

Theſe are the ſubjects of the enſuing work; which, if ſeriouſly purſued by a thoughtful mind, the reader may attain to a competent knowledge in theſe uſeful arts.

Perhaps it may be ſaid, there are books of this kind already, and therefore you are only doing the ſame thing over again.—That there are books publiſhed with the ſame deſign, is acknowledged, but that I have trod in the ſame ſteps with their authors, I muſt beg leave to deny; for the chief reaſon that induced me to write this Treatiſe was, becauſe very few had given the operations worked at full length: this was an article I have heard a great many complain of, even teachers themſelves.

As to the work itſelf, it is laid down upon the beſt foundation I could procure from the moſt celebrated authors; and the rules are built upon the beſt principles now taught and practiſed by the moſt eminent maſters of our private and public academies

mies in this kingdom, every difficulty being explained in the moſt conciſe method, and the whole performance made perfectly eaſy to be underſtood; ſo that, by the help of this Treatiſe, any young man, of a tolerable capacity, may in a ſhort time make himſelf maſter of the moſt difficult parts here laid down.

The inſtruction of youth in ſchools and academies is certainly the moſt expeditious method of forming the minds of young perſons, and of bringing them acquainted with that kind of learning, which their intended ſtation and degree of life ſeems to require; thoſe, therefore, that are bleſt with affluent fortunes, and are under the care of prudent parents and guardians, will ſtand in no need of the aſſiſtance of this Treatiſe, unleſs it be to refreſh their memories with what they have formerly been taught, or to look into ſuch ſubjects as are quite foreign to the inſtitution of thoſe ſeminaries of learning; but there are a great many adult perſons, and grown up youth, who through the narrowneſs of their circumſtances, or the neglect of their friends, are forced to endeavour to improve their loſt time as well as they can.

To ſuch as theſe the following Treatiſe will be of great ſervice; for the variety of the ſubjects here treated of, muſt needs gain the attention of all who have the leaſt inclination to ſtudy arts and ſciences.

Perhaps ſome of our moſt eminent teachers may ſay, by inſerting the operations at length, I have encouraged dull and lazy boys, by this means, to

copy

copy out their anſwers, in order to deceive their teachers; but ſuch kind of piracy may ſoon be detected, by varying the work of the queſtions according to the nature of the ſeveral rules.

But my ſole motive for undertaking this work, was purely for the inſtruction and benefit of the unlearned; ſo I hope every impartial reader will judge with candour of the merits of this performance; and if it meets with their approbation, I ſhall not think I have ſpent ſo much labour in vain, but rejoice at having done any thing for the ſervice and good of my country.

I hope my readers will excuſe all defects, and correct what errors they may occaſionally find herein; and that, as well as all other favours, ſhall be gratefully acknowledged by,

Their moſt obedient humble ſervant,

BIRMINGHAM,
FEB. 20, 1783.

W. TAYLOR.

THE

CONTENTS.

PART I.

BOOK I.

BOOK II.

PART II.

PART III.

Extraction

PART IV.

A table

PART

PART V.

PART VI.

PART VII.

PART VIII.

Subtraction

APPENDIX.

ERRATA.

Page 52, in Example 1, Weights and Measures, for 8 lb. 0 oz. 17 dwts. 14 grs. read 8 lb. 0 oz. 17 dwts. 16 grs.—In E. 2, for 11 A. 2 R. 28 P.—10, read 11 A. 2 R. 35 P.—6.——Page 55, E. 13, for 3428l. read 342l.—Page 191, E. 2, for 588,222, read 488,222.—Page 405, under 11 shillings, in the column under value for 11s. 0$\frac{1}{4}$d. read 11s. 0d. this farthing and the two next below being slipped up too high.—Page 428, in the 2d line of figures, in the second col. for 564, read 364.—Page 448, in the 3d line of tenths, for 0 ,1 ,3 ,0 4, read 0 ,1 ,2 ,3 ,4.—and in the 4th line of tenths, for 0 ,1 ,3 ,3 ,4, read 0 ,1 ,2 ,3 ,4.——Page 454, E. 1. for Auctioneers, read thus: suppose an auctioneer hath sold goods, &c. to the value of 32l. what duty has he to pay for the same, at 6d. in the pound, and 15 per Cent. upon that duty? Against 32l. in the next col. under 6d. is 16s. and the duty at 15 per Cent. by the table, is 2s. 4$\frac{3}{4}$d. ,2 p. which added to 16s. = 18s. 4$\frac{3}{4}$d. 2 parts, the duty required.—Page 457, in the table of Square Measure, under yards, for 27, read 2,7.—Page 458, line last, for = 4 A. 3 R. 21 P. read 14 A. 3 R. 21 P.

CHARACTERS used in this SYSTEM.

CHARACTERS in Arithmetic and the Mathematics.

$\therefore$ Ergo; therefore.

$+$ Plus, or more; the sign of addition; as, $3+4$, is 3 added to 4.

$-$ Minus, or less; the sign of subtraction; as, $4-2$, is 2 from 4.

$\backsim$ The sign expressing the difference between two quantities, when it is not known which is the greater of the two.

$\times$ The sign of multiplication; as, 3×4, is 3 multiplied by 4.

. Is likewise a note of multiplication, and sometimes used for $\times$.

$\div$ The sign of division; as, $9 \div 3$, is 9 divided by 3. This is sometimes written like a fraction; thus, $\frac{9}{3}$.

$=$ The sign of equality; as $a = 8$, is a equal to 8.

$: \; :: \; :$ The sign of arithmetical proportion; as, $3 : 6 :: 4 : 8$; is, 3 to 6 so is 4 to 8.

$\div$ The sign of geometrical proportion.

$\sqsubset$ *or* $>$ The sign of majority; as $a \sqsubset b$, signifies a is greater than b.

$\sqsupset$ *or* $<$ The sign of minority; as, $a \sqsupset b$, signifies a is less than b.

$\sqrt{}$ The square root; as, $\sqrt{16}$, is the square root of 16.

$\sqrt[3]{}$ The cube root; as $\sqrt[3]{8}$, is the cube root of 8. This character sometimes affects several quantities, distinguished by a line drawn over them; thus, $\sqrt{\overline{b+d}}$, denotes the square root of the sum of b and d.

$\overline{1+n}|^2$ Signifies that the number denoted by n is to be added to 1, and the sum squared. $\overline{1+n}|^3$; that $1+n$ is to be raised to the third power, or cubed, &c.

$\overline{\quad}|^{\frac{n}{m}}$ Signifies any root or power in general.

CHARACTERS in Geometry and Trigonometry.

∥ Parallel	⊥ Perpendicular	≚ Equiangular, or similar
△ Triangle	□ Square	
< Angle	▭ Rectangle	≟ Equilateral
∟ Right Angle	○ Circle	

CHARACTERS used in Astronomy.

Of the PLANETS.

♄ Saturn	☉ Sun	☽ Moon
♃ Jupiter	♀ Venus	⊕ Earth
♂ Mars	☿ Mercury	

Of the SIGNS.

♈ Aries	♌ Leo	♐ Sagittarius
♉ Taurus	♍ Virgo	♑ Capricorn
♊ Gemini	♎ Libra	♒ Aquarius
♋ Cancer	♏ Scorpion	♓ Pisces

CHARACTERS of the ASPECTS.

☌ or S Conjunction	□ Quartile	*Vc* Quincunx
S S Semisextile	*T d* Tredecile	☍ Opposition
✱ Sextile	△ Trine	☊ Dragon's Head
Q for Quintile	*B q* Biquintile	☋ Dragon's Tail

CHARACTERS of TIME.

A. M. Ante meridiem; before the Sun comes upon the meridian.

O. or *N.* Noon.

P. M. Post meridiem; when the Sun is past the meridian.

A

SYSTEM

OF

PRACTICAL ARITHMETIC.

PART I.

BOOK I.

DEFINITIONS.

ARITHMETIC is the art of computing by numbers; it is called Vulgar or Common Arithmetic, when it treats of whole numbers.

Unit is any thing considered as one, or 1, and is the beginning of number.

Number is a multitude of units; by this every thing is reckoned.

A whole Number is a precise number, without any parts annexed.

A mixed Number is a whole number, with some part annexed.

A Fraction is a part or parts of a unit.

A proper Fraction is less than a unit.

An improper Fraction is greater than a unit.

An aliquot Part is that which is contained a precise number of times in another.

An aliquant Part is such as is contained in another, some number of times, with some part or parts over.

A prime Number is that which can only be measured by a unit.

Numbers are said to be prime to one another, when only a unit measures both. These are also called co-primes.

A perfect Number is that which is equal to the sum of all its aliquot parts.

Integers, or whole Numbers, are such as express a number or multitude of things, whereof each is considered as a unit. Thus 4 tons, 6 yards, 20 miles, &c. each of which is called an integer, or whole number.

Compound Numbers are such as consist of different denominations, as tons, hundreds, quarters, pounds, ounces, &c. Thus 12 T. 3 C. 1 qr. 12 ℔. &c.

Arithmetic, with regard to art and science, consists both in theory and practice.

Theory

Theory considers the nature and quality of numbers, and demonstrates the reason of practical operations.

The practice is that which shews the method of working by numbers, and consequently becomes the most useful and expeditious for business, and has five principal or fundamental rules for the operation, viz.

1. Numeration, or Notation; 2. Addition; 3. Subtraction; 4. Multiplication; and 5. Division.

SECTION I.

NUMERATION.

TEACHETH to read or express the true value of any number when written down, and consequently, to write down any proposed number according to its true value; and this consisteth of two parts.

1. The true order of placing down figures.
2. The true valuing of each figure in its place; both of which are plainly exhibited in the following

TABLE.

Thousands of millions.	Hundreds of millions.	Tens of millions.	Millions.	C. of thousands.	X. of thousands.	Thousands.	Hundreds.	Tens.	Units.	
									9	Nine.
								9	8	Eight.
							9	8	7	Seven.
						9	8	7	6	Six.
					9	8	7	6	5	Five
				9	8	7	6	5	4	Four.
			9	8	7	6	5	4	3	Three.
		9	8	7	6	5	4	3	2	Two.
	9	8	7	6	5	4	3	2	1	One.
9	8	7	6	5	4	3	2	1	0	Cypher.

In this table each figure from the place of units, increaseth in a tenfold proportion; as 9 in the first place is nine; 9 in the second place is ninety; and so of the rest.

These ten places are as far as any common business will require; but when large numbers are expressed by figures, for the more easy reading of them, divide, or rather distinguish your number into periods, each period to contain six characters or figures, called grand periods; then the first period to the right hand will be units, the second millions, third billions, fourth trillions, fifth quadrillions, sixth quintillions, seventh sextillions, eighth septillions, ninth octillions, tenth nonillions, eleventh deci-millions, &c. As for instance, suppose I would number this train of figures, viz.

Septill.

Septill.	Sextill.	Quint.	Quadr.	Trill.	Billions.	Millions.	Units.
8	7	6	5	4	3	2	1
123456,	124121,	681213,	415682,	181643,	214168,	218164,	823812.

EXAMPLES.

Write down 34167 in words at length? Anſwer, Thirty-four thouſand, one hundred and ſixty-ſeven.

Write down 790684218 in words at length? Anſwer, Seven hun dred ninety millions, ſix hundred eighty-four thouſand, two hundred and eighteen.

Expreſs in figures, ſix thouſand and fifty-five? Anſw. 6055.

Expreſs in figures, nine hundred eighty-ſeven millions, ſix hundred and fifty thouſand? Anſw. 987650000.

Expreſs in figures, ſeventeen millions, ſeventeen thouſand, ſeventeen hundred and ſeventeen?

```
17000000
   17000
    1700
      17
--------
17018717  Anſwer.
```

Expreſs in figures, forty-five billions, four hundred, forty-five thouſand and four millions, ſixty thouſand, ſix hundred and fifteen? Anſw. 45445004060615.

NOTATION, *by Roman Numerical Letters.*

One, five, ten, fifty, hundred, five hundred, thouſand.
I. V. X. L. C. D. M.

When a leſs numerical letter ſtands before a greater, it muſt be taken from it, as I before V or X, and X before L or C, &c. thus:

Four, nine, forty, ninety, &c.
IV. IX. XL. XC.

When a leſs numerical letter ſtands after a greater, it is to be added to it, thus:

Six, eleven, ſixty, one hundred and ten.
VI. XI. LX. CX.

A line drawn over any number leſs than a thouſand, ſignifies ſo many thouſands, as $\overline{\text{XL}}$. is forty thouſand; $\overline{\text{C}}$. is one hundred thouſand; $\overline{\text{M}}$. is one million, &c.

Write down in common figures the following numbers, expreſſed in numerical letters, viz.

XI. C. DC. DXL. MC. MDCCLXXXI. $\overline{\text{MC}}$.
Anſwer, 11, 100, 600, 540, 1100, 1781, 1100000.

Write down in numerical letters the following numbers, expreſſed in common figures, viz.

19, 50, 95, 101, 1000, 1500, 60000.
Anſwer, XIX. L. XCV. CI. M. MD. $\overline{\text{VI}}$.

Write down in numerical letters the following number, expreſſed in common figures, viz. 2060000. Anſw. $\overline{\text{MMLX}}$.

II. ADDITION.

TEACHETH to add sundry numbers together, into one sum, called the total.

RULE. 1. Place all the numbers of a like name so, that units may stand under units, tens under tens, hundreds under hundreds, &c.

2. Begin at the units place, and reckon up all the figures in that place from the bottom to the top, and what overplus there is above even tens, set down, carrying one for every ten to the next row, and so on, continuing to the last row, at which set down the total amount.

PROOF. Begin at the top of the sum, and reckon the figures downwards, in the same manner as you added them upwards; and if the sum comes the same as before, it is undoubtedly supposed to be right.

EXAMPLES.

Let these numbers be added together:

9482
590
307
85
10464

To add up this sum, begin at 5, say the sum of 5 and 7 is 12, and 2 is 14; set down 4 and carry 1. The sum of 1 and 8 is 9, and 9 is 18, and 8 is 26; set down 6 and carry 2. Then 2 and 3 is 5, and 5 is 10, and 4 is 14; set down 4 and carry 1. Lastly, 1 and 9 is 10, which being the last, set it down.

More examples for practice.

126893	12486	368121	21304
81621	2101	16012	2186
2118	652	5168	21
312	18	21	318
604	25	68102	21
24	3	245	5
211572	15285	457669	23855

234563	254312	689876	878761
41234	32168	2101	2180
312	1214	86	815
2	21	42	43
4286	24798	31234	124
302	314	1312	4210
56	45	421	432
		25	25
280755	312872	725097	886590

A Gentleman

A Gentleman had in his nurfery one hundred million of oak, one hundred thoufand afh, and one hundred fruit trees, and alfo fixty-nine elm trees; I demand how many trees were growing in the nurfery?

Oak	100000000
Afh	100000
Fruit	100
Elm	69
Anfwer	100100169 Trees in all.

A perfon was born in the year 1734, when will he (if he lives) be 65 years of age?

This is no more than to -	1734
Add -	65
And we have the year required	1799

Suppofe a man was born in the year of our Lord 1780, in what year will he be 60 years of age?

To -	1780
Add -	60
Anfwer	1840

I was born in the year 1748, when fhall I be 40 years of age?

To	1748
Add	40
Anfwer	1788 the year required.

PARADOXICAL QUESTIONS.

QUEST. 1. *If from fix ye take nine, and from nine ye take ten,*
(Ye wits, now the myft'ry explain)
And if fifty from forty be taken; there then
Shall be juft half-a-dozen remain.

SOLUTION.

If from SIX *you take nine* = IX, *there then remains* S
And if from IX *you take ten* = X, *there remains* I
And if from XL *you take fifty* = L, *there remains* X
} = SIX.

QUESTION 2.

Four things I faw, but what they were,
I beg, dear ladies, you'll declare;
And though there were but four exact,
Thirteen they were full as compact;
I cut off half, and then could find,
Exactly eight were left behind:
What feems more ftrange, tho' very fure,
Thefe eight remaining, were but four.

SOLUTION. *Divide thirteen by line that's ftraight,*
The half is evidently eight.
} ~~XIII~~

III. SUBTRACTION,

TEACHETH to take a leffer number from a greater, to find their difference or remainder.

RULE. 1. Place the greater number uppermoft, and the other under it, fo that units may be under units, tens under tens, &c. and draw a line under them.

2. Begin at the right hand or place of units, and fubftract the lower figure from the upper, and fet down the difference underneath them; do the fame with the reft of the figures.

3. When the under figure exceeds that which ftands over it, you muft borrow ten, and add it to the upper number, from which fubtract the lower, and fet down the remainder; carry one to be added to the

next

next lower figure, and subtract the sum from the upper, setting down the remainder ; and so on from one row to another.

PROOF. The way to prove subtraction is no more than, to the lesser number add the remainder ; if the sum be like the greater, the work is right.

	EXAMPLE 1.	E. 2.	E. 3.		E. 4.		E. 5.
From	94165	368419	86459	Bought	9876	From	869426017
Take	35641	126124	21821	Sold	1213	Take	214981764
Rem.	58524	242295	64638	Rem. unsold	8663	Rem.	654444253
Proof	94165	368419	86459	Proof	9876	Proof	869426017

To work example first, say, 1 from 5 and there remains 4, write down four in the place of units, and say 4 from 6 and there remains 2, which write down in the place of tens ; then say 6 from 1 I cannot, but 10 that I borrow added to 1 is 11, 6 from 11 and there remains 5 ; then 1 that I borrowed and 5 is 6, 6 from 4 I cannot, but 6 from 14, and there remains 8 ; 1 that I borrowed and 3 is 4, 4 from 9 and there remains 5, which set down, and the work is done. But as these things are so easy, I think any farther explanation of the rest would be looked upon as prolixity only.

E. 6. Suppose a person was born in the year of our Lord seventeen hundred and forty-five, how old is he this present year, being 1781 ?

From	1781
Take	1745
Answer	36 Years.

EX. 7.

The age of a lady is fifty and three,
What year was she born in, pray tell unto me ?

From	1781
Take	53
Answer	1728

E. 8. In the year of our Lord 508, Tarquin was banished from Rome, for the ravishing of Lucretia, how many years is it since to this present year, being 1781 ?

	1781
	508
Answer -	1273

E. 9. What is the difference betwixt twice twenty-five, and twice five and twenty ?

This is no more than from twice 25=50, subtract twice 5=10+20=30.

Thus	25	5
	25	5
From	50	10
Take	30	20
	20 the difference.	30

E. 10. The difference between two numbers is 36842, the greater is 864952, what is the lesser ?

	864952
	36842
Answer -	828110 the lesser number.

IV. MUL-

IV. MULTIPLICATION,

TEACHETH how to increase any one number by another, so often as there are units in that number by which the one is increased; and serves instead of many additions.

There are three principal members belonging to this rule, viz.

1. The multiplicand, or number to be multiplied.
2. The multiplier, or number by which the multiplicand is multiplied.
3. The product, or number produced in multiplying.

Note, for the ready performance of this, and all the following rules, it is absolutely necessary the following table should be got by heart.

PROOF. The best way to prove multiplication is by division, but as the learner is supposed not yet to know that rule, he cannot prove by it; he must therefore make the multiplicand the multiplier; then if the product is the same as the other, the work is right.

Note. There is a way of proving multiplication by casting away the nines, which is mostly taught in schools, and is very expeditious, but liable to error; but for the exercise and improvement of the learner, I shall shew him both ways in its proper place.

T A B L E.

1	2	3	4	5	6	7	8	9	10	11	12
2	4	6	8	10	12	14	16	18	20	22	24
3		9	12	15	18	21	24	27	30	33	36
4			16	20	24	28	32	36	40	44	48
5				25	30	35	40	45	50	55	60
6					36	42	48	54	60	66	72
7						49	56	63	70	77	84
8							64	72	80	88	96
9								81	90	99	108
10									100	110	120
11										121	132
12											144

The USE of the TABLE.

Find one of the two digits on the side of the table, and the other at top, then in the angle of meeting is their product sought. Thus, to multiply 5 by 8, seek 8 in the upper line, and under it against 5 on the left hand, is 40, the product; and so of others.

CASE 1. To multiply by a single figure.

RULE 1. Place the multiplier underneath the units place of the multiplicand.

2. Multiply from the right hand to the left, thus; begin with the units or lowest figure of the multiplier, by which multiply the lowest figure of the multiplicand, and set down the overplus, above the tens; and carry the tens: Then multiply the second figure of the multiplicand by the same, adding so many units, as you had tens to carry; set down the overplus, and carry the tens as before. Do thus until you come to the last figure, whose product must be set down entire.

EXAMPLE 1.	Multiplicand	6874
	Multiplier	4
	Product	27496

To

To work this example you muſt ſay, 4 times 4 is 16, ſet down 6 and carry one; then 4 times 7 is 28, and 1 that I carried is 29, ſet down 9 and carry 2; then ſay 4 times 8 is 32, and 2 that I carried is 34, ſet down 4 and carry 3; laſtly, 4 times 6 is 24, and 3 that I carried is 27, which ſet down, and the product will be 27496, as by the work.

To prove the foregoing work by caſting away the nines, make a croſs, and add all the figures of the multiplicand together, as units, thus 6+8+7+4=25, throw away the nines, and ſet the remainder 7 on the ſide of the croſs; do the ſame with the multiplier 4, but as there are no nines to throw away, I ſet down 4 on the other ſide of the croſs. Do the like with the product, 2+7+4+9+6=28, throw away the nines, and there remains 1 to be ſet at the top of the croſs. Laſtly, multiply the figures on the ſides, thus, 4 times 7 is 28, throw away the nines and ſet the remainder 1 at the bottom of the croſs, which is the ſame as the top, and proves the work to be right.

$$\begin{array}{ccc} & 1 & \\ 7 & \times & 4 \\ & 1 & \end{array}$$

The other way to prove multiplication is this; ſee the ſame example proved by making the multiplicand the multiplier:

	E. 2.	E. 3.	E. 4.
4			
6874	32142352	53124564	21684218
16	2	3	4
28	64284704	159373692	86736872
32			
24			
Proof 27496			

E. 5.	E. 6.	E. 7.	E. 8.	E. 9.
16841509	6802124	3214568	3456102	1406178
5	6	7	8	9
84207545	40812744	22501976	27648816	12655602

CASE 2. When the multiplier conſiſts of ſeveral figures;

RULE. Multiply each figure through the line, and write down the products according to their places, that is each reſpective product underneath that figure of the multiplier, by which you multiply, then add them up, and you will have the true product in one line.

E. 10.	E. 11.	E. 12.	E. 13.
31246812	34216812	3841265	21806847
16	23	63	84
187480872	102650436	11523795	87227388
31246812	68433624	23047590	174454776
499948992	786986676	241999695	1831775148

E. 14.

E. 14.	E. 15.	E. 16.	E. 17.
281642	864927	302614	621452
458	653	362	984
2253136	2594781	605228	2485808
1408210	4324635	1815684	4971616
1126568	5189562	907842	5593068
128992036	564797331	109546268	611508768

E. 18.	E. 19.	E. 20.	E. 21.
368121456	460136527	746542	253648
2345	3615	253648	746542
1840607280	2300682635	5972336	507296
1472485824	460136527	2986168	1014592
1104364368	2760819162	4479252	1268240
736242912	1380409581	2239626	1521888
863244814320	1663393545105	3732710	1014592
		1493084	1775536
		189358885216	189358885216

CASE 3. When cyphers are intermixed with the figures in the multiplier.

RULE. Omit the cyphers, and place the first figure of each particular product under its respective multiplier; the following examples, wrought at full length, will sufficiently explain this rule.

E. 22.	E. 23.
804700625	314020065
207008009	200405006
7242305625	1884120390
6437605000	1570100325
5632904375	1256080260
1609401250	628040130
166579474222305625	62931193010445390

CASE 4. When there are cyphers at the right hand of either, or both the multiplier and multiplicand.

RULE. Multiply as before, neglecting the cyphers until all the particular products are added together, and to that sum place the number of cyphers that are on the right hand of both factors.

E. 24.	E. 25.	E. 26.
234000	36840	3684000
2600	230	306000
1404	11052	22104
468	7368	11052
608400000	8473200	1127304000000

If you have any number to multiply by 10, 100, 1000, &c. annex as many cyphers thereto as there are in the multiplier, and the work is done.

Multiply	1781	1781
By - -	10	100
Product -	17810	178100

CASE 5. When the multiplier is such a number as any two figures (in the multiplication table) being multiplied together will produce it.

RULE. Multiply the given number by one of those figures, and that product by the other, and you will have the true product.

E. 27.

Multiply 36421 by 16 ?
$4 \times 4 = 16$

145684
4

Product 582736

E. 28.

Multiply 48612 by 36 ?
$6 \times 6 = 36$

291672
6

1750032

E. 29.

Multiply 4364213 by 72 ?
$8 \times 9 = 72$

34913704
9

Product 314223336

E. 30.

Multiply 32410642 by 144 ?
$12 \times 12 = 144$

388927704
12

Product 4667132448

CASE 6. When the multiplier is any number between 10 and 20.

RULE. Multiply each figure in the multiplicand, by the figure in the unit's place, adding to each its back figure, which stands next on the right hand of that you multiplied, only mind, that the first figure you begin with, is the thing itself, there being none to be added to it.

E. 31.

E. 31. Let it be required to multiply 365 by 12 in one line?

$$\begin{array}{r} 365 \\ 12 \\ \hline 4380 \\ \hline \end{array}$$

To work this example, according to the above rule, ſay, 2 times 5 is 10; ſet down 0 and carry 1; then 2 times 6 is 12, and 1 I carried is 13, and the 5 in the units place of the multiplicand makes 18; ſet down 8 and carry 1; then 2 times 3 is 6, and 1 I carried is 7, and 6 in the multiplicand makes 13; ſet down 3 and carry 1, which is added to 3 in the multiplicand, and it makes 4 to be ſet down: So the product of $365 \times 12 = 4380$, as appeareth by the work.

E. 32.	E. 33.	E. 34.	E. 35.
4263	36124	48965	32145
16	18	17	19
68208	650232	832405	610755

NOTES *upon the* NINE DIGITS.

1. The digit One, hath a property which no other number hath, for it neither multiplieth nor divideth, but leaveth the number to be multiplied or divided the ſame.

2. Multiplying any number by Two, is the ſame with doubling of it; and whether you add two to itſelf, or multiply it in itſelf, the ſum and product are equal. And likewiſe obſerve, that no ſquare number can ever terminate, or end with the digit two.

3. If you would multiply any given number by Three, to that number add the double thereof, and the ſum will be equal to the product.

4. If you would multiply any number by Four, you muſt double the doublication, and the ſum is the product.

5. If you would multiply any number by Five, add a cypher to the given number, and take half that ſum for the product.

6. If you would multiply any number by Six, add a cypher to the given number, and take half thereof, to which add the given number, and the ſum will be equal to the product.

Note. Between theſe two laſt-mentioned digits Five and Six, there is a ſecret property; for if you multiply either of them into themſelves, the number produced by ſuch multiplications will terminate in themſelves.

The number Six hath another eminent property, for all its aliquot parts are equal to itſelf, as its half, its third, and its ſixth, being all added together, make ſix. And of numbers that have this property, there are but ten to be found, between one, and one million of millions, which are thoſe exhibited in this table:

$$\text{Numbers of aliquot parts } \tfrac{1}{2}\ \tfrac{1}{3}\ \tfrac{1}{6} \left\{ \begin{array}{r} 6 \\ 28 \\ 486 \\ 8128 \\ 120816 \\ 2096128 \\ 33550336 \\ 536854528 \\ 8586869056 \\ 137438691328 \end{array} \right.$$

Note, If the number 28, 8128, and several of the other numbers, be divided by 2, 3, and 6, severally, there will remain $\frac{1}{3}$ and $\frac{2}{3}$, which fractions are equal to unity, which one being added to the three several quotients, will make up the given number.

As, suppose the last number in the table;

2)137438691328(68719345664
Remains 0

3)137438691328(45812897109
Remains $\frac{1}{3}$

6)137438691328(22906448554
Remains $\frac{4}{6}$, or $\frac{2}{3}$ ———————
Sum of the quotients 137438691328 = to the dividend.

7. If you would multiply any number by Seven, add a cypher to the given number, and take half thereof, to which half add the double of the number given; the sum of them will be the product of the given number, multiplied by seven.

8. To multiply any number by Eight, to the given number add a cypher, and from thence subtract the double of the number given, the remainder is the product.

9. To multiply by Nine, add a cypher to the given number, and from that number subtract the given number, the remainder is the product.

This digit Nine hath a privilege above all other digits; for if you take any number, the nines taken out of the gross sum of that number, or of all the parts thereof severally, the remaining digit will be still the same.

EXAMPLE. The number 45 hath five nines contained therein; so if you multiply 9 by 5, the product is 45. In like manner, if you take the nines out of this number 843, it is the same as if you took the nines out of the single figures 8, 4, 3, which make 15, from which 9 being taken, there will remain the digit 6; and also if you divide 843 by 9, the quotient will be 93, and 6 remaining.

From hence proceeds the way of proving multiplication, by casting away the nines out of the factors and product, as taught in sect. 4, page 8th.

I might introduce various other methods of contracting and working multiplication by short, though tedious and insignificant rules; but these

thefe I fhall omit for a future work (as they are more curious than ufeful) and give the learner a few queftions for practice and improvement in this rule.

QUESTION 1. If 1 hogfhead of tobacco coft 16 pounds, what will 18 coft?

£.
16
18

Anfwer 288 pounds.

Q. 2. Suppofe 200 men take a prize, and each man's fhare amounts to 160 pounds; what is the value of the prize?

160
200

Anfwer 32000 pounds.

Q. 3. In Egypt, there was an ancient city called Babylon, which ftood upon a fquare of 15 miles each way; how much ground did the whole city ftand on?

Multiply 15 } The length of
By - - 15 } a fide.

75
15

Anfwer 225 miles.

Q. 4. A certain country village, it is faid, hath 500 houfes in it; now, allowing 5 perfons to each houfe, what number of people are there in all?

500
5

Anfwer 2500

Q. 5. What is the difference, and what the fum, of fix dozen dozen, and half a dozen dozen?

12 a dozen
12

144 a dozen dozen
6

From - 864 fix dozen dozen
Take - 72 half a dozen dozen

Difference 792

To 864 fix dozen dozen
Add 72 half a dozen dozen

Sum 936

V. DIVISION,

TEACHETH us to find how often one number, called the divifor, is contained in another called the dividend, and ferves inftead of many fubtractions. In this rule there are three real numbers, and a fourth accidental, viz.

1. The dividend, or number to be divided.

2. The divifor, or number by which you divide.

3. The quotient, or number that fhews how often the divifor is contained in the dividend.

4. The remainder, which is always lefs than your divifor.

CASE 1. When the divifor is not greater than 12.

RULE.

RULE. First, seek how often the divisor is contained in the first figure of the dividend; or if the first figure of the dividend be less than the divisor, then in the two first figures of the dividend, and set the quotient figure down, and if any thing remain, carry it to the next figure in the dividend, where it must be reckoned as so many tens, that is, if 1 remain you must call it 10; if 2, 20; if 3, 30, and so on, bearing in mind the remainder of each figure, and adding it to the next, until you have made use of all the figures in the dividend.

PROOF. Multiply the quotient by the divisor, and as you multiply, add in the remainder (if any) or add the whole remainder to the product at last, and if it comes the same as the dividend, the work is right.

EXAMPLE 1.

```
            8)456789
Quotient     57098 — 5
                 8
Proof    -  456789
```

To work this example, seek how often 8 is in 45, which is 5 times; set down 5 under the dividend, and say 5 times 8 is 40, then 40 from 45, and there remains 5, which makes the following 56; then say, how often 8 in 56, 7 times 8 is 56, from 56, and there remains 0; then say, how often 8 in 7, (no time) set down 0, and there remains 7, which makes the next figure 78; then say, how often 8 in 78, 9 times 8 is 72, from 78, there remains 6, which makes the next figure 69; then, how often 8 in 69, 8 times 8 is 64, from 69, there remains 5, which set down at the end of the quotient, as a remainder, and the work is compleated.

E. 2.	E. 3.	E. 4.
2)368421	3)423968	4)986428
184210 — 1	141322 — 2	246607

E. 5.	E. 6.	E. 7.
5)867501	6)612135	9)819684
173500 — 1	102022 — 3	91076

CASE 2. When the divisor consists of many places of figures;

RULE. 1. Set down the dividend, and the divisor on the left hand of it.

2. Enquire how often the first figure of the divisor is contained in the first figure of the dividend; or in the two first figures when that of the divisor is greater, and place the answer in the quotient, by which multiply the divisor, and place the product under the said figure of the dividend, drawing a line underneath it; subtract it therefrom, and to the remainder annex the following figure of the dividend, proceeding as before; but if this product be greater than that part of the dividend, a less figure must be placed in the quotient.

3. If

3. If the remainder should be so small, that when the figure of the dividend joined with it make a sum less than the divisor, then a cypher is to be placed in the quotient, and another figure brought down; for every figure brought down, a cypher or figure must be placed in the quotient. This is called

LONG DIVISION.

EXAMPLE 1. What is the quotient of 14122 divided by 46?

Divisor. Dividend. Quotient.

```
46 ) 14122 ( 307
     138
     -----
     . . 322
         322
         ---
         . . .
```

To work this example, say how often can I have 4 in one (no times) then, how often 4 in 14, which is 3 times, then place 3 in the quotient, and multiply 46 by it, setting the product 138 under 141, and subtract it therefrom, and there remains 3. Then bring down the next figure 2 from the dividend, and annex it to 3, which makes 32; then enquire how often 4 in 3, the answer is 0, which I place in the quotient, and bring down the next figure 2, the dividual is then 322; then seek how often 4 in 32, the answer would be 8; but 46 multiplied by 8 would exceed 322; therefore I place 7 in the quotient, by which I multiply 46, and the product is 322; that, subtracted from 322, leaves nothing, therefore 307 is the quotient.

SCHOLIUM. There are various ways of proving division, and for the exercise of the learner I shall prove it by three different ways; first, by multiplying the quotient by the divisor; secondly, by casting away the nines; and lastly, by addition, thus:

Take the quotient of the last example, and multiply it by the divisor.

```
  307
   46
 ----
 1842
1228
-----
14122 the same as the dividend.
```

E. 2.

```
47 ) 8460 ( 180
     47
     ---
     376
     376
     ---
     . . 0
```

Take this example, and cast away the nines in the divisor and quotient, which put on each side of the cross, and cast away the nines out of the dividend; put the remainder at the top of the cross; then multiply the side figures thereof into each other, and cast the nines out of the product, and if the work be right, the remainder to be written at the bottom of the cross will be the same as the top, as appears by this example.

```
 0
2×0
 0
```

E. 3.

E. 3.

345)746789(2164

*690

567

* 345

2228

* 2070

1589

* 1380

* 209 Remainder

Proof 746789

To prove this example, add up all the lines marked thus *; and as there is nothing but a cypher to be added to 9 in the remainder, put down 9, and for the same reason put down 8: then say, 2 and 3 is 5, and 7 is 12, and 5 is 17; set down 7 and carry 1. Then 1 and 1 is 2, and 4 is 6; set down 6, and say 2 and 3 is 5, and 9 is 14; set down 4 and carry 1; 1 and 6 is 7, which set down, and the sum is the same as the dividend, which proves the work to be right.

Note. If there be a remainder when you prove by the cross, it must be added to the product on the sides of the cross, and the nines thrown out as before.

E. 4.

6023)1897258(315

18069

9035

6023

30128

30115

13

E. 5. 61745)392628787(6358

370470

221587

185235

363528

308725

548037

493960

54077

E. 6.

684573)3233238699(4723

2738292

4949466

4792011

1574559

1369146

2054139

2053719

420

E. 7.

476085)988390547808(20760S

952170

3622054

3332595

2894597

2856510

3808780

3808680

100

CASE 3. When the divisor has cyphers on the right hand;

RULE. Strike them off, and likewise strike off as many places of the dividend on the right hand; and perform the division by the remaining

maining figures. And when the division is finished, annex the figures cut off to the remainder.

```
        E. 1.                          E. 2.
304|00)7456|78(24               41|00)86498764|32(2109725
       608                            82
       ----                           --
       1376                           44
       1216                           41
       ----                           --
        16078 Remainder.               398
       -----                           369
                                       ---
                                        297
                                        287
                                        ---
                                         106
                                          82
                                         ---
                                          244
                                          205
                                          ---
                                          3932 Remaind.
```

When the dividend has the same number of cyphers on the right hand, as the divisor, strike them off from each, and the remainder will be so many of what you divide by, without annexing the cyphers that were cut off.

```
                                 E. 4. 312|000)98643|000(316
E. 3. 123|00)6397|00(52                        936
             615                               ---
             ---                                504
              247                               312
              246                               ---
              ---                               1923
                1                               1872
              ---                               ----
                                                  51
                                                ----
```

CASE 4. When the divisor is such a number, that any two figures (in the multiplication table) multiplied together, will produce it.

RULE. Divide the given number by those numbers or component parts, which is much easier than dividing by all the divisor at once; see the following examples worked at full length.

Note. If there be a remainder in the last division, it will be so many times the first divisor, which added to the first remainder (if any) will give the true remainder sought.

```
        E. 1.                               E. 2.
8|45876912306÷by 32?                  7|17862493508÷by 42?
 -------------                          -----------
4| 5734614038—2 \                     6| 2551784786—6 \
 -------------   }18 Remr.              -----------   }20 Remr.
   1433653509—2 /                         425297464—2 /
```

To prove by multiplication all examples of this kind, you must add, or take in separately the two remainders, when you multiply by their respective divisors that produced them.

E. 3.

```
8|78906543218 7÷by 72?
 |------------
9| 98633179023—3 }
 |------------    }51
 | 10959242113—6 }
 --------------
```

E. 4.

```
9|819186100212÷by 81?
 |------------
9| 91020677801—3 }
 |------------    }48
 | 10113408644—5 }
 --------------
```

E. 5.

```
12|244801864013÷by 144?
  |------------
12| 20400155334—5 }
  |------------    }77 Remainder
  | 1700012944—6 }
  --------------
```

Those who are well acquainted with the nature of division, may subtract each figure of the product as it is produced, and only write down the remainders; this will shorten the work, and is commonly called Italian division; to perform which the following examples will sufficiently explain:

E. 1.

```
34)368492(10838
   284
    129
     272
     ...
```

E. 2.

```
324)6842189(21117
    362
     381
      578
      2549
       281 Remainder
```

E. 3.

```
6125)86497531(1412
     25247
      7475
      13503
       1253 Remainder
```

E. 4.

```
406502)416903149750(102558
       1040114
        2271109
         2385997
          3534875
           282859 Rem.
```

VI. REDUCTION,

TEACHETH to reduce all great names into small, by multiplying continually the given number with so many of the next lower name, as makes one of the higher, keeping them equivalent in value; this is called Reduction Descending. On the contrary, where the quantity is to be reduced to a higher denomination, divide continually the given number by so many of the lesser name as make one of the greater; this is termed Reduction Ascending.

TABLES

TABLES of ENGLISH COINS.

Marked							q.
q.	4 Farthings	make one	Penny	$\frac{1}{4}$	is wrote for		1
d.	12 Pence		Shilling	$\frac{1}{2}$			2
s.	20 Shillings		Pound £.	$\frac{3}{4}$			3

Note. The reason for placing £. s. d. q. over every denomination, signifies, Libra, Solidi, Denarii, Quadrantes; that is, Pounds, Shillings, Pence, Farthings.

PENCE TABLE.

s.		d.		d.		s.	d.	
1	are	12		20	are	1	8	
2		24		30		2	6	Half a Crown
3		36		40		3	4	
4		48		50		4	2	
5		60		60		5	0	A Crown
6		72		70		5	10	
7		84		80		6	8	Noble
8		96		90		7	6	
9		108		100		8	4	
10		120		110		9	2	
11		132		120		10	0	Angle
12		144		130		10	10	
13		156		140		11	8	
14		168		150		12	6	
15		180		160		13	4	Mark
16		192		170		14	2	
17		204	Pistole	180		15	0	
18		216		190		15	10	
19		228		200		16	8	
20		240	Pound	210		17	6	
21		252	Guinea	220		18	4	
27		324	Moidore	230		19	2	
				240		20	0	Pound.

The WEIGHTS and VALUE of such GOLD and SILVER COINS, as are most commonly used in England.

	WEIGHT.			VALUE.		
	Dwts.	gr.	mites.	£.	s.	d.
A Guinea	5	9	9	1	1	0
Half ditto	2	16	14	0	10	6
A Quarter ditto	1	8	7	0	5	3
SILVER.						
A Crown	19	8	$10\frac{3}{4}$	0	5	0
Half ditto	9	16	$5\frac{1}{6}$	0	2	6
A Shilling	3	20	18	0	1	0
A Six-pence	1	22	9	0	0	6

Note. 20 mites make one grain.

A pound weight avoirdupoise of copper, is coined into twenty-three pence; consequently a halfpenny is nearly $\frac{1}{3}$ of an ounce, and a farthing $\frac{1}{6}$.

EXAMPLE 1.

EXAMPLE 1.

In 24 pounds, how many shillings and pence?

```
 £.
 24
 20
----
480 Shillings
 12
----
5760 Pence
```

E. 2. How many shillings and pounds are there in 5760 pence?

```
      d.
12)5760
--------
2|0)48|0 Shillings
--------
    24  Pounds
```

E. 3. In 36*l.* 10*s.* how many shillings, pence, and farthings?

```
 £.  s.
 36  10
 20
----
 730 Shillings
  12
----
8760 Pence
   4
-----
35040 Farthings
```

In this example multiply as before, but observe to take in the 10*s.* in their proper place, that is, when you multiply by 20, the shillings in a pound.

E. 4. In 35040 farthings, how many pounds?

```
      qrs.
  4)35040
  -------
  12)8760
  -------
  2|0)73|0
  --------
Answer £.36 10s.
```

E. 5. Reduce 302*l.* 16*s.* $4\frac{3}{4}$*d.* to farthings?

```
 £.   s.  d.
 302  16  4¾
  20
-----
 6056
   12
-----
72676
    4
-----
Answer 290707 Farthings
```

E. 6. In 290707 farthings, how many pounds?

```
4)290707
--------
12)72676 ¾qrs.
--------
2|0)605|6 4d.
--------
Answer £. 302 16 4¾
```

WEIGHTS and MEASURES.

TROY WEIGHT.

Marked			
gr.	24 Grains	make one	Penny weight
dwts.	20 Penny weights	make one	Ounce
oz.	12 Ounces	make one	Pound

By Troy weight is weighed gold, silver, jewels, corn, bread, and all liquors; from this weight all measures for wet and dry commodities are taken.

N. B.

N. B. 14*oz.* 11*dwts.* $15\frac{1}{2}$ *grs.* Troy, is equal to one pound Avoirdupoise.

E. 1. In 36*lb.* of silver, how many ounces, penny weights, and grains?

```
lb.
36
12
———
432 Ounces
20
———
8640 Dwts.
24
———
34560
17280
———
207360 Grains
```

E. 2. Reduce 207360 grains to penny weights, ounces, and pounds?

```
grs.
4)207360
————
6) 51840
————
2|0) 864|0
————
12) 432
Answer 36 Pounds
```

E. 3. In an ingot of silver, weighing 14*lb.* 10*oz.* 16*grs.* how many grains?

```
lb. oz. dwts. grs.
14  10   0    16
12
———
178
20
———
3560
24
———
14246
7121
———
Answer 85456 Grains
```

E. 4. Let it be required to reduce 85456 grains to pounds?

```
               grs.
4×6=24 { 4)85456
          ————
         6)21364       }
          ————         } =16
         2|0) 356|0—4  }
          ————
         12) 178
          ————   oz. dts. grs.
Answer  lb. 14  10   0   16
```

APOTHECARIES' WEIGHT.

Marked			
grs.	20 Grains	make one	Scruple
℈.	3 Scruples	make one	Dram
ʒ.	8 Drams	make one	Ounce
℥.	12 Ounces	make one	Pound, *lb.*

Apothecaries, in making up their medicines, use this weight; but they buy and sell their drugs by avoirdupoise weight.

E. 1.

E. 1. In 18 pounds, how many ounces, drams, scruples and grains?

18 Pounds
12
216 Ounces
8
1728 Drams
3
5184 Scruples
20
103680 Grains

E. 2. Reduce 103680 grains to scruples, drams, ounces, and pounds?

2|0)10368|0
3) 5184
8) 1728
12) 216
18 Pounds

E. 3. In 2 *lb.* 4℥. 3ʒ. 2℈. 12 *grs.* how many grains?

lb.	℥.	ʒ.	℈.	*grs.*
2	4	3	2	12

12
28
8
227
3
683
20
Answ. 13672 Grains

E. 4. In 13672 grains, how many pounds?

2|0)1367|2
3) 683 — 12
8) 227 — 2
12) 28 — 3
Answer *lb.* 2 4 3 2 12

AVOIRDUPOISE WEIGHT.

Marked			
dr.	16 Drams	make one	Ounce
oz.	16 Ounces	make one	Pound
℔.	28 Pounds	make one	Quarter of Cwt.
qr.	4 Quarters, or 112 ℔.	make one	Hundred
Cwt.	20 Hundred	make one	Ton

By avoirdupoise weight is weighed all manner of grocery, and chandler's wares, and all metals, except silver and gold; also bread, butter, cheese, butcher's meat, &c.

The denominations in some of which are as follows, viz.

WOOL WEIGHT.

7 Pounds	make one	Clove	6 ½ Todds	make one	Wey
2 Cloves	make one	Stone	2 Weys	make one	Sack
2 Stones	make one	Todd	12 Sacks	make one	Last

HAY.

HAY.

56 Pounds of old hay 60 Pounds of new dit.	are 1 truss
36 Trusses - -	are 1 last

BREAD WEIGHT.

	lb.	oz.	dr.
Peck loaf - -	16	6	1
Half ditto - - -	8	11	$\frac{1}{2}$
Quarter ditto - -	4	5	8

8 Pounds	make one	Stone of butcher's meat
14 Pounds	make one	Stone of horseman's weight
19 $\frac{1}{2}$ Hundreds	make one	Fodder of lead

Note. There are some sorts of silk, which are weighed by a great pound of 24 ounces.

EXAMPLE 1. In 20 *Cwt.* how many quarters, pounds, ounces, and drams?

Cwt.
20
4
80 Quarters
28
640
160
2240 Pounds
16
35840 Ounces
16
573440 Drams

E. 2. Let it be required to reduce 573440 drams to hundreds?

$4\times4=16\begin{cases}4)573440 \text{ Drams} \\ 4)143360\end{cases}$
4) 35840
4) 8960
4) 2240
7) 560
4) 80
Answer 20 *Cwt.*

E. 3. In 10 *T.* 10 *Cwt.* 14 *lb.* 11 *oz.* 5 *drs.* how many drams?

T.	*C.*	*qrs.*	*lb.*	*oz.*	*drs.*
10	10	0	14	11	5

20
210 Hundreds
4
840 Quarters
28
6724
1681
23534 Pounds
16
141205
23535
376555 Ounces
16
2259335
376555
6024885 Drams

E. 4. In 6024885 drams, how many ounces, pounds, quarters, hundreds, and tons?

4)6024885 Drams
4)1506221 — 1 } = 5
4) 376555 — 1 }
4) 94138 — 3 } = 11
4) 23534 — 2 }
7) 5883 — 2 } = 14
4) 840 — 3 }
2|0) 21|0

	T.	*Cwt.*	*qr.*	*lb.*	*oz.*	*drs.*
Answer	10	10	0	14	11	5

CLOTH

CLOTH MEASURE.

			Marked
4 Nails	make one	Quarter of a yard	§ *na. qrs.*
3 Quarters		Ell Flemifh	§ *Ell Fl.*
4 Quarters		Yard	§ *yd.*
5 Quarters		Ell Englifh	§ *Ell Eng.*
6 Quarters		Ell French	§ *Ell Fr.*

Scotch and Irifh linens are bought and fold by the yard; but Dutch linens are bought by the ell Flemifh, and fold by the ell Englifh.

EXAMPLE 1. In a piece of cloth, containing 36 yards, how many quarters and nails?

36 Yards
4
144 Quarters
4
576 Nails

E. 2. In 576 nails, how many yards?

4)576
4)144
Anfwer 36 Yards

E. 3. How many nails are there in 84 *Ell Eng.* 4 *qrs.* 2 *na.*?

Ell. qrs. na.
84 4 2
5
424
4
Anfwer 1698 Nails

E. 4. In 1698 nails, how many ells Englifh?

4)1698
5) 424 — 2
Anfwer 84 4 2

E. 5. In 201 ells Flemifh, how many nails?

201
3
603
4
Anfwer 2412 Nails

E. 6. Reduce 2412 nails to ells Flemifh?

4)2412
3) 603
Anfwer 201 Ells Flemifh

E. 7. How many nails in 64 ells French?

64
6
384
4
Anfwer 1536 Nails

E. 8. In 1536 nails, how many ells French?

4)1536
6) 384
Anfwer 64

LONG

LONG MEASURE.

Marked			
b. c.	3 Barley corns	make one	Inch
in.	12 Inches - - -	make one	Foot
f.	3 Feet, or 36 inches	make one	Yard
yd.	2 Yards, or 6 feet - -	make one	Fathom
	$5\frac{1}{2}$ Yards, or 11 half yards	make one	Pole, rod, or perch
p.	40 Poles, or 220 yards	make one	Furlong
fur.	8 Furlongs, or 1760 yards	make one	Mile
m.	3 Miles - -	make one	League
lea.	$23\frac{1}{6}$ Leagues, or $69\frac{1}{2}$ miles	make one	Degree. *Deg.*

360 Degrees are the circumference of the globe.
5 Feet is a geometrical pace.
$16\frac{1}{2}$ Feet is a pole.

ALSO,

4 Inches	make one	Hand, or hand's breadth
3 Hand's breadth	make one	Foot
$1\frac{1}{2}$ Foot	make one	Cubit
2 Cubits	make one	Yard

By this meafure diftances of places, or any thing elfe, that has length only, are meafured.

EXAMPLE 1. How many yards, feet and inches, are there in 300 miles?

```
   300 Miles
  1760 The yards in a mile
 ------
528000 Yards
     3
 ------
1584000 Feet
     12
 -------
19008000 Inches
```

E. 2. In 19008000 inches, how many miles?

```
    12)19008000
    -----------
      3) 1584000
      ----------
1760) 52800(0)300 Miles
      528
      ---
       000
```

E. 3. Let it be required to reduce 12 leagues, 1 mile, 6 furlongs, 28 poles, and 4 yards, to barley-corns?

```
  Lea. m. fur. p. yds.
   12  1   6  28  4
    3
   --
   37
    8
  ---
  302
   40
 -----
 12108
    5½
 -----
 60544
  6054
 -----
 66598
    36 Inches in a yard
 ------
 399588
199794
-------
2397528
      3
-------
Anfwer 7192584 Barley corns
```

E LAND

LAND MEASURE.

Marked			
	$5\frac{1}{2}$ Yards	make one	Perch, rod, or pole
p.	40 Poles	make one	Rood
r.	4 Roods	make one	Acre
a.	30 Acres	make one	Yard of land
	100 Acres	make one	Hide of land

Land is commonly meaſured by a chain called Gunter's, whereof 10 in length and 1 in breadth, are an acre of land = 4840 yards.

7 Inches 92 parts	make one	Link
25 Links	make one	Pole
4 Poles, or 100 links, or 22 y.	make one	Chain
10 Chains	make one	Furlong

EXAMPLE 1. In 84 acres, how many rods and poles?

84 Acres
4
336 Roods
40
13440 Poles

E. 2. Let it be required to reduce 13440 poles to acres?

4|0)1344|0
4)336

Anſwer - 84 Acres

E. 3.

To meaſure a neighbouring plain,
I took up my croſs ſtaff and chain;
Having found th' content of the whole,
Eighty acres, two roods, and a pole,
What roods and perches were there
Be pleaſed to make to appear?

A.	r.	p.
80	2	1

4
322 Roods
40
12881 Perches

E. 4. In 12881 perches, how many acres?

4|0)1288|1
4)322

Anſwer - - 80 Acres 2 roods 1 perch.

WINE MEASURE.

Marked			
pts.	2 Pints	make one	Quart
qts.	4 Quarts	make one	Gallon
	10 Gallons	make one	Anchor of br. orrum
	18 Gallons	make one	Runlet
	$31\frac{1}{2}$ Gallons	make one	Barrel

Marked			
	42 Gallons	make one	Tierce
tier.	2 Tierce, or 84 gallons	make one	Puncheon, *punch.*
	63 Gallons	make one	Hogſhead
h.	2 H. or 126 ga.	make one	Pipe or butt
p.	2 P. or 152 ga.	make one	Tun

Note. A tun of wine is 18 hundred weight avoirdupoiſe.

A gallon is 231 ſolid inches.

By wine meaſure all ſpirits, mead, perry, vinegar, oil, cyder, and honey, &c. are meaſured.

E. 1.

E. 1. In 10 anchors of brandy, how many gallons and quarts?

10
10
———
100 Gallons
4
———
400 Quarts

E. 2. In 400 quarts, how many anchors?

4)400
———
10)100
———
Anſwer 10 Anchors

E. 3. In 8 hogſheads of wine, how many gallons and pints?

8
63
———
24
48
———
504 Gallons
8
———
4032 Pints

E. 4. In 4032 pints of wine, how many hogſheads?

8)4032
———
$9 \times 7 = 63 \begin{cases} 9)\,504 \\ 7)\,56 \end{cases}$
———
Anſwer 8 Hogſheads

WINCHESTER MEASURE.

Called ALE and BEER MEASURE.

Marked					
pts.	2 Pints			make one	Quart
qts.	4 Quarts, or 8 pints			make one	Gallon
gal.	8 Gallons ale, or 9 gallons beer			make one	Firkin
fir.	2 Firkins			make one	Kilderkin
kil.	2 Kilderkins, or 4 Firkins	or	32 gallons ale, 36 gallons beer	make 1 barrel	
bar.	1½ Barrel, or 3 Kilderkins	or	48 gallons ale, 54 gallons beer	make one hogſhead	
hhds.	2 Hogſheads, or 3 barrels, or 108 gallons			make one	Butt
	2 Butts, or 236 gallons			make one	Tun

Note. $8\frac{1}{2}$ Gallons is a firkin in all parts of England except London, where the ale firkin contains 8 gallons, and the beer firkin 9.

A gallon of ale or beer is 282 ſolid inches.

E. 1. In 14 barrels of ale, how many gallons and quarts?

14
32
———
28
42
———
448 Gallons
4
———
1792 Quarts

E. 2. In 1792 quarts of ale, how many barrels?

4)1792
———
$8 \times 4 = 32 \begin{cases} 8)\,448 \\ 4)\,56 \end{cases}$
———
Anſwer 14 Barrels

E. 3. In 34 barrels of beer, how many pints?

```
      34
      36
     ---
     204
    102
    ----
    1224
       8
    ----
Anſwer 9792 Pints
```

E. 4. In 9792 pints of beer, how many barrels?

```
               8)9792
                 ----
6 × 6 = 36 { 6)1224
           {     ----
           { 6) 204
                 ----
Anſwer           34 Barrels
```

DRY MEASURE.

Marked			make one
pts.	2 Pints - - - -	make one	Quart
qts.	4 Quarts, or 8 pints		Gallon
gal.	2 Gallons - - -		Peck
pks.	4 Pecks, or 8 gallons		Buſhel
bu.	4 Buſhels - - -		Comb
c.	2 Combs, or 8 buſhels		Quarter
qrs.	5 Quarters - - - -		Wey
	2 Weys, or 10 quarters		Laſt

ALSO, 4 Quarters, or 32 buſhels, make 1 chaldron of corn, and 2 buſhels make 1 ſtrike. A cart load of corn is 40 buſhels.

2 Quarts are one pottle, both in liquid and dry meaſure.

A gallon contains $268\frac{4}{5}$ ſolid inches.

In meaſuring ſea coal, 5 pecks are one buſhel, water meaſure.

3 Buſhels - -	make one	Sack
9 Buſhels - - -		Vatt
36 Buſhels, or 12 ſacks		Chaldron
21 Chaldrons - - -		Score

By dry meaſure, corn, ſalt, and all other dry goods are meaſured. The ſtandard buſhel is $18\frac{1}{2}$ inches wide, and 8 inches deep.

EXAMPLE 1. In 36 quarters of corn, how many buſhels, pecks, gallons, and quarts?

```
  36 Quarters
   8
 ---
 288 Buſhels
   4
----
1152 Pecks
   2
----
2304 Gallons
   4
----
9216 Quarts
```

E. 2. In 9216 quarts, how many quarters?

```
        4)9216
          ----
        2)2304
          ----
        4)1152
          ----
        8) 288
          ----
Anſwer     36 Quarters
```

E. 3.

E. 3. Reduce 36 chaldron, 26 bushels of coals, to pecks?

```
cha.  bu.
 36   26
 36
----
 222
110
----
1322
   5
----
6610 Pecks
```

E. 4. In 6610 pecks of coals, how many chaldron?

```
5)6610
  ----
6)1322
  ----
6) 220 — 2 }
   ---     } = 26
    36 — 4 }
   ---
```

Answer 36 chald. 26 bush.

TIME.

Marked				
‴ 60 Thirds	make one	Second	§	*Thirty days hath September,*
sec. 60 Seconds		Minute	§	*April, June, and November;*
m. 60 Minutes		Hour	§	*February hath twenty-eight alone,*
h. 24 Hours		Day	§	*And all the rest have thirty-one;*
d. 7 Days		Week	§	*Except leap-year, and then's the time,*
w. 4 Weeks, or 28*d.*		Month	§	*February's days are twenty-nine.*

52 Weeks, 1 day, 6 hours; or 13 months, 1 day, 6 hours; or 365 days, 6 hours, make one year Julian.

According to the best computation a Solar year contains 365 days, 5 hours, 48 minutes, 57 seconds, 39 thirds.

The year is also divided into 12 unequal calendar months, according to the above verse.

EXAMPLE 1. How many days, hours, minutes and seconds, are there in 4 weeks?

```
      4
      7
    ---
     28 Days
     24
    ---
    112
    56
    ---
    672 Hours
     60
  -----
  40320 Minutes
     60
 -------
2419200 Seconds
```

E. 2. In 2419200 seconds, how many weeks?

```
             6|0)241920|0
                 ------
             6|0) 4032|0
                 ------
4 × 6 = 24 { 4) 672
           {    ---
           { 6) 168
                ---
             7) 28
                --
Answer           4 Weeks
```

E. 3. How many ſeconds are there in a Julian year, or 365 days 6 hours?

365 6
24
1466
730
8766 Hours
60
525960
60

Anſwer 31557600 Seconds

E. 4. In 31557600 ſeconds, how many years?

6|0)3155760|0
6|0) 52596|0
4) 8766
6)2191 — 2 } = 6 *h.*
d. 365 — 1 }

Anſwer 365 *days* 6 *hours.*

SQUARE MEASURE.

144 Square inches	make one	Square foot	
9 —— feet		—— yard	
$30\frac{1}{4}$ —— yards		—— pole	
40 —— poles		—— rood	
4 —— roods		—— acre	
640 —— acres		—— mile.	

$172\frac{1}{4}$ Feet, is 1 rod of brick work. 100 Sq. feet is 1 ſquare of flooring.

EXAMPLE 1. In 32 ſquare yards, how many ſquare inches?

32
9
288
144
1152
1152
288

Anſwer 41472 Inches

E. 2. How many ſquare yards are there in 41472 ſquare inches?

12×12=144 { 12)41472
12) 3456
9)288

Anſwer 32 Squ. yds.

E. 3. Reduce 4 ſquares, 31 feet, 38 inches of flooring to inches?

Sq. *f.* *i.*
4 31 38
100
431
144
1732
1727
431

Anſwer 62102 Inches

E. 4. How many ſquares are there in 62102 ſquare inches?

12×12 =144 { 12)62102
12) 5175—2 } =38*i.*
1|00) 4|31—3 }

Anſwer *Sq.* 4 31*f.* 38*in.*

SOLID

SOLID MEASURE.

1728 Solid inches - - -	make one	Solid foot
27 Feet - - - - -	make one	—— yard
40 Feet of round timber, or 50 of hewn ditto	make one	Ton or load

A ſolid yard of earth is called a load.

A ſtatute cord of wood, is a pile 8 feet long, 4 feet broad, and 4 feet high; conſequently its content is 128 feet; for $8\times4\times4=128$. This ſort of cord is uſed in moſt of the northern counties of England; but in Suſſex, and moſt of the ſouthern counties, a pile of wood 3 feet high, 3 feet wide, and 14 feet long, is called a cord. The content of this is two feet leſs than the other; for $14\times3\times3=126$.

By this meaſure are meaſured all things, in which are conſidered length, breadth, and depth or thickneſs.

EXAMPLE 1. In 28 ſolid yards, how many ſolid inches?

```
      28
      27
     ---
     196
     56
     ---
     756
    1728
    ----
    6048
   1512
  5292
  756
 -------
 1306368 Inches
```

E. 2. In 1306368 ſolid inches, how many ſolid yards?

```
                    { 12)1306368
12×12×12=1728       { 12) 108864
                    { 12)   9072
3×9=27              {  3)    756
                    {  9)    252
Anſwer                 Yards  28
```

PRACTICAL ARITHMETIC.

PART I.

BOOK II.

VII. COMPOUND ADDITION,

TEACHETH to add ſundry ſums or numbers together, having divers denominations, as in money, weights, meaſures, &c.

RULE. 1. Place the numbers ſo, that thoſe of the ſame denomination may ſtand directly under each other, viz. pounds under pounds,

pounds, ſhillings under ſhillings, pence under pence, farthings under farthings, &c.

2. Begin to add at the loweſt denomination firſt, as in integers; then divide that ſum by as many of the ſame denomination, as makes one of the next greater, ſetting down the remainder under the row added, and carry the quotient to the next greater denomination, whoſe ſum you muſt alſo find. Proceed in this manner to the greateſt denomination, which add as integers.

EXAMPLES OF MONEY.

EXAMPLE 1.

£.	s.	d.
41	14	5$\frac{3}{4}$
86	18	11$\frac{1}{4}$
51	19	4$\frac{1}{2}$
67	17	7$\frac{3}{4}$
12	12	3$\frac{1}{4}$
261	2	8$\frac{1}{2}$

To add up this example ſay, 1 and 3 is 4, and 2 is 6, and 1 is 7, and 3 is 10; 10 farthings are two-pence half-penny, ſet down the half-penny thus $\frac{1}{2}$, and carry the two-pence to the pence row; ſaying 2 and 3 is 5, and 7 is 12, and 4 is 16, and 11 is 27, and 5 is 32; 32 pence is 2 ſhillings and 8 pence, ſet down 8 and carry 2. Then proceed to the ſhillings, and ſay, 2 and 2 is 4, and 7 is 11, and 9 is 20, and 8 is 28, and 4 is 32, (which is 2 above 3 tens) ſet down the 2, and go on to the next row) which is compoſed of a number of ones, being ſo many ten ſhillings, as you may ſee by their being placed, or ſet in the place of tens, and carry the three tens thereto; and ſay, 3 and 1 is 4, and 1 is 5, and 1 is 6, and 1 is 7, and 1 is 8; eight ten ſhillings make 4 pounds, which carry to the place of pounds, and ſay, 4 and 2 is 6, and 7 is 13, and 1 is 14, and 6 is 20, and 1 is 21; write down 1, and carry 2, ſaying 2 and 1 is 3, and 6 is 9, and 5 is 14, and 8 is 22, and 4 is 26, which write down; and the total will be £.261 2*s.* 8$\frac{1}{2}$*d.* In the ſame manner, you may proceed with any other examples of the like kind.

E. 2.			E. 3.			E. 4.			E. 5.		
£.	s.	d.	£.	s.	d.	£.	s.	d.	£.	s.	d.
31	16	8$\frac{1}{4}$	21	10	11	14	13	4$\frac{1}{4}$	361	12	8
20	10	4	36	11	2$\frac{1}{2}$	16	10	8	416	18	9
68	11	1$\frac{1}{2}$	41	10	6	62	12	4$\frac{3}{4}$	618	10	4$\frac{1}{2}$
30	13	6	23	13	4$\frac{1}{4}$	71	18	6$\frac{1}{2}$	481	12	2
46	10	2$\frac{1}{2}$	34	18	8	42	16	8	310	10	2$\frac{1}{2}$
84	19	4	42	12	2$\frac{1}{2}$	81	11	6$\frac{1}{4}$	681	10	8
42	12	8$\frac{1}{4}$	32	10	4	30	10	6	322	12	6$\frac{1}{2}$
325	13	10$\frac{1}{2}$	233	7	2$\frac{1}{4}$	320	13	7$\frac{3}{4}$	3193	7	4$\frac{1}{2}$

E. 6. A houfekeeper had difburfed for her lady, in marketing (per memorandum-book) for beef 10*s.* 5½*d.* Mutton 7*s.* 8*d.* Veal 6*s.* 3*d.* Chickens 3*s.* 4½*d.* and for eggs 3¼*d.* What was the fum difburfed?

		s.	d.
Beef - - -		10	5½
Mutton - -		7	8
Veal - - -		6	3
Chickens - -		3	4½
Eggs - - -		0	3¼
Sum	£. 1	8	0¼

E. 7. An affeffment for the highway levy, in the townfhip of B——, and parifh of M——, in the county of Warwick, rated at 6*d.* per pound, from Michaelmas 1779, to Michaelmas 1780.

	£.	s.	d.
Sir J. Fletcher, Bart.	31	12	6½
Thomas Carelefs -	42	10	0
John Ward - -	2	0	8
John Teverill -	6	12	0
Richard Moore	12	10	2½
James Day -	0	16	0
Thomas Farrol	21	8	2
William May -	16	10	0
Thomas Baker	8	1	4
Samuel Hodgetts	2	10	6½
William Swift -	1	0	2
Sarah Dunn -	16	4	3
John Garrifon	0	12	9½
John Howes -	0	10	4
Jofeph Rann -	21	1	0
Thomas Auftin -	12	4	2
	£. 196	4	2

E. 8. *A farmer's bill upon his labourer.*

Thomas Myatt,
1780. To Jof. Latchford, *Dr.*

			£.	s.	d.
May	30,	To a meafure of corn	-	5	6
June	8,	Ditto - -		6	4
	26,	Ditto - -		7	2
	30,	A bufhel of oats		2	3
July	1,	A load of coals	1	0	4
	13,	Beef - -		6	4
	16,	Bacon - -		3	1
Aug.	4,	Cheefe -	1	0	4
	8,	Butter - -		2	3½
		Total	£. 3	13	7½

WEIGHTS and MEASURES.

TROY WEIGHT.

EXAMPLES:

℔.	oz.	dwts.	gr.
8	2	4	1
6	1	2	4
8	9	1	6
4	2	1	8
3	6	4	1
2	1	8	6
3	4	2	1
2	1	4	2
38	3	7	5

℔.	oz.	dwts.	gr.
364	11	16	14
121	10	12	13
22	1	1	6
1	10	12	4
312	10	4	18
24	11	12	6
6	4	10	19
214	1	8	1
1069	1	18	9

APOTHECARIES' WEIGHT.

EXAMPLES:

℥.	ʒ.	℈.	grs.
3	4	1	17
1	2	0	13
6	1	2	14
3	4	1	16
1	1	2	0
3	7	1	18
9	1	0	10
8	6	2	12
4	2	1	4
42	1	0	4

℔.	℥.	ʒ.	℈.	gr.
14	11	4	2	11
16	10	3	0	4
8	1	4	1	16
1	3	1	0	10
18	1	2	1	16
24	10	7	0	17
38	1	2	2	14
61	8	4	1	16
31	4	2	0	12
215	5	1	0	16

AVOIRDUPOISE WEIGHT.

EXAMPLES:

lb.	oz.	drs.
14	11	14
2	10	11
26	4	2
24	9	8
61	10	12
1	11	16
2	4	1
1	2	11
34	12	16
179	14	11

Tons	C.	qrs.	lb.
121	17	2	12
312	14	1	14
421	10	3	16
121	16	1	15
124	10	2	16
181	8	0	4
311	16	1	13
426	11	2	3
801	18	1	14
2824	4	0	23

CLOTH MEASURE.

EXAMPLES:

Yds.	q.	na.
21	2	3
2	1	1
8	2	2
14	3	1
21	1	2
4	1	3
6	3	2
12	1	2
14	2	3
107	0	3

Eng. e.	q.	na.
12	4	2
31	3	1
42	1	3
12	2	1
31	4	0
8	1	2
9	3	3
14	3	1
16	2	2
180	1	3

F. e.	q.	n.
16	2	3
41	1	2
64	2	1
31	1	2
63	2	1
5	1	2
8	2	1
16	1	2
81	1	3
327	5	1

LONG MEASURE.

EXAMPLES:

Lea.	m.	fur.	p.
21	1	7	36
32	2	1	21
61	1	3	26
8	2	6	21
4	1	2	6
61	2	4	3
3	1	2	12
21	0	6	10
6	1	4	9
222	0	6	24

Yds.	f.	in.	b.c.
31	2	11	2
42	1	8	1
80	2	4	0
4	1	3	2
31	2	10	3
6	1	8	1
2	2	4	0
5	1	3	1
16	2	3	2
223	0	10	0

LAND MEASURE.

EXAMPLE:

A.	R.	P.
436	3	26
21	1	34
6	2	27
214	1	2
45	2	1
301	0	14
124	2	12
32	1	28
3	2	16
1184	2	0

WINE MEASURE.

EXAMPLES:

Tu.	p.	hhd.	ga.	qts.
12	1	1	14	2
4	1	1	27	3
10	1	0	61	1
6	1	1	42	2
2	0	0	26	3
13	1	1	4	2
6	0	0	36	3
12	1	0	2	1
3	0	1	15	2
73	0	0	42	3

Punch	gal.	qt.	pt.
14	14	2	1
7	32	3	1
24	51	2	1
14	14	1	1
49	36	3	1
37	17	1	1
8	62	3	1
21	2	1	1
24	6	0	0
200	71	0	0

WINCHESTER MEASURE.

EXAMPLES:

A. hhds.	gal.	qts.
14	12	2
6	41	3
17	27	1
8	34	2
47	40	3
4	27	1
18	11	0
6	12	2
8	10	0
132	25	2

B. hhds.	gal.	pts.
24	51	7
14	17	4
6	8	6
14	10	0
9	47	3
34	36	5
17	11	2
4	29	7
16	12	4
142	9	6

DRY MEASURE.

EXAMPLES:

qrs.	*bu.*	*p.*	*gal.*
14	7	2	0
27	4	3	1
31	4	2	0
62	1	1	1
14	1	2	0
21	0	1	1
31	1	2	0
12	6	1	1
10	1	2	0
225	5	2	0

cha.	*bu.*	*p.*
12	27	2
20	1	1
31	12	0
11	10	1
12	16	0
21	12	1
16	10	0
11	2	1
14	12	0
150	31	2

TIME.

EXAMPLES:

mo.	*w.*	*d.*	*h.*
11	2	4	21
24	3	6	14
12	1	0	23
31	2	5	0
14	1	1	11
6	3	6	17
8	2	1	12
4	1	0	2
3	2	1	14
117	1	0	10

d.	*h.*	*m.*	*sec.*
14	21	14	41
2	16	11	16
1	2	1	4
13	12	16	18
31	11	11	11
42	11	14	11
31	1	2	1
6	2	4	2
1	0	1	16
144	4	16	0

Note. You muſt write down the numbers of the ſame denomination under each other, in all the preceding examples; and add them up as in addition of money; only take care to carry from one denomination to another, according to the table pertaining to each particular weight or meaſure.

QUESTIONS.

QUESTION 1.

Frank Guzzle, Belch, and Soaking Dan,
Muſt have a bottle with Sir John,*
And, topers like, with Trot† prevail,
To fill a jug of nappy ale.
A jug! a mighty jug indeed!
A yard about, was fill'd with ſpeed;
Ten quarts it held, as neighbours tell,
Which pleas'd the landlord mighty well.
Three times b'ing fill'd, the topers they
Could ſcarce conduct themſelves away,
But paid the ſcore, which pleaſed Trot
To think what cuſtomers he'd got,
'Twas fifty-pence a-piece the ſhot.
What was the whole young Tyro tell,
Which pleas'd the landlord Trot ſo well?

By the pence table, 50 pence = 4*s.* 2*d.* which ſet down three times, thus:

	s.	*d.*
Frank Guzzle	4	2
Belch	4	2
Soaking Dan	4	2
The ſum ſpent	12*s.*	6*d.*

Q. 2. How much is A (born 20 years ago) older than B, who will come into the world fourteen years hence?

To	20
Add	14
Anſwer	34 Years

Q. 3. A gentleman by will left the following legacies to be divided amongſt his children, viz. four ſons and three daughters: to Simon 300*l.* to Ralph 160*l.* to John 1*s.* to James 120*l.* to Suſan 93*l.* to Ruth 1*s.* and to Margery 430*l.* Query, the fortune the gentleman left?

* Sir John Barleycorn. † The Landlord.

To anſwer this queſtion, write down each one's fortune, and add them together, thus :

		£.	s.	d.
Simon's	Share,	300	0	0
Ralph's		160	0	0
John's		0	1	0
James's		120	0	0
Suſan's		93	0	0
Ruth's		0	1	0
Margery's		430	0	0
	Fortune left	1103	2	0

Q. 4. A perſon was 16 years of age 29 years ſince ; and it is ſaid he will be drowned 21 years hence : pray in what year of his age will this happen ?

His age 29 years ſince =	16
His preſent age — 16 =	29
Years to come - - -	21
Anſwer	66

Q. 5. A gentleman dying, left his executor a ſum not amounting to 2000*l.* to be ſo divided amongſt his relations ; that his father and mother, his ſon and his grandſon, his brother and his daughter, ſhould each receive a ſum not leſs than 666*l.* 13*s.* 3*d.* Query, the ſcheme of kindred, and exact ſum left ?

SOLUTION. Suppoſe two widows, A and B, no kin to each other, to be left each with a ſon, and that A's ſon marries B, and B's ſon marries A ; and that A's ſon has a ſon by B, and A's ſon is the gentleman that leaves the money. This is the ſcheme of kindred, and to find the ſum left, proceed thus :

	£.	s.	d.
To his father, who in this caſe is the ſame as his ſon,	666	13	3
To his mother, who in the ſame manner is his daughter,	666	13	3
To his grandſon likewiſe, who is the ſame as his brother,	666	13	3
Sum left - -	1999	19	9

Q. 6. A ſheep-fold was robbed three nights ſucceſſively ; the firſt night half the ſheep were ſtolen, and half a ſheep more ; the ſecond night half the remainder were loſt, and half a ſheep more ; the laſt night they took half what were left, and half a ſheep more, by which time they were reduced to twenty ; how many were there at firſt ?

The number left =	20	Sheep.
	21	Stolen the 3d Night
	42	Stolen the 2d Night
	84	Stolen the 1ſt Night
Anſwer - -	167	Sheep at firſt.

Q. 7. How many years is it ſince the creation of Adam to the univerſal deluge in the days of Noah, called Noah's flood ? which may be eaſily found, by the 5th chapter of Geneſis, and the 6th verſe of the 7th chapter.

When

When Seth was born	Adam was	-	130	years old.
Enos —	Seth	—	105	
Cainan —	Enos	—	90	
Mahalaleel —	Cainan	—	70	
Jared —	Mahalaleel	—	65	
Enoch —	Jared	—	162	
Methuſelah	Ecoch	—	65	
Lamech —	Methuſelah	—	187	
Noah —	Lamech	—	182	
And when the flood happened	Noah was		600	
		Anſwer	1656	Years

VIII. COMPOUND SUBTRACTION,

TEACHETH to find the difference between any two ſums of divers denominations.

RULE. Subtract as in integers, only when the lower number in any denomination happens to be the greater, borrow one, that is, add as many to the upper number as makes one of the next ſuperior denomination, and then ſubtract the lower number, and ſet down the remainder; then carry 1, and add it to the lower number of the next denomination, and then ſubtract as before.

EXAMPLES of MONEY.

EXAMPLE 1.

	£.	s.	d.
From - -	19	14	$5\frac{1}{2}$
Take - -	12	16	$4\frac{1}{4}$
Remains -	6	18	$1\frac{1}{4}$
Proof - -	19	14	$5\frac{1}{2}$

To work this example, begin at the leaſt denomination, ſaying 1 from 2, and there remains 1 farthing, which place under the line thus $\frac{1}{4}$; then ſubtract 4 from 5, and there remains 1 penny, which ſet down under its own denomination, and proceed to the ſhillings; 16 from 14 I cannot, but 20 that I borrow to 14 is 34, 16 from 34, and there remains 18; then, becauſe you borrowed 1 pound, or 20 ſhillings, ſay 1 that I borrowed and 2 is 3, 3 from 9 and there remains 6; 1 from 1 and there remains nothing; and the remainder is 6*l.* 18*s.* $1\frac{1}{4}$*d*, which add to the line above it, and the ſum will be equal to the top line, which proves the work to be right.

OBSERVATION. Subtraction of all ſorts and denominations, is performed after the very ſame manner as the preceding example, only you muſt borrow, and add or repay, according to each denomination; therefore to give any further explanations relating thereto would be quite unneceſſary.

E. 2.	£.	s.	d.	E. 3. £.	s.	d.
From -	32	16	$8\frac{1}{4}$	42	10	$6\frac{1}{4}$
Take - -	12	14	$10\frac{3}{4}$	21	19	8
Remains -	20	1	$9\frac{1}{2}$	20	10	$10\frac{1}{4}$
Proof - -	32	16	$8\frac{1}{4}$	42	10	$6\frac{1}{4}$

E. 4.	£.	s.	d.	E. 5.	£.	s.	d.
Borrowed - -	524	14	6	Lent - - -	4861	14	8
Paid at sundry times -	12	5	1	Received at sundry times - -	121	1	2
	11	2	2		21	2	4
	36	1	0		312	6	6
	46	2	6		2	1	2
	31	1	8		6	3	1
	12	4	8		82	4	4
	2	1	6		311	1	1
	3	4	4		8	2	1
Paid in all -	154	2	11	Received in all -	864	1	9
Remains unpaid	370	11	7	Remains unpaid	3997	12	11

E. 6. Suppofe my half-year's rent is 20 guineas, and that I have laid out for the land-tax and other levies 8*l.* 18*s.* $8\frac{1}{2}$*d.* and for feveral repairs 3*l.* 4*s.* 2*d.* what remains due to the landlord?

				£.	s.	d.
Half-year's rent - - - - - - - -				21	0	0
Land-tax, &c. - -	£. 8	18	$8\frac{1}{2}$	12	2	$10\frac{1}{2}$
Repairs - - - - - -	3	4	2			
Balance due to the landlord - - - -				8	17	$1\frac{1}{2}$

E. 7. A carpenter's bill upon a farmer is 86*l.* 18*s.* 8*d.* out of which he has received in cafh 20*l.* in corn 6*l.* 12*s.* 2*d.* in coals 2*l.* 1*s.* 6*d.* and in cheefe and bacon 12*s.* $6\frac{1}{2}$*d.* what remains due to the carpenter?

					£.	s.	d.
The carpenter's bill - - -					86	18	8
Paid in	Cafh - -	£. 20	0	0	29	6	$2\frac{1}{2}$
	Corn - -	6	12	2			
	Coals - -	2	1	6			
	Cheefe, &c. -	0	12	$6\frac{1}{2}$			
Balance due to the carpenter - -					57	12	$5\frac{1}{2}$

WEIGHTS

WEIGHTS and MEASURES.

TROY WEIGHT.

	℔.	oz.	dwts.	grs.
From	81	10	15	18
Take	14	8	12	19
Remains	67	2	2	23

APOTHECARIES' WEIGHT.

	℔.	℥.	ʒ.	℈	grs.
From -	38	10	1	2	4
Take -	2	8	2	1	12
Remains	36	1	7	0	12

AVOIRDUPOISE WEIGHT.

	Tons	C.	qrs.	lb.	lb.	oz.	drs.
From	36	18	2	26	26	0	8
Take	21	19	3	6	12	1	12
Rem.	14	18	3	20	13	14	12

CLOTH MEASURE.

	Yds.	qr	na.	EllEng.	qrs.	na.
From	326	2	3	38	4	2
Take	218	3	1	14	3	3
Rem.	107	3	2	24	0	3

LONG MEASURE.

	Lea.	m.	fur.	p.	Yds.	f.	in.	b.c.
From	281	1	7	26	36	2	8	2
Take	82	2	5	38	18	1	4	1
Rem.	198	2	1	28	18	1	4	1

LAND MEASURE.

	A.	R.	P.	A.	R.	P.
From	864	2	26	38	0	31
Take	318	1	18	21	3	24
Rem.	546	1	6	16	1	7

WINE MEASURE. A nobleman hath two cellars, the larger contains of feveral kinds of liquors 3 tons and 2 hogfheads, and the other 1 ton 3 hogfheads, 32 gallons, and four pints; how much liquor is there in the one more than the other?

	Tons.	hhd.	gal.	pts.
From - -	3	2	0	0
Take - - -	1	3	32	4
Remains - -	1	2	30	4
Proof - -	3	2	0	0

WINCHESTER MEASURE. A brewer delivers to his cuftomers in one day 24 hogfheads and 16 gallons, in another day 18 hogfheads and 48 gallons, what is the difference?

	hhds.	gal.
From -	24	16
Take -	18	48
Remains	5	19
Proof	24	16

DRY MEASURE.

	Qu.	bu.	p.	Cha.	bu.	p.
From	18	14	2	22	26	0
Take	6	10	3	3	34	3
Rem.	12	3	3	18	27	2

T I M E.

	Mo.	w.	d.	h.	D.	h.	m.	fec.
From -	18	2	6	21	8	14	46	31
Take -	10	3	2	23	4	21	18	52
Remains	7	3	3	22	3	17	27	39

QUESTIONS.

QUESTIONS.

QUESTION I. A horſe in his furniture is worth 38*l.* 12*s.* out of it 16*l.* 15*s.* how much does the price of the furniture exceed that of the horſe?

	£.	s.
In furniture - -	38	12
Out of it - - -	16	15
Anſwer -	21	17

Q. 2. When the air preſſes with its full weight, in very fair weather, it may be demonſtrated, that there preſſes upon the human body about 33905 pounds of that fluid matter; and in foul weather, when the air is moſt light, but 30624 pounds; what difference of weight lies on ſuch a body in the two greateſt alterations of the weather?

	33905	Pounds
	30624	
Anſwer -	3281	Pounds

Q. 3. A boy was bound, by indentures, to ſerve his maſter ſeven years; and when he had accompliſhed 6 years, 6 months, 6 weeks, 6 days, 6 hours, 6 minutes, and 6 ſeconds, pray how long had he to ſerve?

	Yrs.	*mo.*	*w.*	*d.*	*h.*	*m.*	*ſec.*
From - -	7	0	0	0	0	0	0
Take - -	6	6	6	6	6	6	6
Anſwer -	5	1	0	17	53	54	
Proof - - - -	7	0	0	0	0	0	0

Q. 4. A ſnail in getting up a may-pole only 20 feet high, was obſerved to climb 8 feet every day; but every night it came down again four feet; in what time, by this method, did it reach the top of the pole?

Goes up the firſt day	8	Feet
Comes down at night	4	
	4	1ſt day
Goes up the 2d day	8	
	12	
Comes down at night	4	
	8	2d day
Goes up the third day	8	
	16	
Comes down at night	4	
	12	3d day
Goes up the 4th day	8	
	20	feet

Anſwer, the fourth day at night.

Q. 5. A was born when B was 21 years of age; how old will A be, when B is 47, and what will be the age of B, when A is 60?

From	47	
Take	21	
Rem.	26	the age of A
To	60	
Add	21	
Sum	81	B's age

Q. 6.

Q. 6. *A lady left her daughter fair*
Twelve thousand pounds in gold,
To be distributed with care,
As underneath is told.
First to a niece there must be paid
Just fourteen hundred pound,
And half that sum to parson Wade,
To make his glass go round;
And to her maid miss Nancy Hare,
Three hundred pounds in cash,
Who swells with pride, and such an air!
She apes my lady Flash.
The stew'rd and butler each must have,
Just twice two hundred more,
And to a tenant, farmer Brave,
In shining pounds six score.
The greasy cook, each other maid,
B'ing three in number, they
Had twenty guineas each one paid,
To make them fine and gay;
The coachman Ralph and footman Dan,
Ten guineas and a crown,*
Which made them toss about the can,
In ev'ry market town.
When all these legacies were paid,
What did remain behind
For miss, that blooming peerless maid,
Whose virtues made her kind?

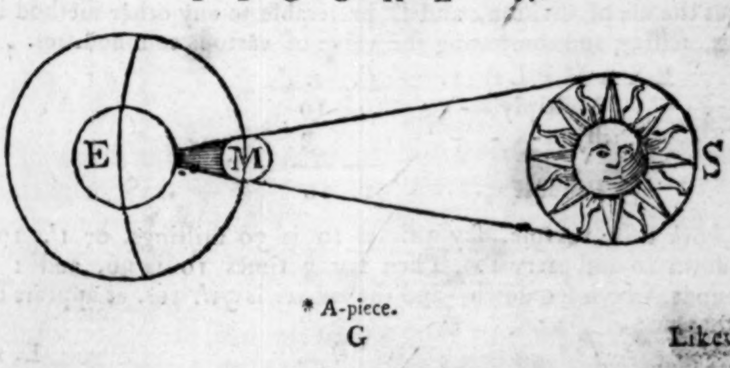
				£.	s.	d.
Sum left - - - - - - - -				12000	0	0
Niece - - - - -	1400	0	0			
Parson - - - - -	700	0	0			
Miss Nancy Hare - - -	300	0	0			
Steward and butler - - - -	400	0	0			
Farmer Brave - - -	120	0	0			
Cook, and the two other maids -	63	0	0			
Coachman and footman - - -	21	10	0			
				3004	10	0
The daughter's share - £.				8995	10	0

Q. 7. If the mean distance between the earth and sun be 81 millions of miles, and between the earth and moon 240 thousands; how far are those two luminaries asunder in an eclipse of the sun, when the moon is lineally between the earth and sun? And in another of the moon when the earth is in a line between her and him?

Suppose E the earth, M the moon, and S the sun; then the eclipse of the sun will be represented by fig. 1, and that of the moon by fig. 2.

Therefore 81000000—240000=80760000=S M fig. 1, or the distance these two luminaries are asunder, in an eclipse of the sun.

FIGURE 1.

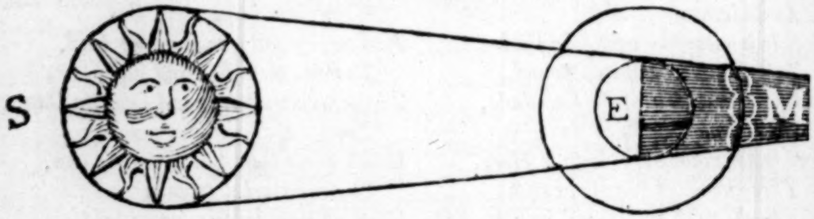


* A-piece.

Likewise

Likewise 81000000+240000=81240000=S M fig. 2, or the distance these two luminaries are asunder in an eclipse of the moon.

FIGURE 2.



Q. 8. B, born 161 years ago, died when C was 47 years of age, who it seems came into the world 180 years since, and out-lived B 43 years: the sum of their ages is required?

First 180
47
133 Years since B died

Then 161
133
28 Years, B's age

Add $\begin{cases}47\\43\end{cases}$
90 C's age. Then 90 + 28 = 118 the answer.

IX. Compound MULTIPLICATION,

TEACHETH to multiply (by one common multiplier) any sum or number consisting of divers denominations.

CASE 1. When the given quantity doth not exceed 12.

RULE. 1. Write the multiplier under the lowest denomination of the multiplicand.

2. Begin at the lowest denomination, and multiply it by the given number, and see how many of the next denomination is contained in the product; set down the odds, and carry so many to the next. Then multiply the next denomination, adding what you carried, and set down the odds; proceed thus till all be multiplied.

N. B. Multiplication is a short way of working the rule of three, without the use of division, and is preferable to any other method in buying, selling and computing the value of various commodities.

EXAMPLE 1.	£.	s.
Multiply - -	10	10
By - - - -		3
Product - -	31	10

To work this example, say 3 times 10 is 30 shillings, or 1*l.* 10*s.* write down 10 and carry 1. Then say 3 times 10 is 30, and 1 is 31 pounds, which set down, and the answer is 31*l.* 10*s.* as appears by the work.

E. 2.

E. 2. What will 4 pounds of sugar come to, at $5\frac{1}{4}d.$ per pound?

	d.
	$5\frac{1}{4}$
	4
Answer	1 9

E. 3. What come 5 hundred of cheese to, at 1*l.* 12*s.* $6\frac{1}{2}d.$ per Cwt?

	£.	s.	d.
	1	12	$6\frac{1}{2}$
			5
Answer	8	2	$8\frac{1}{2}$

Note. Cheese-factors, and many other dealers, who buy goods wholesale, are allowed 120 pounds to 1 Cwt. but sell them out at 112 pounds per Cwt.

E. 4. What will 6 ells of holland come to, at 6*s.* 10*d.* per ell?

		s.	d.
		6	10
			6
Answer	£. 2	1	0

E. 5. What will 8 gallons of brandy come to, at 2*l.* 1*s.* 3*d.* per gallon?

	£.	s.	d.
	2	1	3
			8
Answer	16	10	0

E. 5. What come 10 anchors of rum to, at 3*l.* 4*s.* 2*d.* per anchor?

	3	4	2
			10
Answer	£. 32	1	8

E. 7. What will 11 barrels of small beer come to, at 10*s.* 6*d.* per barrel?

		10	6
			11
Answer	£. 5	15	6

E. 8. What will 12 dozen of candles come to, at 6*s.* 8*d.* per dozen?

		6	8
			12
Answer - -	£. 4	0	0

CASE 2. When the given quantity exceeds 12, and is such a number that any two figures (in the multiplication table) being multiplied together, will produce it;

RULE. Multiply the given price by one of those numbers, and that product by the other, and if you make use of any more numbers, proceed in like manner, and the final product will be the answer.

E. 9. What will 15 bushels of wheat come to, at 6*s.* $9\frac{1}{2}d.$ per bushel?

		6	$9\frac{1}{2}$
			$5 \times 3 = 15$
	1	13	$11\frac{1}{2}$
			3
Answer -	£. 5	1	$10\frac{1}{2}$

To work the preceding example, say 5 times 2 is 10 farthings, or $2\frac{1}{2}d.$ set down $\frac{1}{2}$, and carry 2; then say 5 times 9 is 45, and 2 is 47 pence, = 3*s.* 11*d.* set down 11, and carry 3; then 5 times 6 is 30 shillings, and 3 is 33, = 1*l.* 13*s.* The first product being finished, multiply that by the other number, saying 3 times 2 is 6 farthings, or $1\frac{1}{2}d.$ set down $\frac{1}{2}$ and carry 1, and say 3 times 11 is 33, and 1 is 34 pence, = 2*s.* 10*d.* set down 10 and carry 2; then 3 times 3 is 9, and 2 is 11, set down 1 and carry 1, saying 3 times 1 is 3, and 1 is 4,

4 ten shillings, or 2 pounds; then 3 times 1 is 3, and 2 is 5*l.*—and the answer is 5*l.* 1*s.* 10½*d.* as appears by the work.

E. 10. What will 18℔. of butter come to, at 4½*d.* per pound?

```
   d.
   4½
    3×6=18
 ________
 1  1½
     6
 ________
Answer 6 9
```

E. 11. What will 30*lb.* of cheese come to, at 3¼*d.* per pound?

```
      d.
      3¼
      10×3=30
 ________
   2  8½
       3
 ________
Answer 8 1½
```

E. 12. What will 45 ℔. of bacon come to, at 3¼*d.* per pound?

```
      5¾
       9×5=45
 ________
   4  3¾
       5
 ________
Answer £. 1 1 6¾
```

E. 13. How much is the sterling value of 50 moidores, at 27*s.* each?

```
   1  7
      10×5=50
 ________
  13 10
      5
 ________
Answer £. 67 10
```

E. 14. What do 56 hogs come to, at 1*l.* 5*s.* 4*d.* per hog?

```
   1  5  4
         8×7=56
 ___________
  10  2  8
         7
 ___________
Answer £. 70 18 8
```

E. 15. What come 64 firkins of butter to, at 1*l.* 8*s.* per firkin?

```
   1  8
      8×8=64
 ________
  11  4
      8
 ________
Answer £. 89 12
```

E. 16. What come 72 reams of paper to, at 13*s.* 8*d.* per ream?

```
  13  8
       9×8=72
 ________
   6  3  0
         8
 ________
Answer £. 49 4 0
```

E. 17. What is the price of 80 yards of Irish cloth, at 10½*d.* per yard?

```
      10½
      10×8=80
 ________
   8  9
      8
 ________
Answer £. 3 10 0
```

E. 18. What will 88 gallons of ale come to, at 1*s.* 4*d.* per gal.?

```
   1  4
     11×8=88
 ________
  14  8
      8
 ________
Answer £. 5 17 4
```

E. 19. What come 96 bushels of barley to, at 3*s.* 2½*d.* per bushel?

```
   3  2½
      12×8=96
 ___________
 1 18  6
       8
 ___________
Answer £. 15 8 0
```

E. 20.

E. 20. What will 144 ℔. of tea come to, at 4*s*. 6*d*. per pound?

4 6
12×12=144

2 14 0
12

Anfwer £. 32 8 0

E. 21. How much will 132 dozen feet of fawing come to, at $4\frac{1}{2}$*d*. per dozen? $4\frac{1}{2}$
12×11=132

4 6
11

Anfwer £. 2 9 6

CASE 3. When the given quantity cannot be produced by the multiplication of two fmall numbers.

RULE. Find the neareft number to it lefs, by which multiply as before, then for what is wanting multiply the price by that number, and add it to the laft product, and the total will be the anfwer required.

E. 22. What come 17 Cwt. of raifins to, at 1*l*. 4*s*. 2*d*. per Cwt?

1 4 2
4×4+1=17

4 16 8
4

19 6 8=16
1 4 2=1

Anf. £. 20 10 10

E. 23. What come 29℔. of fine hyfon tea to, at 19*s*. 6*d*. per ℔?

19 6
4×7+1=29

3 18 0
7

27 6 0=28
19 6=1

Anfwer £. 28 5 6

E. 24. What will 37 grofs of buttons come to, at 1*l*. 10*s*. $6\frac{1}{2}$*d*. per grofs?

1 10 $6\frac{1}{2}$
6×6+1=37

9 3 3
6

54 19 6=36
1 10 $6\frac{1}{2}$=1

Anfwer £. 56 10 $0\frac{1}{2}$

E. 25. What will 42 yards of fine holland come to, at 10*s*. $2\frac{1}{2}$*d*. per yard?

10 $2\frac{1}{2}$
5×8+2=42

2 11 $0\frac{1}{2}$
8

20 8 4=40
1 0 5= 2

Anfwer £. 21 8 9

E. 26. Bought 65 fheep, at 1*l*. 5*s*. 4*d*. per fheep, what do they come to?

1 5 4
8×8+1=65

10 2 8
8

81 1 4=64
1 5 4= 1

Anfwer £. 82 6 8

E. 27. What come 75 dozen of foap to, at 6*s*. $3\frac{1}{2}$*d*. per dozen?

s. *d*.
6 $3\frac{1}{2}$
9×8+3=75

2 16 $7\frac{1}{2}$
8

22 13 0 =72
0 18 $10\frac{1}{2}$= 3

Anfw. £. 23 11 $10\frac{1}{2}$

E. 28. What will 86 dozen of men's common hose come to, at 2*l*. 4*s*. 2*d*. per dozen?

```
      2   4   2
          8×11—2=86
    ---------------
     17  13   4
             11
    ---------------
    194   6   8=88
      4   8   4— 2
    ---------------
Ans. £.189  18   4
```

E. 29. What will 104 copies of Taylor's complete system of arithmetic come to, at 5*s*. each?

```
           5
       10×10+4=104
    ------------
         2 10
           10
    ------------
        25  0 =100
         1  0 =  4
    ------------
Answ. £. 26  0
```

In the 28th example I have taken the two nearest numbers above the given quantity, whose product is 88; by which I found the value of that number of dozens of hose to be 194*l*. 6*s*. 8*d*. from which I subtracted twice the price of one dozen to find the price of 86, the answer to the question.

CASE 4. When the given quantity consists of $\frac{1}{4}$, $\frac{1}{2}$, or $\frac{3}{4}$.

RULE. Divide the upper line (the price of one) by 4 for $\frac{1}{4}$, by 2 for $\frac{1}{2}$; and for $\frac{3}{4}$, by 2 first for $\frac{2}{4}$, then divide that quotient by 2 for $\frac{1}{4}$; add them to the product, and the sum will be the answer required.

E. 30. What will 16$\frac{1}{2}$ lb. of raisins come to, at 6$\frac{1}{2}$*d*. per pound?

```
    ½)  6½
        4×4=16
    ----------
      2  2
         4
    ----------
      8  8 =16
         3¼=½
    ----------
Answer  8 11¼
```

E. 31. What come 36$\frac{1}{4}$ tons of hay to, at 3*l*. 4*s*. 6*d*. per ton?

```
    4)  3   4   6
                6×6=36
    --------------
       19   7   0
                6
    --------------
      116   2   0 =36
           16   1½=¼
    --------------
Answer £. 116  18   1½
```

E. 32. Bought 8$\frac{3}{4}$ butts of strong beer, at 8*l*. 4*s*. 8*d*. per butt?

```
        2)  8   4   8
                    8
        --------------
           65  17   4 = 8
        2)  4   2   4 = ½
            2   1   2 = ¼
        --------------
Answer   £. 72   0  10
```

When your given quantity happens to be very large, so as to consist of hundreds, thousands, &c. it may be wrought by the continual product of three or more numbers; and if your given quantity is thousands, multiply the price of 100 by 10 for 1000, and the product by the number of thousands, and for the lower quantities proceed as before. The following examples will make this sufficiently clear to be understood.

E. 33.

E. 33. What will 120 ounces of fine ſilver come to, at 5*s.* 3*d.* per ounce?

	5	3	
		10×4×3=120	
2	12	6	
		4	
10	10	0	
		3	
Anſwer £31	10	0	

E. 35. What comes 1 Cwt. of hops to, at 1*s.* 2½*d.* per pound?

	1	2½
		8×7×2=112
	9	8
		7
3	7	8
		2
Anſwer £. 6	15	4

E. 34. What will 8462 ℔. of iron come to, at 2¾*d.* per pound?

		2¾	
		10	
	2	3½=the price of 10 ℔.	
		10	
1	2	11	=100
		10	
11	9	2	=1000
		8	
91	13	4	=8000
4	11	8	= 400
	13	9	= 60
		5½	= 2
£. 96	19	2½	=8462

Anſwer 96*l.* 19*s.* 2½*d.*

WEIGHTS and MEASURES.

EXAMPLE 1. A ſilverſmith has 4 bars of ſilver, each 4 ℔. 6 *oz.* 8 *dwts.* 3 *grs.* what is the weight of the whole?

	℔.	*oz.*	*dwts.*	*grs.*
	4	6	8	3
				4
Anſwer -	18	1	12	12

Note. Weights, meaſures, &c. are multiplied after the ſame manner as money, only you muſt remember to carry according to each denomination that reſpectively pertain thereto.

E. 2. An apothecary has 6 mixtures, each 3 pounds, 1 ounce, 3 drams, 1 ſcruple, and 10 grains, what is the weight of the whole?

	℔.	℥.	ʒ.	℈.	*grs.*
	3	1	3	1	10
					6
Anſwer	18	8	5	0	0

E. 3. What is the weight of 10 caſks of raiſins, when each caſk weighs 5 *C.* 2 *qrs.* 18 ℔?

	C.	*qrs.*	℔.
	5	2	18
			10
Anſwer	54	2	12

E. 4.

E. 4. If a perfon hath 12 bales of filk, each 2 pounds, 10 ounces, and 4 drams, what is the whole weight?

	℔.	oz.	drs.
	2	10	4
			12
Anfwer	31	11	0

E. 5. A fhopkeeper bought 40 pieces of Irifh cloth, each piece containing 42 yards, 2 quarters, 2 nails, what quantity did he buy?

	Yds.	qrs.	na.
	42	2	2
			4×10=40
	170	2	0
			10
Anfwer	1705	0	0

E. 6. Multiply 30 miles, 2 furlongs, and 18 poles, by 2?

	Miles	fur.	p.
	30	2	18
			2
Anfwer	60	4	36

E. 7. Multiply 300 yards, 1 foot, and 4 inches, by 8?

	Yds.	ft.	in.
	300	1	4
			8
Anfwer	2403	1	8

E. 8. Multiply 68 acres, 2 roods, and 4 perches, by 9?

	A.	r.	p.
	68	2	4
			9
Anfwer	616	2	36

E. 9. Multiply 4 b. hogfheads, 3 gallons, and 6 pints, by 4?

	B.hhd.	gal.	pts.
	4	3	6
			4
Anfwer	16	15	0

E. 10. Multiply 82 lafts, 6 qrs. 4 bufhels, and 1 peck, by 7?

	Lafts	bu.	qrs.	p.
	82	6	4	1
				7
Anfwer	578	5	5	3

E. 11. Multiply 9 months, 2 weeks, 4 days, 12 hours, and 4 minutes, by 12?

	Mo.	w.	d.	h.	m.
	9	2	4	12	4
					12
Anf.	115	3	5	0	48

QUESTIONS.

Q. 1. If I fpend $1\frac{1}{2}d.$ per day, how much is that per year, allowing 365 days to the year?

		$1\frac{1}{2}$	10×10×3+60+5=365
	1	3	
		10	
	12	6	
		3	
1	17	6	=300
	7	6	= 60
		$7\frac{1}{2}$	= 5
2	5	$7\frac{1}{2}$	=365 Anfwer.

Q. 2. The *Silk-mill* at *Derby* contains 26586 wheels, and 97746 movements, which wind off or throw 73726 yards of filk every time the great water wheel, which gives motion to all the reft, goes about, which is three times a minute; the queftion is, how many yards of filk

may

may be thrown by this machine in a day, reckoning ten hours to a day's work? And how many in the compass of a year, deducting for Sundays and holidays 63 days, provided no part of it stands still?

	73726	
	3	
	221178	Yards in a minute
	60	Minutes in an hour
	13270680	
	10	Hours to a day
	132706800	Yards in a day
	302	
	265413600	
	3981204000	
Answer	40077453600	Yards in a year.

Q. 3. There are 7 chests of drawers, in each of which are 18 drawers, and in each of these are 6 divisions, in each of which there is 16*l.* 6*s.* 8*d.* how much is there in the whole?

	£.	s.	d.	
	16	6	8	
			6	
	98	0	0	in each drawer
	18			
	1764			in each chest
	7			
Ans.	12348			Pounds in the whole

Q. 4. A lady's caterer bought 10 birds of two sorts, viz. turkies and geese, for 24 shillings; the turkies cost 4 shillings, and the geese 2 shillings a-piece; how many did he buy of each sort?

2 Turkeys	8 Geese
4	2
8*s.*	16*s.* $+8=24$*s.* the price.

Or thus; $2\times4=8$*s.* the price of the turkies, and $8\times2=16$*s.* the price of the geese; consequently 2 turkies $+8$ geese $=10$, the answer.

Q. 5. Suppose a gentleman has an estate of 800*l.* per annum, and he pays land-tax 150*l.* also for repairs 38*l.* 14*s.* 2*d.* what is his neat estate per annum?

				£.	s.	d.
Estate per annum — — —				800	0	0
Land-tax — —	150	0	0	188	14	2
Repairs, &c. — —	38	14	2			
Neat estate per annum — —				611	5	10

Q. 6. In a company S had 3*l.* 17*s.* 2*d.* more than T, who had 6 guineas less than R, who had within 16*s.* 8*d.* as much as W, who was known to have 100 guineas, wanting ten marks, of 13*s.* 4*d.* each; pray what money had they among them?

	£.	s.	d.		£.	s.	d.
First, 100 guineas =	105	0	0				
Ten marks =	6	13	4				
W had - - -	98	6	8	- -	98	6	8
Subtract - - -	0	16	8				
R had - - -	97	10	0	- - -	97	10	0
Subtract - - -	6	6	0				
T had - - -	91	4	0	- - -	91	4	0
Add - - - -	3	17	2				
S had - - - -	95	1	2	- - -	95	1	2
				Answer - £.	382	1	10

X. COMPOUND DIVISION,

TEACHETH to divide by one common divisor, either a simple or compound number, into any proposed number of equal parts, whereof each shall be a compound number.

CASE 1. When the divisor doth not exceed 12.

RULE. Begin at the highest denomination, which divide by the given divisor, and set the answer in the quotient, and to be of the same denomination; what remains must be multiplied by the number of parts in the next inferior denomination, and added to the given number of that denomination, and then divide as before. Proceed thus through all the denominations.

EXAMPLE 1. Suppose there was 1*l.* 15*s.* $1\frac{3}{4}$*d.* to be divided amongst 7 men; what is each man's share?

	£.	s.	d.
7)	1	15	$1\frac{3}{4}$
Answer - -		5	$0\frac{1}{4}$
			7
Proof - -	1	15	$1\frac{3}{4}$

To work this example, ask how oft 7 in 1? never a time; then 1*l.*=20*s.* added to 15*s.* is 35*s.* Then ask how oft 7 in 35? 5 times, put down 5 in the quotient, and say, 35 from 35, and there remains nothing; then ask, how oft 7 in 1? never a time, and there remains 1, one penny is 4 farthings, and 3 is 7; how often 7 in 7? once; set down 1 farthing, and the answer is 5*s.* $0\frac{1}{4}$*d.* as appears by the work.

E. 2. Bought 4 *Cwt.* of cheese, for which I gave 8*l.* 10*s.* 4*d.* at what rate did I give per *Cwt.*?

4)	8	10	4
Answer £.	2	2	7

E. 3. If 10 dozen of candles cost 3*l.* 17*s.* 1*d.* what does 1 dozen cost at that rate?

10)	3	17	1
Answer £.	0	7	$8\frac{1}{2}$

CASE

CASE 2. When the diviſor exceeds 12, and is ſuch a number that any two figures (in the multiplication table) being multiplied together, will produce it.

RULE. Divide by its component parts, as in diviſion of integers. See ſection 5, caſe 4.

EXAMPLE 1. Let it be required to divide 45*l.* 12*s.* 8*d.* into 16 equal parts?

$$4\times4=16\left\{\begin{array}{l} 4)45\quad 12\quad 8 \\ \hline 4)11\quad 8\quad 2 \end{array}\right.$$

Anſwer £. 2 17 $0\frac{1}{2}$

E. 2. Divide 7*l.* 6*s.* equally amongſt 24 perſons?

$$6\times4=24\left\{\begin{array}{l} 6)7\quad 6\quad 0 \\ \hline 4)1\quad 4\quad 4 \end{array}\right.$$

Anſwer £. 0 6 1

E. 3. If I ſell 81 buſhels of wheat for 30*l.* 7*s.* 6*d.* what is that per buſhel?

$$9\times9=81\left\{\begin{array}{l} 9)\,30\quad 7\quad 6 \\ \hline 9)\,3\quad 7\quad 6 \end{array}\right.$$

Anſwer £. 0 7 6

E. 4. If I ſell 100 quarters of barley for 90*l.* what is that per quarter?

$$10\times10=100\left\{\begin{array}{l} 10)\,90\quad 0\quad 0 \\ \hline 10)\,9\quad 0\quad 0 \end{array}\right.$$

Anſwer £. 0 18 0

CASE 3. When the diviſor cannot be produced by the multiplication of two ſmall numbers, divide as in ſection IV.

E. 1. Divide 21*l.* 17*s.* equally amongſt 17 perſons?

```
     £.  s.   £.  s.  d.
17)21  17 (1   5   8¼ Anſw.
   17
    4
   20
17)97(5s.
   85
   12
   12
17)144(8d.
   136
     8
     4
17)32(¼ qrs.
   17
   15 Remainder.
```

E. 2. Required to divide 214*l.* 17*s.* 3*d.* equally amongſt 34 perſons?

```
      £.   s.  d.
34)214  17  3(6l.
   204
    10
    20
34)217(6s.
   204
    13
    12
34)159(4d.
   136
    23
     4
34) 92(½
    68
    24 Remainder.
```

Anſwer 6*l.* 6*s.* $4\frac{1}{2}d.$ $\frac{24}{34}$ each.

E. 3. If 1 *Cwt.* of cheese cost 1*l.* 15*s.* 4*d.* what is the price of 1 ℔?

$$2\times7\times8=112\left\{\begin{array}{rrr} 2)1 & 15 & 4 \\ \hline 7)0 & 17 & 8 \\ \hline 8)0 & 2 & 6\frac{1}{4} \\ \hline \end{array}\right.$$

Answer - £. 0 0 $3\frac{3}{4}$

E. 4. If 52 tons of hay cost 167*l.* 14*s.* what will 1 ton cost?

52)167 14(3 *l.*
156
―
11
20
―
52)234(4*s.*
208
―
26
12
―
52)312(6*d.*
312
―
0
―

Answer 3*l.* 4*s.* 6*d.* per ton.

E. 5. Let it be required to divide 200*l.* 9*s.* 1*d.* into 104 equal parts?

104)200 9 1(1*l.*
104
―
96
20
―
104)1929(18*s.*
104
―
889
832
―
57
12
―
104)685(6*d.*
624
―
61
4
―
104)244($\frac{1}{2}$
208
―
36

Answer 1*l.* 18*s.* $6\frac{1}{2}$*d.*

WEIGHTS and MEASURES.

EXAMPLE 1. Divide 16*lb.* 1*oz.* 15*dwts.* 8*grains*, by 2?

	lb.	*oz.*	*dwts.*	*grs.*
2)	16	1	15	8
Answer	8	0	17	14

E. 2. Divide 140 acres, 2 roods, 26 poles, by 12?

	A.	*r.*	*p.*
12)	140	2	26
Answer -	11	2	28 — 10

Division of weights and measures, &c. is performed in the same manner as the above (with ease and accuracy) paying a due regard to their several denominations.

QUESTIONS.

Q. 1. *Says Hodge to his grandmother, Grannum, I see,*
That the money and purse which you've given to me,

Is

Is worth ſixteen and eight-pence, its well its no worſe,
For the caſh is in value worth nine times the purſe;
What ſum, then, had Roger, now Tyro, pray tell,
Which tickled his fancy, and pleas'd him ſo well?

	s.	d.
	16	8
		9
10)7	10	0
Anſwer 0	15	0

	s.	d.	
Then from	16	8	
Take - -	15	0	
Remains	1	8	Price of the purſe

Q. 2. The Spectator's club of fat people, though it conſiſted but of 15 perſons, is ſaid to weigh no leſs than 3 tons; how much on an equality was that per man?

3
20
5) 60
3) 12
Anſwer 4 Cwt. each man

Q. 3. The remainder of a diviſion is 325, the quotient 467, the diviſor is 43 more than the ſum of both; what is the dividend?

325
467
43

Multiply 835 Diviſor
by 467 + 325 the remr.

5850
5012
3343

Anſwer 390270 = Diviſor.

Q. 4. By ſelling 240 oranges at 5 for two-pence, half of which coſt me two a penny, and the other half three a penny, I evidently loſt a groat; pray how comes that about?

Firſt, 240
2
5)480
96 = what ſold for.

Secondly, 2)120
60*d.*

3)120
40*d.*

Then 60 + 40 = 100 what bought for
And 100 — 96 = 4*d.* the money loſt.

XI. REDUCTION,

TEACHETH to bring two or more numbers of different denominations, into one denomination; or it ſerveth to change or alter numbers, money, weight, meaſure, or time, from one denomination to another, and is generally performed by multiplication, or diviſion, as in ſection VI.

EXAMPLES

EXAMPLES of MONEY.

E. 1. In 20 pounds, how many shillings, pence, and farthings?

20
20
400 Shillings
12
4800 Pence
4
19200 Farthings

E. 2. How many pounds in 19200 farthings?

4(19200
12) 4800
2|0) 40|0
Answer £. 20

E. 3. In 321*l.* 14*s.* $6\frac{3}{4}$*d.* how many farthings?

321 14 $6\frac{3}{4}$
20
6434
12
77214
4
Answer 308859 Farthings

E. 4. In 298859 farthings, how many pounds?

4)298859
12) 74714 — 3
2|0) 622|6 — 2
Answer £. 311 6 $2\frac{3}{4}$

The third example is multiplied the same as the first, and the 14*s.* $6\frac{3}{4}$*d.* are taken in, in their proper places, viz. the 14 to the product of shillings, the 6 in the pence, and the $\frac{3}{4}$ in the farthings.

E. 5. In 12*l.* 10*s.* 8*d.* how many four-pences?

12 10 8
20
250
12
4)3008
Answer 752 Four-pences

E. 6. Let it be required to reduce 752 four-pences, to pounds?

752
4
12)3008
2|0) 25|0 — 8
Answer £. 12 10 8

E. 7. In 48 guineas, how many shillings, pence, and farthings?

48
21
48
96
1008 Shillings
12
12096 Pence
4
48384 Farthings

E. 8. In 48384 farthings, how many guineas?

4)48384
12)12096
$3\times7=21\left\{\begin{array}{l}3)\ 1008\\7)\ 336\end{array}\right.$
Answer 48 Guineas

E. 9.

E. 9. In 4321*l.* 14*s.* $0\frac{1}{2}$*d.* how many pieces of $13\frac{1}{2}$*d.* per piece?

4321 14 $0\frac{1}{2}$
20
86434
12
1037208
4
qrs 54 { 9) 4148834
6) 460981—5 } = 14
76830—1

Answer 76830 pieces of $13\frac{1}{2}$*d.* each, and 14 farthings rem.

E. 10. How many half-crowns, [redacted], crowns and pounds, are there in 14400 pence?

3|0) 1440|0
2) 480 Half-crowns
4) 240 Crowns
60 Pounds

E. 11. In 60*l.* how many crowns, half-crowns, and pence?

60
4
240 Crowns
2
480 Half-crowns
30
14400 Pence

E. 12. How many crowns, half-crowns, and shillings, are there in 426*l.* 15*s.* 6*d.* and of each an equal number?

s.	*d.*
5	0
2	6
1	0
8	6

12
102

£. *s.* *d.*
426 15 6
20
8535
12
102) 102426 (1004
102
426
408
18

Ans. 1004 pieces of each sort, and 18*d.* remains.

Operations of this sort are proved by multiplying the parts or pieces into which the given number was reduced, and adding their several products together; and if their sum be equal to the given number, the answer is right; thus,

1004	Crowns	make	251	0	0
	Half-crowns		125	10	0
	Shillings		50	4	0
	Remainder		0	1	6
	Proof		£. 426	15	6

In things of this nature, the divisor and dividend must be of one name.

E. 13. In 342*8l.* 18*s.* how many shillings and moidores?

342 18
20
$3 \times 9 = 27$ { 3) 6858 Shillings
9) 2286
254 Moidores

E. 14. Reduce 480 guineas to shillings, crowns and pounds?

First $480 \times 21 = 10080$ Shillings
Then 5) 10080
4) 2016 Crowns
504 Pounds

OF

OF COINS.

To reduce foreign and Engliſh coins to pounds ſterling,

RULE. Multiply the given number by the loweſt denomination of the price or value of 1; and divide the product by ſuch terms as will bring out the value in pounds.

EXAMPLE 1. In 1178 dollars, at 4*s.* 3*d.* each, how many pounds ſterling?

```
s. d.        1178 Dollars
 4 3           51 Pence in 1 dollar
12           ----
--           1178
51           5890
            -----
         12)60078
           -----
      2|0) 500|6 — 6
Anſwer £.250  6  6
```

E. 2. In 340 piſtoles, each 17*s.* 6*d.* how many pounds ſterling?

```
 s. d.           340
17  6            210
12              ----
---             3400
210             680
               -----
            12)71400
              -----
          2|0) 595|0 —
Anſwer     £. 297  10
```

Note. After the ſame manner may any foreign coin be brought into Engliſh ſterling.

To reduce pounds ſterling into foreign and Engliſh coin.

RULE. Reduce both the ſterling money and foreign coin into their loweſt denomination; then divide one by the other, and the quotient will be the anſwer.

EXAMPLE 1. A merchant is to pay 296*l.* 12*s.* 3*d.* with dollars of 4*s.* 3*d.* each, how many will do it?

```
 4  3        296  12  3
12             20
--           ----
51           5932
               12
           ------
       51)71187(1395 Anſw.
          51
          --
          201
          153
          ---
           488
           459
           ---
            297
            255
            ---
             42
```

E. 2. In 774*l.* 18*s.* 4*d.* how many florins, at 3*s.* 2*d.* each?

```
s. d.        £.   s.  d.
3  2        774   18  4
12           20
--         -----
38         15498
              12
          ------
       38)185980(4894 Anſw.
          152
          ---
           339
           304
           ---
            358
            342
            ---
             160
             152
             ---
               8
```

E. 3.

E. 3. How many marks, each 13*s.* 4*d.* are in 248*l.* 9*s.* 2*d* ?

```
s.  d.                  £.   s.  d.
13  4                   248  9   2
12                       20
---                     ----
160                     4969
---                       12
                        ---------
                        1|0) 5962'0
                        4) 5962
10 × 4 × 4 = 160        4) 1490 — 2
                        -----------   } = 10d.
        Answer           372 — 2
```

To reduce one kind of coin into another kind.

RULE. Divide one by the other, in their lowest terms, and the quotient will be the answer.

EXAMPLE 1. How many moidores are equal to 198 guineas ?

```
                       198
                        21
                       ---
                       198
                      396
                      ----
Shill. in a moidore  9)4158
   9×3=27            ------
                     3) 462
                     ------
          Answer       154
```

E. 2. How many crowns, 5*s.* 4*d.* each, are in 474 pistoles of 18*s.* 6*d.* each ?

```
s. d.   s. d.
5  4    18 6     474 Pistoles
12      12       222d. in 1 pistole
--      ---      ---
64      222      948
                948
               948
               ------
          8)105228
     64   --------
          8) 13153—4 }
                      } = 12d.
  Crowns    1644—1   }
```

Having sufficiently shewn how money is changed from one denomination to another, I shall now proceed to weights, measures, &c.

WEIGHTS and MEASURES.

EXAMPLE 1. In 12 lb. of silver, how many ounces, penny-weights, and grains ?

```
   12 Pounds
   12
  ---
  144 Ounces
   20
 ----
 2880 Penny-weights
   24
 -----
11520
5760
-----
69120 Grains
```

E. 2. In 69120 grains, how many pounds ?

```
     4(69120
 24  -------
     6)17280
     -------
   2|0) 288|0
   ----------
     12) 144
     -------
Answer  12 Pounds
```

E. 3. A gentleman ſent 4 ℔. 2 oz. 8 dwts. of old plate, to his ſilverſmith, with orders to make it into the following articles, viz. tankards each 19 oz. 18 dwts.—cups each 14 oz. 10 dwts.—ſalts 11 oz. 15 dwts.—and ſpoons 2 oz. 4 dwts. how many of each ſort muſt he make?

		oz.	*dwt.*
The weight of each	Tankard	19	18
	Cup	14	10
	Salt	11	15
	Spoon	2	4
		48	7
		20	
		967	

lb.	*oz.*	*dwts.*
4	2	8
12		
50		
20		

967)1008(1
967
41

Anſwer 1 of each ſort, and 41 *dwts.* over.

E. 4. In 18 pounds, 2 ounces, 4 drams, 2 ſcruples, and 12 grains, how many grains?

℔.	℥.	ʒ.	℈.	*grs.*
18	2	4	2	12

12
218
8
1748
3
5246
20
104932 Grains, Anſwer.

E. 5. In 104932 grains, how many pounds?

2|0)10493|2
3) 5246—12
8) 1748—2
12) 218—4
Anſwer ℔. 18—2 4 2 12

E. 6. How many pounds of ſilver are there in one dozen of diſhes, each weighing 25 ounces, 15 dwts. and one dozen of plates, each weighing 15 ounces, 15 dwts. 22 grains?

	lb.	*oz.*	*dwts.*	*grs.*
One dozen of diſhes, each weight	2	1	15	0
Ditto — Plates,	1	3	15	22
One of each	3	5	10	22
				12
Anſwer	41	6	11	0

E. 7. In a medicinal compoſition of 25 pounds, 7 ounces, and 6 drams, how many papers of powder may be made thereout, each weighing 2 ſcruples and 16 grains, allowing an ounce and a half to be loſt in levigating and weighing, and admitting theſe powders were to be equally divided amongſt 175 perſons, how many muſt each of of them have?

From

	lb.	oz.	drs.
From the weight	25	7	6
Subtract the loss		1	4
	25	6	2

Scr. grs.
2 16
20
~~66~~
56

25 6 2
12
306 Ounces
8
2450 Drams
3
7350 Scruples
20
7)147000
8) 21000
175) 2625(15 Ans.
175
875
875
0

E. 8. In 16 tons, 12 hundred, 2qrs. 12 pounds, 8 ounces, and 6 drams, how many drams?

Tons.	C.	q.	lb.	oz.	drs.
16	12	2	12	8	6

20
332 Hundreds
4
1330 Quarters
28
10642
2661
37252 Pounds
16
223520
37252
596040 Ounces
16
3576246
596040
Ans. 9536646 Drams

E. 9. In 9536646 drams, how many tons?

16 { 2)9536646
8)4768323 } =6 *drs.*
16 { 4) 596040—3
4) 149010 } =8 *oz.*
28 { 4) 37252—2
7) 9313 } =12 *lb.*
4) 1330—3
2|0) 33|2—2 *qrs.*

Answer *Tons* 16 12 2 12 8 6

E. 10. In 12 hogsheads of raisins, each weighing 8 *Cwt.* 2 *qrs.* 6 *lb.* how many pounds and tons?

C.	*qrs.*	*lb.*
8	2	6

4
34 Quarters
28
278
68
958 Pounds in 1 hhd.
12 No. of hogsheads
28= { 4)11496 Pounds in all
7) 2874 } =16 *lb.*
4) 410—4
2|0)10|2—2

Answer *Ton* 5 2 *C.* 2 *qrs.* 16 *lb.*

E. 11. A grocer bought 18 hogsheads of sugar, each weighing 5 *Cwt.* 3 *qrs.* 14 *lb.* out of which he has sold 5 *Cwt.* 1 *qr.* $16\frac{1}{2}$ *lb.* and orders the remainder to be made up into parcels of 27 pounds each; how many will there be, allowing 6 pounds to be lost in weighing them up?

	C.	*qrs.*	*lb.*		*C.*	*qrs.*	*lb.*
To	5	1	$16\frac{1}{2}$		5	3	14
Add			6		4		
	5	1	$22\frac{1}{2}$		23 Quarters		

4
21
28
170
44
610
2
1221

28
188
47
658 Pounds
2
1316 Half-pounds in one hogshead
18 Number of hogsheads
10528
1316

From - - 23688 } Half-pounds in the { 18 hogsheads
Take - - 1221 } Half-pounds in the { Quant. sold

Half-pounds in 1 parcel 54 { 6)22467
9) 3744—3
416—0 } =3 Half-pounds

Answer 416 parcels, and $1\frac{1}{2}$ *lb.* over.

E. 12. In 146 yards of cloth, how many quarters and nails?

146
4
584 Quarters
4
2336 Nails

E. 13. In 2336 nails, how many yards?

4)2336
4) 584
Answer 146 Yards

E. 14. In $864\frac{1}{4}$ yards, how many ells English?

$864\frac{1}{4}$
4
5)3457
Answer *Ells* 691 2 *qrs.*

E. 15. In 691 English ells, and 2 quarters, how many yards?

691 2
5
4)3457
Answer *Yds.* 864 1 *qr.*

E. 16.

E. 16. In 86 pieces of cloth, each piece containing 30 yards, how many suits of clothes may be be made thereout of $6\frac{3}{4}$ yards to the suit?

$6\frac{3}{4}$
4
27

86 Pieces
30 Yards in 1 piece
2580 Yards in all
4
3)10320 Quarters
9) 3440
382—2 } = 6 *qrs.*

Answer 382 suits, and $1\frac{1}{2}$ *yd.* over

E. 17. In one mile, how many poles, yards, feet, inches and barley-corns? 1 Mile
8
8 Furlongs
40
320 Poles
$5\frac{1}{2}$
1600
160
1760 Yards
3
5280 Feet
12
63360 Inches
3
190080 Barley-corns

E. 18. In 190080 barley-corns, how many miles?

3)190080
12)63360 Inches
3) 5280 Feet
1760 Yards
2
$\frac{1}{2}$ *yds.* in 1 *p.* 11)3520
4|0) 32|0 Poles
8) 8 Furlongs
Answer 1 Mile

E. 19. How many barley-corns will reach from Birmingham to London, being 109 miles?

109
1760 Yards in 1 mile
654
763
109
191840 Yards in all
3
575520 Feet
12
6906240 Inches
3
20718720 Barley-corns

E. 20. How many times doth the wheel which is $5\frac{1}{2}$ *yds.* in circumference, turn round between London and Liverpool, being 202 miles?

First, 202 × 1760 (yards in 1 mile) = 355520 yards

$5\frac{1}{2}$ yards
2
11

Then 355520
2
11)711040 Half-yards in 202 miles
Answer 64640 Times

E. 21.

E. 21. How many Barley-corns will reach round the terrestrial globe, which is 360 degrees, and each degree $69\frac{1}{2}$ miles?

```
      360 Degrees
      69½ Miles in a degree
     ----
     3240
    2160
    -----
    24840
      180
    -----
    25020 Miles
   190080 Barley-corns in one mile
  -------
  2001600
 225180
 25020
 -------------
Answer 4755801600 Barley-corns
```

E. 22. In 21 acres of land, how many roods and poles?

```
  21
   4
  --
  84 Roods
  40
 ----
3360 Poles
```

E. 23. In 3360 poles, how many roods and acres?

```
4|0)336|0
   ------
   4) 84 Roods
      --
      21 Acres
```

E. 24. A person rents a farm, which contains 400 acres of land, but he is to till no more than $196\frac{1}{2}$ acres; I desire to know how many perches there are in the remainder?

```
 400 Acres
   4
----
1600
  40

From 64000 Perches in the whole
Take 31440                          A. R.
                                   196 2
Ans. 32560 Per. rem.                 4
                                   ---
                                   786
                                    40
                                 -----
               Perches tilled    31440
```

E. 25. In 12 hogsheads, 46 gal. 3 qts. of wine, how many quarts?

```
Hhd. gal. qts.
 12   46   3
 63
---
 42
76
---
802 Gallons
  4
----
3211 Quarts
```

E. 26. In 3211 quarts of wine, how many hogsheads?

```
       4)3211
     { 7) 802—3 Quarts
63 = {    ---
     { 9) 114—4 }
           ---   } = 46 Gal.
            12—6 }
```

Answer 12 *hhds.* 46 *gal.* 3 *qts.*

E. 27.

E. 27. A gentleman ordered his butler to bottle off 2 pipes of red port, into quart bottles, how many dozens will the 2 pipes fill?

2 Pipes
2
4 Hogsheads
63
252 Gallons
4
12) 1008 Quarts
Answer 84 Dozens

E. 29. In 8 lasts, 3 quarters, 2 bu. of corn, how many gallons?

L. q. bu.
8 3 2
10
83 Quarters
8
666 Bushels
8
Answer 5328 Gallons

E. 30. In 5328 gallons of corn how many lasts?

8) 5328
8) 666
1|0) 8|3 — 2
Answer *Lasts* 8 3 *qrs.* 2 *bu.*

E. 31. How many minutes are there in a Julian year?

W. d. h.
52 1 6
7
365
24
1466
730
8766
60
Answer 525960

E. 28. In 8 hogsheads of beer, how many pints?

8
54
32
40
432 Gallons
8
Answer 3456 Pints

E. 32. How many days, hours, minutes, and seconds, is it since the birth of our Saviour, to Christmas, 1781 (allowing Julian years?)

4)1781
445--1=6*

1781
365
8905
10686
5343
650065 *h.*
445 6
650510 Days
24
2602046
1301020
15612246 Hours
60
936734760 Minutes
60
56204085600 Seconds

* 6 Hours being a quarter of a natural day, or 24 hours; therefore dividing the number of years by 4, the quotient will be days, as above, and $\frac{1}{4}$ or six hours over.

XII. The

XII. The RULE of THREE DIRECT,

TEACHETH, by having three numbers given, to find a fourth, in the fame proportion to the third, as the fecond is to the firft; or as the firft is to the fecond, fo is the third to the fourth, for which reafon it is called the Rule of Proportion, as it is called the Rule of Three, from its having three numbers given; and becaufe of its excellent ufe in arithmetic, it is often named the *Golden Rule*.

To perform which obferve the following

RULE. 1ft. Place the three given terms in fuch order, that the firft and third may be of one name; and the fecond muft be of the fame name with the fourth term fought.

2. If your firft and third terms confift of divers denominations, reduce them into one, and the fecond into the loweft name mentioned.

3. Multiply the fecond and third terms together, and divide that product by the firft; the quotient will be the anfwer in the fame denomination you left your fecond term in.

4. If there happens to be a remainder, it will either make a fractional part, or it muft be reduced to a lower denomination, and divided by the fame divifor, the quotient will be fo many of the faid next name; proceed in this manner to the leaft name, and all the quotients together will be the anfwer to the queftion.

Before I fhew how to work any queftions in this rule, it will be neceffary to give the learner the following inftructions. Firft, obferve that the firft and fourth numbers are called extremes, and the fecond and third means; the product of the extremes, is equal to the product of the means.

EXAMPLE 1. As 4 is to 12, fo is 36 to a certain number; what is that number?

$$
\begin{array}{rcrcr}
4 & : & 12 & :: & 36 \\
 & & & & 12 \\
\hline
 & & & 4) & 432 \\
\hline
 & & \text{Anfwer} & & 108
\end{array}
$$

Now it may eafily be proved, that the product of the two extremes is equal to the product of the two means, for $4\times108=12\times36=432$.

CONTRACTION 1. When the fecond term can be divided by the firft, multiply that quotient into the third term, and the product will be the anfwer.

Take the laft example to prove this; thus, 12 divided by $4=3$; and 3 multiplied by $36=108$, the anfwer, as before.

E. 2.

E. 2. As 12 is to 18, so is 24 to a certain number; what is that number?

12 : 18 : : 24
18
—
192
24
—
12)432
—
Answer 36

The preceding example at full length.

4 : 3 17 6 : : 28
20
—
77
12
—
930
28
—
7440
1860
—
4)26040
—
12)6510
—
2|0) 54|2—6d.

Answer £. 27 2 6 as before.

Thus you may see that these contractions being considered, the work may oftentimes be performed much shorter than by the common methods.

CONTRACTION 2. When the third term can be divided by the first, multiply that quotient by the second term, and the product will be the answer.

E. 3. If 4 yards of broad cloth cost 3*l.* 17*s.* 6*d.* what will a piece, containing 28 yards, come to, at the same rate?

Yds.	£.	*s.*	*d.*	
4 :	3	17	6 : :	28
			7	4)28
Answer	27	2	6	7

CONTRACTION 3. When the first term can be divided by the second, and the third term by that quotient; the last quotient will be the answer.

E. 4. As 24 is to 8, so is 36 to a certain number; query that number?

24 : 8 : : 36
8)24 3)36
— —
3 Ans. 12 No. required.

The work at length.

24 : 8 : : 36
8
—
$24\left\{\begin{array}{l}4)288 \\ 6)\ 72\end{array}\right.$
—
Answer 12

E. 5. If 48 yards of cloth cost 6*l.* what will 64 yards cost?

Yds. £. *Yds.* £.
48 ÷ 6 = 8; and 64 ÷ 8 = 8, the Answer.

CONTRACTION 4. When the first term can be divided by the third, and the second by that quotient; the last quotient will be the answer. See the preceding example.

E. 6. If 6 yards of cloth cost 1*l.* 16*s.* what will be the value of 34 yards, at the same rate?

Yds. £. *s.* *Yds.*
6 : 1 16 : : 34
20
36
34
144
108
6)1224
2|0) 20|4
Anſwer £. 10 4 0

E. 7. If 34 yards coſt 10*l.* 4*s.* what will 6 yards of the ſame coſt?

34 : 10 4 : : 6
20
204
6
34) 1224 (36*s.* Anſwer.
102
204
204

E. 8. If 1 *Cwt.* of cheeſe coſt 26*s.* what will 40 *Cwt.* of the ſame coſt?

Cwt. *s.* *Cwt.*
1 : 26 : : 40
40
2|0) 104|0
Anſwer £. 52

E. 9. If 40 *Cwt.* of cheeſe coſt 52*l.* what will 1 *Cwt.* coſt?

40 : 52 : : 1
20
4|0) 104|0
2|0) 2|6
Anſwer £. 1 6 0

Note. To prove examples in this rule, is only varying the operation, as may be ſeen by the preceding examples.

E. 10. Suppoſe I buy 1 *oz.* of tea for $7\frac{1}{2}$*d.* how much muſt I pay for 1 *Cwt.* of the ſame?

oz. *d.* *lb.*
1 : $7\frac{1}{2}$: : 112 = 1 *Cwt.*
2 16
15 672
112
1792
15
8960
1792
2)26880
12)13440
2|0) 112|0
Anſwer £. 56

E. 11. Bought 36*oz.* of ſilver, at the rate of 5*s.* 4*d.* per ounce, what does the whole come to?

oz. *s.* *d.* *oz.*
1 : 5 4 : : 36
12
64
36
384
192
12)2304
2|0) 19|2
Anſwer £. 9 12 0

The ſame by multiplication.

5 4
6×6=36
1 12 0
6
Anſwer £. 9 12 0 as before.

This

This example plainly ſhews the extenſive uſe of multiplication, and how much preferable, in ſome caſes, it is to the rule of three, by ſolving queſtions in a more conciſe manner, and therefore it is very neceſſary for all perſons to be thoroughly acquainted with thoſe moſt uſeful rules, viz. compound multiplication and diviſion.

E. 12. If a ſoldier's pay be 6*d*. per day, how much is that per year?

day. *d.* *days.*
1 : 6 : : 365
6
12) 2190
2|0) 18|2 — 6
Anſwer - £. 9 2 6

If you would know at what rate you muſt ſell your goods by retail, ſo as to make a propoſed gain by the whole; add the money you would gain to the ſum the goods coſt you, and then ſtate your queſtion as before. Thus, if the whole be ſold for the total of the coſt and gain, at what rate muſt any part of it be ſold for?

E. 13. Bought 75 *Cwt.* 1 *qr.* 13 *lb.* of tobacco, which coſt 387*l.* 15*s.* 8*d.* and the charges upon it amounted to 6*l.* 5*s.* 8*d.* how much did it lie me in per pound?

	£.	s.	d.
Prime coſt	387	15	8
Charges -	6	5	8

C. *qr.* *lb.* *lb.*
75 1 13 : 394 1 4 : : 1
4 20
301 7881
28 12
2411 8441)94576(11*d.*
603 8441
8441 10166
8441
1725

Anſwer 11*d.* $\frac{1725}{8441}$ per pound

E. 14. If 34 *Cwt.* 3 *qrs.* 25 *lb.* of tobacco coſt 111*l.* 15*s.* 6*d.* what will 1 pound come to at that rate?

Cwt. *qr.* *lb.* £. *s.* *d.* *lb.*
34 3 25 : 111 15 6 : : 1
4 20
139 2235
28 12
1117 26826
280 4
3917) 107304(27*qrs.*=6¼*d.*
7834
28964
27419
1545

Anſwer 6¼*d.* $\frac{1545}{3917}$ per pound

E. 15. If 1*lb.* of tobacco cost $6\frac{3}{4}d.\ \frac{1545}{3917}$, how much may be bought for 111*l.* 15*s.* 6*d.*

d. *qrs.* — *lb.* — £. *s.* *d.*

6 3 $\frac{1545}{3917}$: 1 : : 111 15 6

4 — 20

27 — 2235

3917 — 12

189 — 26826

27 — 4

243 — 107304

81 — 3917

105759 — 751128

1545 — 107304

107304 — 965736

321912

28) 4)

107304) 420309768 (3917 (139

321912 — 28 — *qr.* *lb.*

Cwt. 34 3 25

983977 — 111

965736 — 84

182416 — 277

107304 — 252

751128 — 25 *lb.*

751128

. . . . . .

Answer 34 *Cwt.* 3 *qrs.* 25 *lb.*

E. 16. A grocer bought 2 *Cwt.* 1 *qr.* 14 *lb.* weight of cloves, which cost him 32*l.* 4*s.* and he gained 5*l.* by the bargain, at what rate must he sell them per pound?

Cwt. *qr.* *lb.* — £. *s.* — *lb.*

32 4

5

2 1 14 : 37 4 : : 1

4 — 20

9 — 266)744(2*s.*

28 — 532

76 — 212

19 — 12

266 — 266)2544(9*d.*

2394

150

4

266)600(2 *qrs.*

532

68

Answer 2*s.* $9\frac{1}{2}d.\ \frac{68}{266}$ per pound

If

If at any time damage has happened to goods, ſo as to make a propoſed loſs by the whole, then the ſaid loſs muſt be ſubtracted from the coſt, and the remainder made the ſecond term as before.

E. 17. Suppoſe I have by me 300 yards of holland, which coſt me 80*l.* but ſome damage having happened to it, I am willing to loſe 6*l.* 10*s.* by the whole; at what rate then muſt I ſell it per yard?

	£.	s.	
	80	0	
	6	10	
Yds.			*Yd.*
300 :	73	10 ::	1
	20		

3|00)14|70

*s.*4 270
12

3|00)32|40

*d.*10 240
4

3|00) 9|60

3 *qrs.*

Anſwer 4*s.* 10¾*d.* $\frac{60}{300}$ per yard

E. 18. *If ten pounds of bacon juſt coſt me a crown,*
For a flitch of ſix ſcore what muſt I pay down?*

lb.		*s.*		*lb.*
10	:	5	::	120
				5

1|0) 60|0

2|0) 6|0

Anſwer £.3

* Pounds.

E. 19. An oilman bought 3 tons of oil, which coſt him 151*l.* 14*s.* and ſo it chanced, that it leaked out 85 gallons, but he is deſirous to ſell it again ſo that he may be no loſer; how muſt he ſell it per gallon? Firſt 3 tons=756 gallons—85=671; and 51*l.* 14*s.*=3034*s.* then,

gal.		*ſhil.*		*gal.*
If 671	:	3034	::	1
		1		

671)3034(4*s.*
2684

350
12

671)4200(6*d.*
4026

174
4

671)696(1 *qr.*
671

25

Anſwer 4*s.* 6¼*d.* $\frac{25}{671}$ per gallon

E. 20. *If four ſtrike of corn coſt a guinea, not more,*
Pray what muſt I give for one hundred and four?

bu.		*s.*		*bu.*
4	:	21	::	104
				21

104
208

4) 2184

2|0) 54|6

Anſwer £. 27 6 0

E. 21.

E. 21. Suppose a bankrupt owes 3000*l.* and has in money, goods, and recoverable debts 800*l.* 12*s.* $9\frac{3}{4}$*d.* now suppose these things delivered to his creditors, what do they get per pound?

```
£.          £.  s. d.     £.
3000   :   800  12 9¾  ::  1
            20
          -----
          16012
             12
          ------
          192153
               4
        --------------
 3|000)768|615
      ------
     4) 256 — 615 Remains
      -----
    12) 64
      -----
   Answer  5s. 4d.
```

In the above example, though there is a remainder of 615, yet the part of a pound each one is to receive, can be no more than 5*s.* 4*d.*

E. 22. A person owes 296*l.* 17*s.* but not being able to pay the whole, compounds with his creditors to pay them 7*s.* 6*d.* in the pound, how much money doth he pay his creditor's?

```
£.      s. d.       £.   s.
1   :   7  6  ::   296   17
20      12          20
--      --         ----
20      90         5937
--      --           90
                  ------
            2|0)53433|0
              12)2671|6  { —10=
                ------   { 10/20=½
             2|0) 222|6 — 4
      Answer  £. 111  6  4½
```

E. 23. Suppose a person fails in trade, and compounds with his creditors to pay them 12*s.* 6*d.* in the pound; how much doth he owe, when all his cash and effects amount only to 700*l*?

```
s.  d.       £.       £.
12  6    :   1   ::   700
12                     20
---               --------
150                 14000
---                    12
                  --------  £.
            15|0)16800|0(1120
                 15
                 --
                  18
                  15
                  --
                   30
                   30
                   --
                    0
                   --
Answer  £. 1120
```

E. 24. If I spend 2*s.* 6*d.* a day, and lay up 20*l.* at the year's end, how much is my yearly income?

```
Day      s. d.        days.
1   :    2  6   ::    365
        12             30
        --          -------
        30       12)10950
        --         -------
              2|0) 91|2—6
                  ---------
                   45  12  6
                   20   0  0
                  ----------
      Answer  £.   65  12  6
```

E. 25. Bought coals at $4\frac{1}{2}d.$ per *Cwt.* how much will 30 tons come to at that rate?

Cwt. *d.* *Tons.*

1 : $4\frac{1}{2}$ :: 30

2 20

9 600

9

2)5400

12)2700

2|0)22|5

Anſwer £. 11 5 0

E. 26. Bought a quantity of timber by the lump, for which I gave 147*l.* 16*s.* 4*d.* it is ſuppoſed to contain 70952 feet, how much did it lie me in per foot?

Ft. £. *s.* *d.* *ft.*

70952 : 147 16 4 :: 1

20

2956

12

35476

4

70952)141904(2 *qrs.* per ft.

141904

0

E. 27. A woman bought 496 eggs at 2 a penny, and 294 at 3 a penny, which ſhe ſold out together at 5 for two-pence; I would know whether ſhe gained or loſt by the bargain, and how much?

First 2 : 1 :: 496

1

2)496

12)248

2|0) 2|0—8

1 0 8

0 8 2

The ſum the eggs coſt £. 1 8 10

Secondly 3 : 1 :: 294

1

3)294

12) 98

s. 8—2*d.*

Thirdly *e.* 5 : *d.* 2 :: *e.* 790

2

5)1580

12)316

2|0 2|6—4*d.*

£. 1 6 4

£. *s.* *d.*

From 1 8 10 the ſum the eggs coſt

Take 1 6 4 for which they were ſold

Rem. 0 2 6 loſt thereby, Anſwer

E. 28. If 100*l.* principal, gain 5*l.* intereſt in 12 months, what will 40*l.* gain in the ſame time?

£. £. £.

100 : 5 :: 40

5

1|00) 2|00

Anſwer £. 2

E. 29. What will 145 sheep come to at 11*l.* 13*s.* 6*d.* per score?

```
Sh.      l.  s. d.     Sh.
20  :    11 13  6  ::  145
         20
        ----
        233
         12
        ----
       2802
        145
      ------
      14010
     11208
     2802
     ------
   2|0)40629|0
     -------
   12)20314—10=10/20=1/2
     -------
  2|0) 169|2—10d.
        ---
Anf. £. 84 12 10½
```

E. 30. One who had sold a parcel of cloaths for 2*s.* 10*d.* per yard on 3 months credit, found he gained 25*l.* per cent. by them; what did they cost him per yard?

First 25*l.* ÷ by 4 = 6*l.* 5*s.* then 6*l.* 5*s.* + 100 = 106*l.* 5*s.*

```
   £.  s.    s.  d.       £.
If 106 5  :  2  10  ::   100
    20       12           20
  ----      ----         ----
  2125       34          2000
  ----      ----           34
                         ----
                         8000
                        6000
                       ------ 12)
                2125)68000(32
                     6375   ----
                    ------   2—8
                      4250
                      4250
                     ------
                         0
```

Answer 2*s.* 8*d.*

E. 31. A person bought 448 eggs at 3 a penny, another sort at 2 a penny, which together were sold out for 1*l.* 17*s.* 6*d.* how many eggs were bought at 2 a penny?

```
e.   d.    e.          £.  s.  d.
3 :  1 :: 448          1  17  6
            1          0  12  5
          ---         ---------
        3)448          1   5  1
         ----             20
     12)149—1            ----
        ----              25
         12 5             12
       ------            ----
                          301
                            2
                          ---
Answer, eggs at 2 a penny 602
                          ---
```

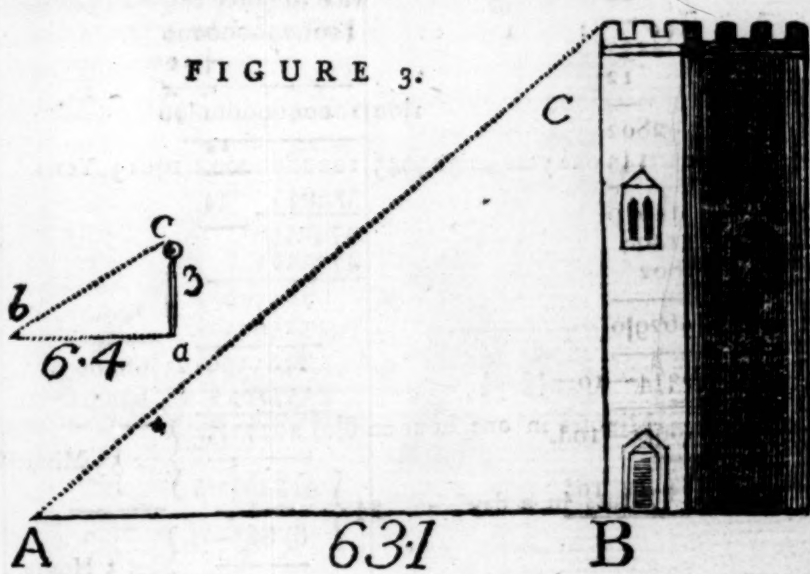
E. 32. *If* 13 *marks and* 14 *groats buy* 15 *loads of hay*; *How many pounds, with* 16 *crowns, for* 90 *loads will pay?*

First, a mark = 13*s.* 4*d.* then 13 marks + 14 groats = 178*s.* then

```
   loads.     s.        loads.
If 15   :    178   ::    90
              90
            -----
     ┌ 5)16020
  15 ┤  -------
     └ 3) 3204
        -------
          1068 Price of 90 loads
            80 = 16 crowns
        -------
      2|0)98|8
        -------
Answer £. 49  8
```

E. 33. A certain tower projected upon level ground a shadow, to the distance of 63 yards 1 foot, when a staff, 3 feet in length, perpendicularly erected, cast a shadow of 6 feet 4 inches, from hence the height of the tower is required?

FIGURE 3.



In the above figures, $ac = 3$ feet, the length of the staff; $ab = 6$ feet 4 inches, or 76 inches, length of its shadow. Also $BC =$ the height of the tower, and $AB = 63$ yards 1 foot, or 2280 inches distance of its shadow; then,

	in.		*ft.*		*in.*
If	76	:	3	::	2280
					3
					76) 6840 (90 Feet
					684
					0

Answer 90 feet = 30 yards, the height of the tower

E. 34. Suppose a person travels 228 miles in 6 days, 4 hours, at what rate is that per hour, (allowing 12 hours to the day?)

D. *h.*		*m.*		*h.*
6 4	:	228	::	1
12				
76)228)3 Miles, the answer				
228				
...				

E. 35. In how long time would a million of millions of money, be in counting, ſuppoſing one hundred pounds to be counted every minute without intermiſſion, and the year to conſiſt of 365 days, 5 hours, and 45 minutes?

£.		M.		£.
100	:	1	::	1000000000000
				1
				1|00) 10000000000|00

Minutes in one year = 525945) 10000000000 (19013 Years
525945
4740550
4733505
704500
525945
1785550
1577835
Minutes in one hour = 6|0) 20771|5 } = 55 Minutes
Hours in a day - 24 { 4) 3461--5
6) 865—1 } = 5 Hours
144—1

Anſwer 19013 years, 144 days, 5 hours, and 55 minutes.

E. 36. *If thirteen tuns of claret wine, coſt nineteen Engliſh pounds. How many pints of the ſame wine are worth a thouſand crowns?*

Firſt 13 tuns reduced to pints=26208, and 19*l.*=380 ſhillings; and 1000 crowns=5000 ſhillings; then

s.		pints		s.
If 380	:	26208	::	5000
		5000		

Pints
38|0) 13104000|0 (344842$\frac{4}{38}$, or $\frac{2}{19}$ [Anſwer
114
170
152
184
152
320
304
160
152
80
76
4

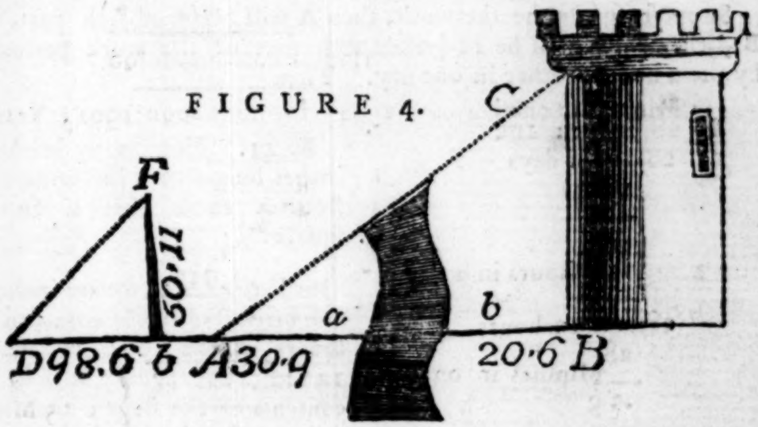
E. 37. By laying out a ſum of money, one ſhilling brings 17$\frac{1}{2}$*d.* what is gained per cent?

s.		d.		£.
1	:	5$\frac{1}{2}$	::	100
		2		20
		11		2000
				11
				2) 22000
				12) 11000
				2|0) 91|6—8

Anſwer £. 45 16 8

E. 38. A may-pole 50 feet 11 inches long, at a certain time of day casts a ſhadow 98 feet 6 inches long; I would thereby find the breadth of a river, that running due E. and W. within 20 feet 6 inches of the foot of a ſteeple 300 feet 8 inches high, which throws the extremity of its ſhadow 30 feet 9 inches beyond the ſtream?

FIGURE 4.



In figure 4, $Fb = 50$ feet 11 inches, $= 611$ inches, $=$ the height of the may-pole; and $Db = 98$ feet 6 inches, or 1182 inches, length of its ſhadow. Alſo $BC = 300$ feet 8 inches, or 3608 inches, the height of the ſteeple; and AB the length of its ſhadow; then

in. *in.* *in.*
If 611 : 1182 :: 3608
3608
9456
70920
3546
12)
611)4264656(6979
3666
5986
5499
4875
4277
5986
5499
487

581 $7\frac{487}{611} = AB$
20 6 fr. ſt. to r.
561 1
30 9 projection
530 $4\frac{487}{611} = ab$, [Br. of the river

E. 39. B and C together can build a boat in 18 days; with the aſſiſtance of A they can do it in 11 days; in what time would A do it by himſelf?

Firſt, $18-11 = 7$th part performed by A alone—then

If 7 : 1 :: 198 the whole work
7)198
d. 28—2
12 hours in a day
7)24
h. 3—3
60 min. in an h.
7)180
25—$\frac{5}{7}$ minutes

Anſwer 28 days, 3 hours, $25\frac{5}{7}$ minutes, by A himſelf.

E. 40. *A working alone in twelve days can compleat,*
The making a veſſel of copper quite neat;
Which would take ſixteen days to be made up by B,
He working more ſlowly than A, *you may ſee.*
Now working together, what time will they take,
Before the ſaid veſſel compleatly they'll make?

Suppoſe 192 to be the work, then A will perform $\frac{1}{12}$th part, and B $\frac{1}{16}$th, which will be 12+16=28th part of the work performed by them both together in one day. Then

```
   w.   d.    w.
If 28 : 1 :: 192
   28)192(6 days
      168
      ---
       24
       12 hours in a day
      ---
   28)288(10 hours
      28
      --
      ..8
       60
      ---
   28)480(17 minutes
      28
      ---
      200
      196
      ---
        4
        d. h.  m.
Anſwer  6  10  17 4/28
```

E. 41. How many bricks, 9 inches long and 4 in. wide, will floor a room that is 20 feet ſquare?

Firſt, 9×4=36 ſquare inches in one brick, and 20×20=400 feet ſquare in the floor, which × by 144=57600 ſquare inches, the contents of the floor; then

```
   inch. b.   inch.
If  36 : 1 :: 57600
      ⎧ 6)57600
   36 ⎨  ------
      ⎩ 6) 9600
         ------
Anſwer   1600 bricks
```

E. 42. Bought a pipe of port wine, for which I gave 25*l.* 4*s.* but it leaked out 12 gallons; the remainder I ſold at the rate of 18*d*, per quart: what was my gain or loſs in the whole?

Firſt by reduction, a pipe=126 gallons, from which take 12 gallons; remains 114 gallons=456 quarts, at 18d. per quart—then

```
   qt.  d.   qts.
If 1 : 18 :: 456
              18
            ----
            3648
            456
            ----
         12)8208
         2|0) 68|4
            ------
         £. 34 4 what ſold for
            25 4
            ----
Anſwer   £. 9 0 gained.
```

E. 43. If 2 men earn 15*s.* in 3 days, how much will 7 men earn in the ſame time?

```
M.    s.    M.
2  :  15 :: 7
       7
     ---
   2)105
     ---
  2|0 5|2—½=6d.
Anſwer 2l. 12s. 6d.
```

E. 44. A. sets out from London to Birmingham at the very same time that B. at Birmingham sets forwards for London, distance 109 miles; at 8 hours end they met on the road, and it then appeared that A. had rode $2\frac{1}{2}$ miles an hour more than B. at what rate an hour did each of them travel?

	h.		*m.*		*h.*
First, if	8	:	109	: :	1
			8)109		
			m. 13 5 furlongs, what both rode per hour		

	m.	*f.*	*p.*	
Then	13	5	0	
Less	2	4	0	the distance per hour that A. over-rode B.
2)	11	1	0	
	5	4	20	B. rode per hour
+	2	4	0	
	8	0	20	A. rode per hour

$$\therefore \left\{\begin{array}{ccc} m. & f. & p. \\ 5 & 4 & 20 \\ 8 & 0 & 20 \end{array}\right\} \times \text{by } 8 = \left\{\begin{array}{ccl} m. & f. & \\ 44 & 4 & \text{B.} \\ 64 & 4 & \text{A.} \end{array}\right\} \text{travelled}$$

Proof 109 miles

E. 45.

As I was beating on the forest grounds,
Up starts a hare before my two greyhounds:
The dogs being light of foot, did fairly run
Unto her fifteen roods, just twenty-one.
The distance that she started up before,
Was fourscore sixteen rods, just and no more:
Now this I'd have you unto me declare,
How far they ran before they caught the hare?

First, from 21 take 15, remains 6 rods; the dogs gained in running 21 rods, and fourscore=80 +16=96 rods the hare started before the dogs—then

	r.		*r.*		*r.*
If	6	:	21	: :	96
					21
					96
					192
					6)2016

Dist. the hare started before the dogs	336	r. dogs ran
	−96	
	240	r. hare ran

E. 46. If the sun moves every day one degree, and the moon thirteen, and at a certain time the sun be at the beginning of Cancer, and in three days after, the moon in the beginning of Aries, the place of their next following conjunction is required?

First,

First, $13^\circ - 1^\circ = 12^\circ$ moon gains of the sun per day.
And $30^\circ \times 3 = 90^\circ$ from the first of Aries to the first of Cancer.
Also $90^\circ + 3 = 93^\circ$ sun before the moon; then

$$\text{If } 12^\circ : \overset{D.}{1} :: 93^\circ$$

$$12)93$$

$7\frac{9}{12}$ days, in which time the sun will be overtaken by the moon.

∴ $7\frac{3}{4} + 3 = 10\frac{3}{4}$ degrees of Cancer, the answer.

Note $\frac{9}{12}$ in its lowest terms $= \frac{3}{4}$.

SOUND not interrupted, is by experiments found uniformly to move about 1150 feet in one second of time.

E. 47. How long after the firing of a cannon at Birmingham may the report be heard at Worcester, distance 25 miles?

	ft.		sec.		miles.
If	1150	:	1	::	25

5280 feet in 1 mile
25
―――
26400
10560
―――
1150)132000(114 seconds
1150
―――
1700
1150
―――
5500
4600
―――
900
60
―――
1150)54000(46 thirds
4600
―――
8000
6900
―――
1100

Answer 1 minute 54 seconds, 46 $\frac{1100}{1150}$ thirds.

E. 48. If I see a flash of a piece of ordnance, fired by a vessel in distress at sea, which happens, we will suppose, nearly at the instant of its going off, and hear the report a minute and two seconds afterwards, how far is she off, reckoning for the passage of sound as before?

First, 1 minute 2 seconds = 62 seconds; then

	″		ft.		″
If	1	:	1150	::	62

62
―――
2300
6900
―――

		m.	f.	p.	y.	ft
Answer	71300 feet =	13	2	0	6	2

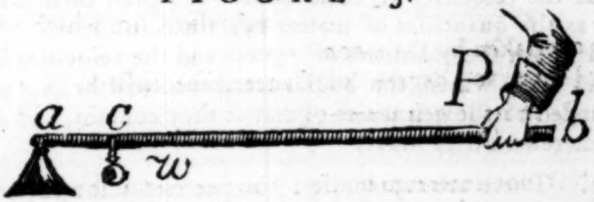
LEAVERS, of the second order, are such sort where the power acts at one end, the prop fixed directly at the other, and the weight somewhere between them.

In

In this order of leavers, their force is in a contra-proportion to their length.

E. 49. If a leaver be 120 inches long, what weight, lying $8\frac{1}{2}$ inches from the end, resting on a pavement, may be moved with a force of 182 ℔. lifting at the other end of the leaver?

FIGURE 5.



Let $ab = 120$ inches, and $ac = 8\frac{1}{2}$, P the power, or 182 ℔. and w, the weight to be moved.

Inches.		*in.*		*lb.*		*in.*
120	If	$8\frac{1}{2}$	:	182	::	$111\frac{1}{2}$
$8\frac{1}{2}$		2				2
$111\frac{1}{2}$ Longest end		17				223
						182
						446
						1784
						223
						17) 40586 ($2387\frac{7}{17}$ lb.
						34
						65
						51
						148
						136
						126
						119
						7

$\therefore 2387\frac{7}{17}$ *lb.* $= w$.

In leavers of the third order, the prop is planted at one end of the bar, the weight at the other end, and the moving force somewhere between.

E. 50. A water-wheel turns a crank, working three pump-rods, fixed just six feet from the joint or pin; by which their several leavers, each nine feet in length, are fastened, for the sake of the intended motion, at one end, the suckers of the pumps being worked by the other, shews them to be leavers of the third order: now I would know what the length of the stroke in each of the barrels will be, if the crank be made to play just nine inches round its centre?

First, $9 \times 2 = 18$ inches, the diameter of the crank; then

	Feet.		*in.*		*ft.*
If	6	:	18	::	9
			9		
			6)162		
	Answer		27	Inches	

MOTION

MOTION of BODIES, with their Velocities.

1. If the quantities of matter in any two or more bodies put in motion, be equal, the forces wherewith they are moved will be in proportion to their velocities.

2. If the velocities of these bodies be equal, their forces will be directly as the quantities of matter contained in them.

3. If both the quantities of matter and the velocities be unequal, the forces with which the bodies are moved will be in a proportion, compounded of the quantities of matter they contain, and of the velocities wherewith they move.

E. 51. There are two bodies, the one contains 25 times the matter of the other (or 25 times heavier) but the lesser moves with 100 times the swiftness of the greater; in what proportion are the forces by which they are moved?

If 25 : 100 :: 1
1

25)100(4
100

Answer 4, the less is moved with a force so much greater than the other.

E. 52. There are two bodies, the greater contains 9 times the quantity of the matter in the less, and is moved with a force 48 times greater; the ratio of the velocity of these two bodies is required?

As 9 : 48 :: 1
1

9)48

$5\frac{3}{9}$

Answer, Lesser than the greater as 1 to $5\frac{1}{3}$.

1. Note, In comparing the motion of bodies, if their velocities be equal, the spaces described by them are in direct proportion of the times in which they are described.

2. If the times be equal, then the spaces described will be as their velocities.

3. If the times and the velocities be unequal, the spaces will be in a proportion compounded of the times and velocities.

E. 53. There are two bodies, one of which moves 80 times swifter than the other, but the swifter body has moved but one minute, whereas the other has been in motion two hours: the ratio of the spaces described by these two bodies is required?

First 2 hours×60=120 minutes; then

As 80 : 120 :: 1 : $1\frac{1}{2}$.

Answer, The swifter to the slower, as 1 to $1\frac{1}{2}$.

E. 54.

E. 54. There are two bodies, one whereof has described 50 miles, the other only 5, but the first has moved with 5 times the velocity of the second; what is the ratio then of the times they have been describing those spaces?

First, $50 \div 5 = 10$; then

$$\begin{array}{lcccc} \text{As} & 5 & : & 10 & :: & 1 \\ & & & 1 & & \\ \hline & & & 5)10 & & \\ \hline & & \text{Answer} & 2 & & \end{array}$$

So that the first body hath been in motion double the time of the second.

XIII. RECIPROCAL PROPORTION:

OR, THE

RULE of THREE INVERSE.

RECIPROCAL PROPORTION is, when of four numbers, the third beareth the same proportion to the first, as the second doth to the fourth; consequently, the less the third term is in respect to the first, the greater will the fourth term be in respect to the second.

RULE. Multiply the first and second terms together, and divide their product by the third term, the quotient will be the answer required.

EXAMPLE 1. If 24 men can perform a piece of work in 12 days, how many men can do the same in 36 days?

$$\begin{array}{ccccc} d. & & m. & & d. \\ 12 & : & 24 & :: & 36 \\ & & 12 & & \\ \hline & & & & \\ 36\left\{\begin{array}{l} 6)\,288 \\ \hline 6)\,48 \end{array}\right. & & & & \\ \hline \text{Answer} & & 8 \text{ Men} & & \end{array}$$

E. 2. If 36 days require 8 men to perform a piece of work in, how many men will 12 days require?

$$\begin{array}{ccccc} d. & & m. & & d. \\ 36 & : & 8 & :: & 12 \\ 8 & & & & \\ \hline 12)288 & & & & \\ \hline \end{array}$$

Ans. 24 Men, which proves the above work to be performed right.

Therefore, it is only varying the operations, and you have a proof to all questions of this nature.

E. 3. If 24 men can perform a piece of work in 6 days, how many men can do the same in 36 days?

```
d.       m.       d.
6   :   24   ::   36
         6
      ------
   ( 6) 144
36{    -----
   ( 6) 24
       -----
Answer  4 Men
```

E. 4. *If a board be eight inches*
in breadth, pray declare,
What length of the board will just
make a foot square?

```
in.      in.      in.
12   :   12   ::   8
         12
       -----
     8) 144
       -----
Answer  18 Inches
```

E. 5. How many yards of paper that is three-quarters wide, will hang a room that is 30 yards round, and $3\frac{1}{4}$ yards high?

```
yds.      yds.     qrs.
3 1/4  :   30   ::   3
4          13
---       -----
13       3(390
          -----
Answer    130 Yards
```

E. 6. If I lend a person 300*l.* for a year, how long ought he to lend me 500*l.* to requite me?

```
£.         d.        £.
300   :   365   ::   500
          300
       ----------
   5|00) 1095|00
         -------
Answer    219 Days
```

E. 7. If when the price of a bushel of wheat is 4*s.* 6*d.* the penny-loaf weighs 12 ounces, what must the penny-loaf weigh, when the said bushel is worth only 4*s*?

```
s.  d.      oz.          s.
4   6   :   12    ::     4
2            9           2
----       -----       -----
9        8)108           8
           -----
       oz. 13 4
              16
           -----
          8)64
           -----
            8 drs.
           -----
Answer  13 oz. 8 drs.
```

E. 8. Suppose 275 yards of cloth, which is 5 quarters wide, make coats for 130 men; how many yards of shalloon, of 3 quarters wide, will line the said coats?

```
qrs.    yds.        qrs.
5   :   275    ::     3
          5
      3)1375
        458—1
              4
           ------
           3) 4
           ------
            1—1/3
Answer  458 yds. 1 qr. 1/3 na.
```

E. 9.

E. 9. A garrison consisting of 1500 men, being besieged, have provisions only for three months, but it being necessary they should stand out five months, how many men must depart, that the said provisions may serve that time?

	mo.		*men.*		*mo.*		
First	3	:	1500	::	5	Then from	1500
			3			Take	900
			5)4500			Remains	600 M. to depart
			900 Men to continue				

E. 10. What weight will a man be able to raise, who presses with the force of a hundred and half on the end of an equipoised hand-spike, 100 inches long, which is to meet with a convenient prop exactly $7\frac{1}{2}$ inches above the other end of the machine?

FIGURE 6.

In fig. 6, $ab = 100$ inches, ac $7\frac{1}{2}$, P the power, or $1\frac{1}{2}$ *Cwt.* and w the weight.

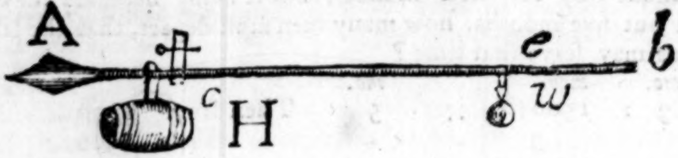
in.		*in.*		*lb.*		*in.*
100	If	$92\frac{1}{2}$	:	168	::	$7\frac{1}{2}$
$7\frac{1}{2}$		2		185		2
$92\frac{1}{2}$		185		840		15
				1344		
				168		
		15		3)31080		
				5)10360		

Answer 2072 lb. or $18\frac{1}{2}$ *cwt.*

A leaver of the first order, equally divided and justly poised, is the balance-beam; to this, if a power be applied at one end, it will always move an equal weight at the other: in like manner, a leaver equally poised, and unequally divided, having a power applied at one end, will move a weight at the other which will be reciprocally proportionable to the distances of those ends from the fulcrum, or point supported; of this kind is the steel-yard.

E. 11. What weight, hung at 70 inches distance from the fulcrum of a steel-yard, will equipoise a hogshead of tobacco weighing $9\frac{1}{2}$ *cwt.* freely suspended at two inches distance on the contrary side?

FIGURE 7.



In the above figure $c\,e = 70$ inches, $A\,c = 2$, H the hogshead, and w the weight.

```
in.   c.     in.
 2 : 9½  ::  70
     4
    ---
    38
      4×7=28
    ---
    152
      7
    ---
   1064
      2
    ---
 7|0)212|8
    ---
   30—28
      16
    ---
   7|0)44|8
    ---
      6—28
       16
    ---
    7|0)44|8
    ---
      6—28
```

Answer 30lb. 6 *oz.* 6 $\frac{28}{70}$ *drs.* the weight required.

E. 12. Suppose the battering ram of Vespasian weighed 100,000lb. and was moved, let us admit, with such a velocity, by strength of hands, as to pass through 20 feet in one second of time, and this was found sufficient to demolish the walls of Jerusalem; with what velocity must a bullet, that weighs but 30lb. be moved in order to do the same execution?

```
  lb.       ft.      lb.
100000  :   20   ::  30
            20
      ------
3|0)200000|0
    ------
    66666—20
    ------ 12
          ---
     3|0)24|0
          ---
           8
```

Answer 66666 feet 8 inc. per sec.

E. 13. A body weighing 200 *lb.* is impelled by such a force, as to send it 100 feet in a second; with what velocity would a body of 8 *lb.* move, if it were impelled by the same force?

```
     lb.        feet.     lb.
As   200   :    100   ::   8
                200
             -------
             8)20000
             -------
```

Answer - - 2500 Feet per second

In comparing the motion of bodies, the ratio, or proportion between their velocities, will be compounded of the direct ratio of the forces wherewith they are moved, and the reciprocal of their quantities of matter they contain.

E. 14.

E. 14. Suppose that, in a room where two men, A and B, are sitting, there is a fire, from which A is three feet, and B six feet distant, it is required to find how much hotter it is at A's seat than B's?

To answer this question, it must first be philosophically considered and learnt, that the effects or degrees of light, heat, and attraction, are reciprocally proportional to the squares of their distances, from the centre whence they are propagated.

A's distance is 3 feet $\times 3 = 9$; and B's distance is 6 feet, which $\times$ by $6 = 36$; then

As 36 : 1 :: 9

$$9)\ \underline{36}$$

Answer 4

So that it is evident A's place is 4 times as hot as B's.

XIV. COMPOUND PROPORTION:

OR,

The RULE of FIVE,

IS so called, from its having five numbers or terms given to find a sixth, which if the proportion is direct, the sixth term must bear such a proportion to the fourth and fifth, as the third bears to the first and second. But if the proportion is inverse, then the sixth term must bear such proportion to the fourth and fifth, as the first bears to the second and third, or as the second bears to the first and third.

The three first terms are a supposition, the two last a demand.

RULE. 1. Let the principal cause of gain, loss, or action, &c. be put in the first place.

2. Let that which denotes time, distance of place, &c. be in the second place, and the remaining one in the third place.

3. Place the other two terms which move the question underneath those of the same name.

4. If the blank, or term sought, fall under the third term, multiply the two first terms together for a divisor, and the three last for a dividend, the quotient arising from them will be the answer, or sixth term.

5. If the blank fall under the first or second term, multiply the third and fourth terms together for a divisor, and the other three for a dividend? the quotient arising from them will be the answer.

PROOF. Is by two statings in the single rules of three.

E. 1.

E. 1. If 6 men can mow 72 acres of grafs in 12 days, how many men can mow 120 acres in 4 days?

m.		d.		a.
6	:	12	:	72
	:	4	:	120

$$\begin{array}{r} 72 \\ 4 \\ \hline 288 \end{array} \qquad \begin{array}{r} 120 \\ 12 \\ \hline 1440 \\ 6 \\ \hline 288)8640(30 \text{ Men, Anfwer} \\ 864 \\ \hline 0 \end{array}$$

Proof. By two ftatings in the fingle rule.

$$\begin{array}{ccccc} d. & & a. & & d. \\ 12 & : & 72 & :: & 4 \end{array} \qquad 24 : 6 :: 120$$

$$\begin{array}{r} 72 \\ 4 \\ \hline 12|288 \\ \hline 24 \end{array} \qquad \begin{array}{r} 120 \\ 6 \\ \hline 24\begin{cases} 4)720 \\ 6)180 \end{cases} \\ \hline \text{Anfwer } \underline{30} \text{ Men as before} \end{array}$$

E. 2. A ufurer put out 120*l.* to receive intereft for the fame; but when it had continued 9 months he took it up, and received for the principal and intereft 125*l.* 8*s.* I demand at what rate per cent. per annum he received?

$$\begin{array}{rcl} \pounds. & & s. \\ 125 & : & 8 \\ 120 & : & 0 \\ \hline \pounds 5 & : & 8s. = 108s. \text{ Intereft.} \end{array}$$

£.		m.		s.
120	:	9	::	108
100	:	12	::	

$$\begin{array}{r} 120 \\ 9 \\ \hline 1080 \end{array} \qquad \begin{array}{r} 108 \\ 100 \\ \hline 10800 \\ 12 \\ \hline 108|0)\ 12960|0\ (120s = 6l. \text{ Anfwer} \\ 108 \\ \hline 216 \\ 216 \\ \hline 0 \end{array}$$

E. 3. Suppose the salary of 6 persons, for 21 weeks, is 120*l.* what will be the salary of 14 persons for 46 weeks?

p. : *w.* : *l.*
6 : 21 : 120
14 : 46 :

21
6
126

120
46
720
480
5520
14
126)77280(613£.
756
168
126
420
378
42
20
126)840(6*s.*
756
84
12
126)1008(8*d.*
1008
0

Answer 613*l.* 6*s.* 8*d.*

E. 4. What is the interest of 259*l.* 13*s.* 5*d.* for 20 weeks, at 5 per cent. per annum?

First 100*l.* = 24000 pence, and 259*l.* 13*s.* 5*d.* = 62321*d.* then

d. : *w.* : £.
24000 : 52 : 5
62321 : 20 : 0

52
24000
208
104
12480|00

62321
5
311605
20
) 62321|00(4 £.
49920
12401
20
12480)248020(19*s.*
12480
123220
112320
10900
12
12480)130800(10*d.*
12480
6000
4
12480)24000($\frac{1}{4}$
12480
11520

Answer 4*l.* 19*s.* 10$\frac{1}{4}$*d.* $\frac{11520}{12480}$

E. 5. If a sack of coals be the allowance of 7 poor people for a week, how many poor belonged to that parish, which, when coals were 36*s.* per chaldron, had 41*l.* to pay in six weeks on that account?

First, 36*s.* ÷ 12 (the sacks in a chaldron) = 3*s.* what the coals cost per week, and 41*l.* = 820*s.* then

p. : *w.* : *s.*
7 : 1 : 3
: 6 : 820

3
6
18

820
7
18)5740(318$\frac{8}{9}$ Poor, Answer
54
34
18
160
144
16

XV.

XV. COMPOUND PROPORTION:

OR, THE

RULE of THREE REPEATED.

ALL questions in the foregoing rule of five, may be resolved by two operations in the rule of three repeated; but there are some questions that cannot be solved by the rule given there, for one stating, yet may be answered by two or more statings in the rule of three repeated.

EXAMPLE 1. A and B are on opposite sides of a wood, 134 toises or fathoms about; they begin to go round it both the same way at the same instant of time; A goes 11 toises in 2 minutes, and B 17 in 3; the question is, how many times will they surround this wood, before the nimbler overtakes the slower?

First, As 2 : 11 :: 3
3
2)33
$16\frac{1}{2}$ A goes, while B goes 17

Then 17
$16\frac{1}{2}$
$\frac{1}{2}$ Toise, B gains of A [in going 17 times round

Again, $\frac{1}{2}$: 17 :: $\frac{1}{2}$: 17 Times round gone by A, and $16\frac{1}{2}$ by B, the Answer.

E. 2. A merchant bought hats in London, which cost him 4*l.* 16*s.* per dozen, and 6*d.* carriage; he is to gain 20 per cent. by the bargain; what must he sell them at a-piece to do it?

	hats.		*£. s. d.*		*hat.*
First,	12	:	4 16 6	::	1

20
96
12
12) 1158
12) 96—6
8 $0\frac{1}{2}$
1 $7\frac{1}{4}$
Answer £. 0 9 $7\frac{3}{4}$ $\frac{20}{100}$

Again, as 100 £. : 20 £. :: 8 s. $0\frac{1}{2}$ d.
12
96
4
386
20
1|00) 77|20
4) 77
12) 19--$\frac{1}{4}$
1 $7\frac{1}{4}$

E. 3. If a leaver, 40 effective inches long, will, by a certain power thrown successively thereon, in 13 hours raise a weight 104 feet, in what

what time will two other leavers, each 18 effective inches long, raise an equal weight 73 feet; the force of strait leavers being in direct proportion of their lengths?

First, $18 \times 2 = 36$ inches, length of the leaver; then

	in.		ft.		in.
As	40	:	104	::	36
			36		
			624		
			312		
		4|0)	374|4		
			$93 - \frac{24}{40} = \frac{3}{5}$		

Again, as $93\frac{3}{5}$ ft. : 13 h. :: 73 ft. : 10 hours, 8 minutes, 20 sec. Answer

E. 4. A weight of $1\frac{1}{2}$ lb. laid on the shoulder of a man, is no greater burthen to him than its absolute weight, or 24 ounces; what difference will he feel between the said weight applied near his elbow, at 12 inches from the shoulder, and in the palm of his hand, 28 inches therefrom; and how much more must his muscles then draw to support it at right angles; that is, have his arm extended right out?

First, as 1 in. : $1\frac{1}{2}$ lb. :: 12 in.

2	3
3	2)36
	18 *lb. wt.* 12 inches from the shoulder

Then, as 1 in. : $1\frac{1}{2}$ lb. :: 28 in.

2	3
3	2)84
	42 *lb. wt.* 28 inches from the shoulder

Consequently, $42 - 18 = 24$ *lb.* the Answer

E. 5. If when port wine is 17 guineas the hogshead, a company of 45 people will spend 20*l.* therein, in a certain time; what is wine a pipe, when 13 persons more will spend 63*l.* in twice the time, drinking with equal moderation?

First, $45 + 13 = 58$ persons, and 17 guineas $= 357$ shillings; then

	p.		£.		p.
As	45	:	20	::	58
					20
	45 {				5) 1160
					9) 232
					£. $25 - \frac{7}{9}$ in the same time

And in twice that time the 58 persons will spend $25\frac{7}{9}$, which multiplied by $2, = 51\frac{5}{9}$ pound's worth, at 17 guineas per head.

Then, As $51\frac{5}{9}$ £. : 17 G. :: 63 £. : 436 s. $2\frac{3}{4}$ d. $\frac{416}{464}$ = 21 £. 16 s. $2\frac{3}{4}$ d. $\frac{416}{464}$ per hogshead, which $\times$ by $2 = 43$*l.* 12*s.* $5\frac{3}{4}$*d.* $\frac{416}{464}$ per pipe, the answer.

E. 6. Suppose a person to travel 152 miles in 7 days, when the days are 12 hours long; how many days will he be in travelling 576 miles, when the days are 16 hours long?

h. d. h.
As 12 : 7 :: 16
12

16 { 4)84
4)21

$5\frac{1}{4}$

m. d. m.
Then, as 152 : $5\frac{1}{4}$ :: 576
4 21
21 576
1152

152) 12096(79
1064
1456
1368
88

4)79
$19-\frac{3}{4}$

Answer 19 days $\frac{3}{4}$ $\frac{88}{152}$

E. 7. *If twenty dogs for thirty groats,*
Go forty weeks to grass;
How many hounds, for sixty crowns,
May winter in that place?

First, from 52 (the weeks in a year) subtract 40, and there remains 12; then 30 groats = 2 crowns ∴

c. dogs. c.
As 2 : 20 :: 60
60

2)1200

600

w. dogs. w.
Again, as 40 : 600 :: 12
40

12) 24000

Answer 2000 dogs

E. 8. My water-tub holds 147 gallons; the pipe usually brings in 14 gallons in 9 minutes; the tap discharges, at a medium, 40 gallons in 31 minutes. Supposing these both carelessly to be left open, and the water to be turned at 2 in the morning; the servant at 5, finding the water running, shuts the tap, and is solicitous to know in what time the tub will be filled after this accident, in case the water continues flowing from the main?

m. gal. m.
As 9 : 14 :: 31
31

14
42

9) 434

$48\frac{2}{9}$ Gallons, fills in 31 minutes

Then, $48\frac{2}{9} - 40 = 8\frac{2}{9}$ gallons in the tub at the end of 31 minutes.

And 2 — 5 = 3 hours, or 180 minutes.

m. gal. m. gal.
Again, as 32 : $8\frac{2}{9}$ :: 180 : $47\frac{23}{31}$ fills in 3 hours.

And $147 - 47\frac{23}{31} = 99\frac{8}{31}$ gallons wants of being full.

gal. m. gal. m. sec.
Also, as 14 : 9 :: $99\frac{8}{31}$: 63 $48\frac{228}{434}$, the tub will be full; which added to 5 o'clock, will give 3 minutes $48\frac{228}{434}$ seconds after 6, the tub will be full.

E. 9.

E. 9. In giving directions for making an Italian chair, the shafts whereof were settled at 11 feet between the axle-tree, whereon the principal bearing is, and the back-band, by means of which the weight is partly thrown upon the horse; a dispute arose where-about on the shafts the centre of the body of this machine should be fixed. The coach-maker advised this to be done at 30 inches from the axle: others were of opinion, that at 24, it would be a sufficient incumbrance to the horse. Now, admitting the two passengers, with their baggage, ordinarily to weigh 2 *Cwt.* a-piece, and the body of the vehicle to be about 70 pounds more; pray what will the beast, in both those cases, be made to bear more than his harness?

First, 30 inches $= 2\frac{1}{2}$ feet, $24 = 2$ ditto, and 4 *Cwt.* 70 *lb.* $=$ 518 *lb.* then $11 - 2\frac{1}{2} = 8\frac{1}{2}$ feet; also $11 - 2 = 9$.

	ft.	*lb.*	*ft.*	*lb.*	
Then, as	11 :	518 ::	$8\frac{1}{2}$:	$400\frac{3}{11}$	force in the former case.
And contra, as	$8\frac{1}{2}$:	$400\frac{3}{11}$ ::	$2\frac{1}{2}$:	$117\frac{8}{11}$	pressure.
Again, as	11 :	518 ::	9 :	$423\frac{9}{11}$	force in the latter case.
Also, as	9 :	$423\frac{9}{11}$ ::	2 :	$94\frac{2}{11}$	pressure.

E. 10. There is an island 73 miles round, and three footmen all start together, to travel the same way about it; A travels 5 miles a day, B 8, and C 10; when will they all come together again?

First, $8 - 5 = 3$ miles B ⎫ gained of A, in one day
And $10 - 5 = 5$ —— C ⎭

Then, as 3 *m.* : 1 *d.* :: 73 *m.*   Again, as 5 *m.* : 1 *d.* :: 73 *m.*

3)73 — $24\frac{1}{3}$ when A and B meet   5)73 — $14\frac{3}{5}$ when A and C meet

So that B nor C can never meet with A, but at the end of these periods, when A and C. will have travelled 219 miles.

Therefore, as $14\frac{3}{5}$ *d.* : 219 *d.* :: $24\frac{1}{3}$ *d.* : 365

$\left.\begin{matrix}219\\365\end{matrix}\right\} \times$ by $\left\{\begin{matrix}24\frac{1}{3}\\14\frac{3}{5}\end{matrix}\right\} = 5329$ days, the 73d time of their general meeting $\therefore 5329 \div 73 = 73$ days, their first general meeting.

For, as $73 : \left\{\begin{matrix}24\frac{1}{3}\\14\frac{3}{5}\end{matrix}\right. :: \left.\begin{matrix}219\\365\end{matrix}\right\} : 73$ days, the Answer.

E. 11. A certain man hires a labourer on this condition, that for every day he worked he should receive 1*s.* but for every day he was idle, he should be mulcted 8*d.* When 390 days were past, neither of them were indebted to one another; how many days did he work, and how many days was he idle?

First, for every day he worked he received 12 pence,
And for every day he played, he paid - 8 pence.

Sum 20

Likewise, as his idle days came to the same money as those he worked, therefore the proportion will be as follows:

	days.		*days.*		*d.*		*days.*		*days.*		*d.*
As	20	:	390	::	8	Again, as	20	:	390	::	12
			8						12		
			2|0)312|0						2|0) 468|0		
			156 Days he worked			Days he played			234		

For 156 days, at 12*d.* per day, comes to the same money as 234 at 8*d.* per day, viz. 7*l.* 16*s.* proof.

E. 12. A man hired a labourer for 40 days, on condition that he should have 20*d.* for every day he worked, and forfeit 10*d.* for every day he idled; at last he received 2*l.* 1*s.* 8*d.* for his labour; how many days did he work, and how many was he idle?

First 2*l.* 1*s.* 8*d.* $=$ 500 pence, and $500 \div 20 = 25$ days' wages; then $40 - 25 = 15$ days more.

For every day he worked he had - - 20 pence
And for every day he played he forfeited 10

Sum 30 pence

$$\begin{array}{lccccl} & d. & & days. & & \\ \text{Then, as} & 30 & : & 15 & :: & \left\{\begin{array}{lcl} 10 & : & 5 \text{ worked} \\ 20 & : & 10 \text{ idle} \end{array}\right. \end{array}$$

Therefore he was idle 10 days, and worked $(5 + 25) = 30$ days.

E. 13. If 248 men, in $5\frac{1}{2}$ days of 11 hours each, dig a trench of 7 degrees of hardness, and $232\frac{1}{2}$ yards long, $3\frac{2}{3}$ wide, and $2\frac{1}{3}$ deep; in how many days of 9 hours, will 24 men dig a trench, of 4 degrees of hardness, and $337\frac{1}{2}$ yards long, $5\frac{3}{5}$ wide, and $3\frac{1}{2}$ deep?

248 *m.*	——	$5\frac{1}{2}$*d.*	——	24 *m.*
11 *h.*	——	——	——	9 *h.*
7 *deg.*	——	——	——	4 *deg.*
$232\frac{1}{2}$ *£.*	——	——	——	$337\frac{1}{2}$ *£.*
$3\frac{2}{3}$ *w.*	——	——	——	$5\frac{3}{5}$ *w.*
$2\frac{1}{3}$ *deep.*	——	——	——	$3\frac{1}{2}$ *deep.*

$$\begin{array}{ll} \text{Dividend} & 248 \cdot 11 \cdot 7 \cdot 5\tfrac{1}{2} \cdot 337\tfrac{1}{2} \cdot 5\tfrac{3}{5} \cdot 3\tfrac{1}{2} \\ \hline \text{Divisor} & 232\tfrac{1}{2} \cdot 3\tfrac{2}{3} \cdot 2\tfrac{1}{3} \cdot 24 \cdot 9 \cdot 4 \end{array} = \text{(by reducing)}$$

$$\frac{248, 11, 7 \cdot \frac{11}{2} \cdot \frac{675}{2} \cdot \frac{7}{2} \cdot \frac{28}{5}}{\frac{465}{2} \cdot \frac{11}{3} \cdot \frac{7}{3} \cdot 24 \cdot 9 \cdot 4}$$

248,

$$= \frac{248.\,11.\,7.\,11.\,675.\,7.\,28}{465.\,11.\,7.\,24.\,9.\,4} \times \frac{2.\,9}{8.\,5} = \frac{248.\,11.\,675.\,7.\,28.\,2}{465.\,24.\,4.\,8.\,5} =$$

$$\frac{31.\,11.\,675.\,7.\,28}{465.\,24.\,2.\,5} = \frac{31.\,11.\,135.\,7.\,7}{93.\,6.\,2.\,5} = \frac{11.\,27.\,7.\,7}{6.\,3.\,2} = \frac{11.\,9.\,7.\,7}{6.\,2}$$

$= \frac{1617}{4} = 404\frac{1}{4}$ days, the anſwer. All this by throwing equal quantities out of both numerator and denominator.

Having given ſuch a variety of examples and explanations in this rule, I ſhall now proceed to that expeditious rule called practice.

XVI. PRACTICE:

SO called from the general uſe it is of to all perſons concerned in trade and buſineſs.

When a queſtion in the rule of three has 1 for the firſt term, it is more expeditiouſly reſolved, by taking ſome aliquot part, or parts of the thing propoſed; by which means many tedious reductions may be avoided.

In order to perform this rule expeditiouſly, let the learner get by heart the following

TABLES of ALIQUOT PARTS.

The even Parts of MONEY.

Of a pound s.	d.		Of a ſhill. d.		Of two ſh. d.		Of a penny qr.	
1	8	= $\frac{1}{12}$	1	= $\frac{1}{12}$	$1\frac{1}{2}$	= $\frac{1}{16}$	1	= $\frac{1}{4}$
2	0	$\frac{1}{10}$	$1\frac{1}{2}$	$\frac{1}{8}$	2	$\frac{1}{12}$	2	$\frac{1}{2}$
2	6	$\frac{1}{8}$	2	$\frac{1}{6}$	3	$\frac{1}{8}$	3	$\frac{3}{4}$
3	4	$\frac{1}{6}$	3	$\frac{1}{4}$	4	$\frac{1}{6}$		
4	0	$\frac{1}{5}$	4	$\frac{1}{3}$	6	$\frac{1}{4}$		
5	0	$\frac{1}{4}$	6	$\frac{1}{2}$	8	$\frac{1}{3}$		
6	8	$\frac{1}{3}$			12	$\frac{1}{2}$		
10	0	$\frac{1}{2}$						

The even Parts of WEIGHT.

Of a ton. Cwt.		Of a hundred. qr.		lb.		Of a quarter of a hundred. lb.	
2	= $\frac{1}{10}$	1	or	28	= $\frac{1}{4}$	$3\frac{1}{2}$	= $\frac{1}{8}$
$2\frac{1}{2}$	$\frac{1}{8}$	2	or	56	$\frac{1}{2}$	4	$\frac{1}{7}$
4	$\frac{1}{5}$	0		16	$\frac{1}{7}$	7	$\frac{1}{4}$
5	$\frac{1}{4}$	0		14	$\frac{1}{8}$	14	$\frac{1}{2}$
10	$\frac{1}{2}$						

CASE 1. When the price is less than a penny.

RULE. Divide by the aliquot parts that are in a penny, then by 12 and 20, which will give the answer.

EXAMPLES.

$\frac{1}{4}$ | $\frac{1}{4}$ | 2067 Yards of tape, at one farthing per yard?

12 | $516\frac{3}{4}$

2|0 | 4|3

Answer £. 2 3 $0\frac{3}{4}$

The price being a farthing a yard, the given quantity is consequently so many farthings, and a farthing being $\frac{1}{4}$ of a penny, and a penny the $\frac{1}{12}$ of a shilling; and a shilling the $\frac{1}{20}$ of a pound; therefore the divisors are 4, 12 and 20.

$\frac{1}{2}$ | $\frac{1}{2}$ | 416 lb. at $\frac{1}{2}d.$ per lb?

12)208

Answer 17*s.* 4*d.*

$\frac{1}{2}$ | $\frac{1}{2}$ | 2067 at $\frac{3}{4}d.$ per lb.

$\frac{1}{4}$ | $\frac{1}{2}$ | 1033 — $\frac{1}{2}$

516 — $\frac{1}{2}$

12)1550

2|0) 12|$9\frac{2}{12}$

£. 6 9 $2\frac{1}{4}$ Answer

The last example may be done by multiplying the given quantity by the farthings in the price, and reducing the product into shillings, pounds, &c. thus:

2067

3

4)6201

12)$1550\frac{1}{4}$

2|0) 12|$9\frac{2}{12}$

Ans. £. 6 9 $2\frac{1}{4}$ the same as above

CASE 2. When the given price is a fraction.

RULE. Multiply the given number or quantity by the numerator, and divide the product by the denominator; the quotient will be the answer in the same denomination with the whole number, of which the price is a part.

EXAMPLES.

What will the carriage of 8372 lb. come to, at $\frac{7}{8}$ of a penny per lb?

8372

7

8)58604

12)$7325\frac{4}{8}$

2|0)61|$0\frac{5}{12}$

Answer £. 30 10 $6\frac{1}{4}$

CASE

CASE 3. When the price is less than a shilling.

RULE. Take the aliquot part or parts that are in a shilling, and add them together, and the sum will be the answer in shillings, &c. which, divided by 20, will give pounds, &c.

EXAMPLES.

12)24501 Pieces, at 1*d*. per piece?
2|0)204|1—9
£. 102 1 9 Answer

1/4 | 1/4 | 1400 Yards, at 1¼*d*. each?
350
12 | 1750
2|0 | 14|5—10
£. 7 5 10 Answer

1½ | 1/8 | 1231 ℔. at 1½ per ℔?
2|0 | 15|3—10½
£. 7 13 10½ Answer

1½ | 1/8 | 1041 ℔. at 1¾*d*. per ℔?
1/4 | 1/6 | 130—1½
21—8¼
2|0 | 15|1 9¾
£. 7 11 9¾ Answer

2 | 1/6 | 736 ℔. at 2*d*. per ℔?
2|0 | 12|2—4=8
£. 6 2 8 Answer

2 | 1/6 | 2408, at 2¼*d*?
1/4 | 1/8 | 401—4
50—2
2|0 | 45|1 6
£. 22 11 6 Answer

2 | 1/6 | 604, at 2½*d*?
1/2 | 1/4 | 100—8
25—0
2|0 | 12|5 8
£. 6 5 8 Answer

2 | 1/6 | 1740, at 2¾*d*?
1/2 | 1/4 | 290
1/4 | 1/2 | 72—4
36
2|0 | 39|8 4
£. 19 18 4 Answer

746 ℔.

3 | $\frac{1}{4}$ | 746 ℔. at 3*d*. per ℔?
2|0 | 18|6—6
£. 9 6 6 Answer

4 | $\frac{1}{3}$ | 961, at 4*d*?
2|0 | 32|0—4
£. 16 0 4 Answer

3 | $\frac{1}{4}$ | 1417, at $3\frac{1}{4}d$.
$\frac{1}{4}$ | $\frac{1}{12}$ | 354—3
29—$6\frac{1}{4}$
2|0 | 38|3 $9\frac{1}{4}$
£. 19 3 $9\frac{1}{4}$ Answer

3 | $\frac{1}{4}$ | 569, at $4\frac{1}{4}d$?
1 | $\frac{1}{3}$ | 142—3
$\frac{1}{4}$ | $\frac{1}{4}$ | 47—5
11 $10\frac{1}{4}$
2|0 | 20|1 $6\frac{1}{4}$
£. 10 1 $6\frac{1}{4}$ Answer

4 | $\frac{1}{3}$ | 5674, at $4\frac{3}{4}d$?
$\frac{1}{2}$ | $\frac{1}{8}$ | 1891—4
$\frac{1}{4}$ | $\frac{1}{2}$ | 236—5
118
2|0 | 224|5 9
£. 112 5 9 Answer

4 | $\frac{1}{3}$ | 814 Ells, at 5*d*. per ell?
1 | $\frac{1}{4}$ | 271—4
67—10
2|0 | 33|9 2
£. 16 19 2 Answer

4 | $\frac{1}{3}$ | 2147, at $5\frac{1}{4}d$?
1 | $\frac{1}{4}$ | 715—8
$\frac{1}{4}$ | $\frac{1}{4}$ | 178—11
44—$8\frac{3}{4}$
2|0 | 93|9 $3\frac{3}{4}$
£. 46 19 $3\frac{3}{4}$ Answer

4 | $\frac{1}{3}$ | 674, at $5\frac{1}{2}d$?
224—8
$1\frac{1}{2}$ | $\frac{1}{8}$ | 84—3
2|0 | 30|8 11
£. 15 8 11 Answer

3 | $\frac{1}{4}$ | 1746, at $5\frac{3}{4}d$?
2 | $\frac{1}{6}$ | 436—6
$\frac{3}{4}$ | $\frac{1}{4}$ | 291 0
109 0
2|0) 83|6 6
£. 41 16 6 Anf.

6 | $\frac{1}{2}$ | 1741, at 6*d*?
2|0) 87|0—6
£. 43 10 6

4 | $\frac{1}{3}$ | 2142, at $6\frac{1}{4}d$?
2 | $\frac{1}{2}$ | 714
$\frac{1}{4}$ | $\frac{1}{8}$ | 357
44—$7\frac{1}{2}$
2|0) 111|5 $7\frac{1}{2}$
£. 55 15 $7\frac{1}{2}$ Anf.

1401,

6 | $\frac{1}{2}$ | 1401, at $6\frac{1}{2}d$?	6 | $\frac{1}{2}$ | 1631, at $7d$?	6 | $\frac{1}{2}$ | 112 lb. at $11d$?
$\frac{1}{2}$ | $\frac{1}{12}$ | 700—6	1 | $\frac{1}{6}$ | 815 — 7	3 | $\frac{1}{2}$ | 56
58—$4\frac{1}{2}$	135—10	2 | $\frac{1}{3}$ | 28
		18—8
2|0) 75|8 $10\frac{1}{2}$	2|0) 95|1 5	2|0) 10|2 8
£. 37 18 $10\frac{1}{2}$ Anf.	£. 47 11 5 Anf.	£. 5 2 8 Anf.

Sometimes the value may be eafily found by reckoning the price of fome even number above what is given, which done, take fome aliquot part for what it is above, and fubtract it from the former.

Take the laft example, thus:

		£.	s.	d.
112 lb. (at 1s.) =	- -	5	12	0
112 lb. (at 1d. is $\frac{1}{12}$) =	Subt.	0	9	4
Anfwer -		£. 5	2	8 the fame as above.

When the price is 2, 3, 4, 6, or 8 pence, you may make ufe of this method; thus, for 2*d*. divide the given quantity by 120; for 3*d*. by 80; for 4*d*. by 60; for 6*d*. by 40, and for 8*d*. by 30, which will give the anfwer in pounds, &c.

CASE 4. When the given price is more than a fhilling, but lefs than two,

RULE. Leave the top line, or given quantity, for fhillings, and take your parts as before for the remaining pence and farthings, which add to the given quantity, and the fum will be the anfwer in fhillings, &c. which divided by 20, will give pounds.

EXAMPLES.

1 | $\frac{1}{12}$ | 261 Ells, at 1*s*. 1*d*. per ell?	3 | $\frac{1}{4}$ | 578 Yards, at 1*s*. 3*d*. per yd?
21—9	144—6
2|0) 28|2 9	2|0) 72|2 6
£. 14 2 9 Anfwer	£. 36 2 6 Anfwer.
4 | $\frac{1}{3}$ | 2140, at 1*s*. 5*d*?	6 | $\frac{1}{2}$ | 1453, at 1*s*. $7\frac{1}{2}d$?
1 | $\frac{1}{4}$ | 713—4	$1\frac{1}{2}$ | $\frac{1}{4}$ | 726—6
178—4	181—$7\frac{1}{2}$
2|0) 303|1 8	2|0) 236|1 $1\frac{1}{2}$
£. 151 11 8 Anfwer	£. 118 1 $1\frac{1}{2}$ Anfwer

CASE 5. If at fuch a profit in the fhilling, you would know what is gained per Cent.

RULE. Divide 100 by the parts that the proposed profit is of a shilling, the quotient or total of the quotient is the answer.

EXAMPLES. At $1\frac{1}{2}d$. profit in the shilling, what is gained per cent?

$1\frac{1}{2}$ | $\frac{1}{8}$ | 100

£. 12 10 Answer

At $1\frac{3}{4}d$. profit in the shilling, what is gained per cent?

$1\frac{1}{2}$ | $\frac{1}{8}$ | 100

$\frac{1}{4}$ | $\frac{1}{6}$ | 12—10
2— 1—8

£. 14 11 8 Answer

CASE 6. When the given price is two shillings,

RULE. Double the unit figure for shillings, the rest are pounds.

EXAMPLES

5168 Ells, at 2s. per ell?

£. 516 16s. Answer

8429 Yards, at 2s. per yard?

£. 842 18s. Answer

8164 ℔. at 2s?

£. 816 8s. Answer

4213 ℔. at 2s?

£. 421 6s. Answer

CASE 7. When the given price is such pence as want an aliquot part of two shillings,

RULE. Work for two shillings, as taught before, and then take for that part, and subtract it from what it comes to at two shillings.

EXAMPLES.

| | 532 ℔. at 16d. per ℔?
8 | $\frac{1}{3}$ | 53 —4
17—14—8

£. 35 9 4 Answer

| | 465, at 22d?
2 | $\frac{1}{12}$ | 46—10
3—17—6

£. 42 12 6 Answer

CASE 8. When the given price is any number of pence above 12, and under 20,

RULE. Multiply by the said pence at once, as taught in the multiplication table, and divide that product by 12 and 20 for the answer.

EXAMPLES.

541 ℔. at 13d?
13

12) 7033

2|0) 58|6—1

£. 29 6 1 Answer

743 ℔. at 14d?
14

12)10402

2|0) 86|6—10

£. 43 6 10 Answer

CASE

CASE 9. When the price consists of any even number of shillings under 20,

RULE. Multiply the given quantity by half the price, doubling the first figure of the product for shillings, and the rest of the product will be pounds.

EXAMPLES.

182 Yards, at 4*s.* per yard?
2
36*l.* 8*s.* Answer

642, at 6*s*?
3
192*l.* 12*s.* Answer

536, at 10*s*?
5
268*l.* 0*s.* Answer

1267, at 18*s*?
9
1140*l.* 6*s.* Answer

CASE 10. When the price is any odd number of shillings under 20,

RULE. Multiply the given quantity by the price, and the product will be the answer in shillings, which divided by 20 will give pounds.

EXAMPLES.

648 lb. at 7*s*?
7
2|0)453|6
226*l.* 16*s.* Answer

312, at 12*s*?
12
2|0)374|4
187*l.* 4*s.* Answer

662, at 17*s*?
17
2|0)1125|4
562*l.* 14*s.* Answer

764, at 19*s*?
19
2|0)1451|6
725*l.* 16*s.* Answer

CASE 11. When the price is shillings, or shillings and pence, or shillings, pence, and farthings, and no even part of a pound,

EXAMPLE.

RULE. Multiply the given quantity for the shillings, and take parts for the pence, &c. and add them together, the same will be the answer in shillings, which divided by 20, will give pounds.

6	$\frac{1}{2}$	2470 lb. at 11*s.* 8$\frac{1}{2}$*d.* per lb?
		11
		27170
2	$\frac{1}{3}$	1235
$\frac{1}{2}$	$\frac{1}{4}$	411 — 8
		102 — 11
		2|0) 2891|9 7
		£. 1445 19 7 Answer

CASE 12. When the price is shillings, or shillings and pence, and they an aliquot part of a pound.

RULE. Divide by the aliquot part, and the quotient will be the answer.

EXAMPLES.

1*s*. 8*d*. is $\frac{1}{12}$)132 Yards, at 1*s*. 8*d*. per yd? — 11*l*. Answer

6*s*. 8*d*. is $\frac{1}{3}$)831 at 6*s*. 8*d*? — 277*l*. Answer

3*s*. 4*d* is $\frac{1}{6}$)736, at 3*s*. 4*d*? — 122*l*. 13*s*. 4*d*. Answer

5*s*. is $\frac{1}{4}$)736, at 5*s*? — 184*l*. Answer

CASE 13. When the price is shillings and pence, and such shillings and pence as are the same figure.

RULE. Multiply the given quantity by the shillings, and take $\frac{1}{12}$ of the product for the pence; the total divided by 20, gives the answer in pounds.

EXAMPLES.

144 ℔. at 6*s*. 6*d*. per ℔?
6
12)864
72
2|0)93|6
£.46 16 Answer

784, at 11*s*. 11*d*?
11
12)8624
718—8
2|0)934|2 8
£.467 2 8 Answer

CASE 14. When the given price is any even number of shillings, and you would know what quantity of any thing may be bought for any even number of pounds.

RULE. Add a cypher to the given pounds, and divide that sum by half the proposed price, and the quotient is pounds.

EXAMPLES. How many pounds of tea may be bought for 86*l*. at 4*s*. per pound?
2)860
Answer 430 ℔.

How many yards, at 14*s*. per yard, may be bought for 50*l*?
7)500
Answer 71 *yds*. 1 *qr*. 3 *na*.

CASE 15. When the price is pounds only.

RULE. Multiply the given quantity by the price, and the product will be the answer.

EXAMPLES. 260 Tons, at 7*l*. per ton?
7
Answer 1820 *l*.

364, at 4*l*?
4
Answer 1456*l*.

405, at 6*l*?
6

2430*l*. Answer

96 *Cwt.* at 26*l*. per *Cwt*?
26

576
192

2496*l*. Answer

CASE 16. When the price is pounds and shillings.

RULE. Multiply the given quantity by the number of pounds, and for the shillings take aliquot parts, and add them together; the sum will be the answer. Or reduce the given price to shillings, by which multiply the given quantity, and divide by 20 will give the answer.

EXAMPLES.

10	$\frac{1}{2}$	164, at 4*s*. 17*s*?
		4
		656
5	$\frac{1}{2}$	82
2	$\frac{1}{5}$	41
		16—8
		£. 795 8 0 Answer

Or thus, 164, at 4*l*. 17*s*?
97   20

1148   97
1476

2[0)1590[8

Answer £. 795 8 0 as before

CASE 17. When the given price be such a fractional part of a pound, shilling, &c. that the numerator is more than a unit,

RULE. Multiply the given quantity by such numerator, or top figure, and divide the product by the denominator, or lower figure, the quotient is the answer in pounds.

EXAMP. 15*s*. $=\frac{3}{4}$) 516, at 15*s*?
3

4)1548

£. 387 Answer

12*s*. 6*d*. $=\frac{5}{8}$)876, at 12*s*. 6*d*.
5

8)4380

£. 547 10 Ans.

CASE 18. When the price is pounds, shillings, and pence, and the shillings and pence be an aliquot part of a pound,

RULE. Multiply the given quantity by the pounds, as in case 16, and take parts for the shillings and pence as in case 12; add them together, and the sum will be the answer.

EXAMPLES.

2*s*. 6*d*. $=\frac{1}{8}$) 247, at 3*l*. 2*s*. 6*d*?
3

741
30—17—6

£. 771 17 6 Answer

6*s*. 8*d*. $=\frac{1}{3}$) 274, at 7*l*. 6*s*. 8*d*?
7

1918
91—6—8

£. 2009 6 8 Answer

CASE

CASE 19. When the price is pounds, ſhillings, pence and farthings, and the ſhillings and pence be not an aliquot part of a pound.

EXAMPLE.

RULE. Reduce the pounds and ſhillings into ſhillings, multiply the given quantity by the ſhillings, and take parts for the pence and farthings as before.

6	$\frac{1}{2}$	267 *Cwt.* at 2*l.* 12*s.* $6\frac{3}{4}d$?	
		52	20
		534	52*s.*
		1335	
$\frac{3}{4}$	$\frac{1}{8}$	133—6	
		16—$8\frac{1}{2}$	
		2|0) 1403|4 $2\frac{1}{2}$	
		£. 701 14 $2\frac{1}{2}$ Anſwer	

Note. When the given quantity doth not exceed 100, proceed as in ſection IX.

CASE 20. When the price hath a fraction annexed,

RULE. Work for the pounds, ſhillings or pence, by the ſhorteſt of the foregoing rules, and value the fraction as directed in caſe 2; or if an aliquot part of the money foregoing, take ſuch part for it.

EXAMPLES.

454 lb. at $19\frac{5}{8}d.$ per lb?
19
8626
$283\frac{3}{4}$
12) $8909\frac{3}{4}$
2|0) 74|2—5
£. 37 2 $5\frac{3}{4}$ Anſwer

454
5
8) 2270
$283\frac{3}{4}$ or $\frac{6}{8}$

1496 French crowns, at $5\frac{3}{9}d.$ per [crown?
3
9) 4488
$498\frac{6}{9}=\frac{1}{3}$

1496
5
7480
$498\frac{1}{3}$
12) $7978\frac{1}{3}$
2|0) 66|4—10
Anſwer £. 33 4 $10\frac{1}{3}$

CASE 21. When the price and quantity given are of ſeveral denominations,

RULE. Multiply the price of one by the quantity given, and take parts for quarters, pounds, &c. add them together, and the ſum will be the anſwer.

EXAMPLES. Sold 8 *Cwt.* of raiſins, at 1*l.* 16*s.* per *Cwt*?

1 *qr.* = $\frac{1}{4}$) 1 16
8
14 8
0 9
Anſwer £. 14 17

In this example the price is multiplied by 8, and divided by 4, the aliquot part for one quarter.

Bought

Bought 7 *Cwt.* 3 *qrs.* 18 *lb.* of sugar, at 17*s.* 6*d.* per *Cwt.* what does it come to?

		£.	s.	d.
2 *qrs.*	$\frac{1}{2}$		17	6
				7
		6	2	6
1 *qr.*	$\frac{1}{2}$		8	9
16 *lb.*	$\frac{1}{7}$		4	$4\frac{1}{2}$
2 *lb.*	$\frac{1}{8}$		2	6
			0	$3\frac{3}{4}$
		£. 6	18	$5\frac{1}{4}$ Anſwer

What is the value of 24 *lb.* of double-refined sugar, at 4*l.* 17*s.* per *Cwt*?

lb.		£.	s.	d.
16	$\frac{1}{7}$	4	17	
8	$\frac{1}{2}$	0	13	$10\frac{1}{4}$
		0	6	11
		£. 1	0	$9\frac{1}{4}$ Anſwer

Bought tobacco at 3*l.* 17*s.* $4\frac{1}{2}$*d.* per hundred weight, what is the worth of 72 *Cwt.* 3 *qrs.* 19 *lb*?

qrs.	*lb.*		£.	s.	d.
2	0	$\frac{1}{2}$	3	17	$4\frac{1}{2}$
					9×8=72
			34	16	$4\frac{1}{2}$
					8
			278	11	0
1	0	$\frac{1}{2}$	1	18	$8\frac{1}{4}$
	16	$\frac{1}{7}$		19	4
	2	$\frac{1}{8}$		11	$0\frac{1}{2}$
	1	$\frac{1}{2}$		1	$4\frac{1}{2}$
				0	$8\frac{1}{4}$
			282	2	$1\frac{1}{2}$ Anſwer

CASE 22. When the given quantity hath a fraction annexed,

RULE. Value the whole number as before, and for the fraction multiply the price by the numerator, and divide that product by the denominator; the quotient is the value of the fraction, and muſt be added to the value of the whole number.

EXAMPLE. What will $358\frac{3}{8}$ ells of holland come to, at 6*s.* 11*d.* per ell?

6	$\frac{1}{2}$	358	
		6	
		2148	
3	$\frac{1}{2}$	179	
$1\frac{1}{2}$	$\frac{1}{2}$	89—6	
$1\frac{1}{2}$	$\frac{1}{3}$	44	9
		14	11
	2|0)	247|6	2
		123 16	2
		2	$7\frac{1}{8}$
		£. 123 18	$9\frac{1}{8}$ Anſwer

	s.	d.
	6	11
		3
8)1	0	9
0	2	$7\frac{1}{8}$

CASE 23. When the given number is not of the ſame name with that on which the price is ſet,

RULE

RULE. Reduce it into the same as taught in reduction, and then find the amount by the shortest of the foregoing rules.

EXAMPLES.

48 *Thous.* at 17*s.* 6*d.* per *Cwt*?

10

6 | $\frac{1}{2}$ | 480

17

8160

240

2|0)840|0

£. 420 0 Answer

1811 *lb.* 11 *oz.* at 2*s.* 10*d.* [per *oz.*

16

28987

8 | $\frac{1}{3}$ | 2898—14 at 2*s.*

2 | $\frac{1}{4}$ | 966 4 8

241 11 2

£. 4106 9 10 Answer

CASE 24. When the given quantity is of less denomination than that on which the price is set,

RULE. Divide the price by the part or parts the quantity given is of; the quotient, or sum of the quotient, is the answer.

EXAMPLES. At 14*l.* 14*s.* per hogshead, what will one gallon of wine come to?

£. s.

63 { 7) 14 14

9) 2 2

£. 0 4 8 Answer

At 8*l.* 12*s.* per *Cwt.* what will 42 *lb.* come to?

lb. £. *s.*

28 | $\frac{1}{4}$ | 8 12

14 | $\frac{1}{2}$ | 2 3

1 1 6

£. 3 4 6 Answer

At 42*s.* per *Cwt.* what will 35 *lb.* come to?

lb. *s.*

28 | $\frac{1}{4}$ | 42

7 | $\frac{1}{4}$ | 10—6

2 $7\frac{1}{2}$

13 $1\frac{1}{2}$ Answer

Note. It often happens in business, that by inverting a question; that is, by calling your price the quantity, and the quantity your price; you may find the answer much easier, and sooner than by the common method.

EXAMPLES. What will 29 yards come to, at 1*s.* 6d. or 18 yards at 2*s.* 5*d*?

s. *d.*

2 5

2

4 10

9

£. 2 3 6 Answer

What will 26*lb.* come to at $6\frac{1}{2}$*d.* per *lb.* or $6\frac{1}{2}$ *lb.* at 2*s.* 2*d*?

s. *d.*

$\frac{1}{2}$) 2 2

6

13 0

1 1

14 1 Answer

PRACTICAL

PRACTICAL METHODS, for casting up particular goods and quantities, some in the wholesale way.

METHOD 1. In goods sold by six-score to the hundred,

RULE. Half the pence in the price of one, is the value of the hundred in pounds.

EXAMPLES.

What will 120 deal boards come to, at $22\frac{1}{2}d.$ per dozen?

2)$22\frac{1}{2}$

£. 11 5 Answer

What will 120 yards of cloth come to, at $9\frac{1}{2}d.$ per yard?

2) $9\frac{1}{2}$

£. 4 15 Answer

METHOD 2. If the quantity given happens to be 240, the pence in the price of one is the value of the whole in pounds.

Note. A farthing must be reckoned as 5*s.*—a half-penny 10*s.*—and three-farthings 15*s.*

EXAMPLES.

What will 240 ℔. come to, at $10\frac{1}{2}d.$ per ℔?

£. 10 10 Answer

What will 240 ℔. of double-refined sugar come to, at $13\frac{1}{4}d.$ per ℔?

£. 13 5 Answer

METHOD 3. If the given quantity be 160, take $\frac{1}{3}$, and multiply that product by 2 for the answer.

EXAMPLES. What will 160 yards come to, at $10\frac{1}{2}d.$ per yard?

3) 10 10

3 10
2

£. 7 0 Answer

What will 160 ells of cloth come to, at $14\frac{3}{4}d.$ per ell?

3) 14 15

4 18 4
2

£. 9 16 8 Answer

METHOD 4. If the given number be 96, multiply $\frac{1}{5}$ of the price by 2.

EXAMPLES. What will 96 yards come to, at $15\frac{1}{2}d.$ per yard?

5) 15 10

3 2
2

£. 6 4 Answer

What will 96 ℔. come to, at $13\frac{1}{4}d.$ per pound?

5) 13 5

2 13
2

£. 5 6 Answer

If the given number be 80, the $\frac{1}{3}$ is the answer.

EXAMPLE. What will 80 yards come to, at $19\frac{3}{4}d.$ per yard?

3) 19 15

£. 6 11 8 Answer

If the given number be 60, the $\frac{1}{4}$ is the anſwer.

EXAMPLE. What will 60 yards come to, at $22\frac{1}{2}d.$ per yard?

£.	s.	
4)22	10	
£. 5	12	6 Anſwer

Note. If the price be given in ſhillings and pence, bring them into pence.

If the given number be 48, take $\frac{1}{5}$ for the anſwer.

EXAMPLE. What will 48 yards come to, at 3s. 6d. per yard?

s.	d.
3	6
12	
5)42	
£. 8	8 Anſwer

If the given number be 40, take $\frac{1}{6}$ for the anſwer.

EXAMPLE. What will 40 ells of holland come to, at 44d. per ell?

£.	s.	d.
6)44		
7	6	8 Anſwer

If 30 is your quantity, take $\frac{1}{8}$ for the anſwer.

EXAMPLE.

s.	d.
8	6
12	
102	

£.	s.
8) 102	0
12	15 Anſwer

GOODS ſold by the THOUSAND.

RULE. Multiply the pence that one coſt by 50, and divide that product by 12; the quotient is the value of a thouſand in pounds.

Note. The reaſon of the above rule is $\frac{1000}{240} = \frac{50}{12}l.$

EXAMPLES. What will a thouſand Dutch tiles come to, at 2d. each?

d.		
2		
50		
12)100		
£. 8	6	8 Anſ.

What will 1000 ells of cloth come to, at 8s. 6d. per ell?

8	6
12	
102	
50	
12) 5100	
£. 425 Anſwer	

GOODS ſold by the great groſs of 144 dozen, are caſt up by the following

RULE. Multiply the price that one coſt by 3, and divide the product by 5, the quotient is the value of the great groſs in pounds.

EXAMPLES.

EXAMPLES. What will a great groſs of buttons coſt, at $7\frac{1}{2}d.$ per dozen?

£.	s.
7	10
	3
5)22	10
4	10 Anſwer

What will a great groſs come to, at $14\frac{1}{2}d.$ per dozen?

£.	s.
14	10
	3
5)43	10
8	14 Anſwer

GOODS ſold by the ſmall groſs of twelve dozen, may be caſt up by the preceding rule; ſeeing there are as many particulars in a ſmall groſs, as dozens in a great one.

EXAMPLES. What will a ſmall groſs of tobacco boxes come to, at $6\frac{1}{2}d.$ per box?

£.	s.
6	10
	3
5)19	10
3	18 Anſwer

What will a ſmall groſs of buttons come to, at $7\frac{1}{2}d.$ per pair?

£.	s.
7	10
	3
5)22	10
4	10 Anſwer

To value the common HUNDRED WEIGHT, of 112 ℔.

RULE. Multiply the price that 1 ℔. coſt by 14, and divide the product by 30, or multiply by 7 and divide by 15; in both caſes the quotient is the anſwer in pounds: $\frac{112}{240} = \frac{14}{30} = \frac{7}{15}$ of a pound.

EXAMPLE. What will 1 hundred weight come to, at $6\frac{1}{2}d.$ per pound?

	£.	s.	
	6	10	
		7	
15 {	5)45	10	
	3) 9	2	
	3	0	8 Anſwer

Note. There are ſeveral other methods and contractions which might have been added, but as they are more curious than uſeful, I ſhall here conclude this rule, and proceed to tare and trett.

XVII. TARE and TRETT.

TARE and TRETT, are allowances made in buying and ſelling commodities that are liable to loſs or waſte.

In this rule there are ſix things to be obſerved, viz. 1, The Groſs weight. 2, Tare. 3, Trett. 4, Suttle. 5, Cloff. 6, Net weight.

1. The groſs weight, is the whole weight of any commodity, be what it will, and that which it is packed up in.

2. Tare is an allowance made for the weight of any box, cask, &c. that contains any commodity.

3. Trett is an abatement of 4 ℔. per 104 ℔. and is the twenty-sixth part allowed for waste, dust, &c. made by the merchant to the buyer.

4. Suttle is the weight of the goods, when only the tare is taken out, and not the trett.

5. Cloff is an allowance of 2 ℔. to the citizens of London, on every draught above 3 *Cwt.* on some sorts of goods, as beaver, galls, madder, &c.

6. Net weight is the weight of any goods, when all allowances are deducted from the gross.

CASE 1. When the net weight of any goods is required, and only tare allowed,

RULE. Subtract the tare from the gross, and the remainder is the net weight.

EXAMPLE 1. Suppose I buy 194 *Cwt.* 2 *qrs.* 18 *lb.* of tobacco, and am allowed 13 *Cwt.* 1 *qr.* 12 *lb.* tare, what is the net weight?

	Cwt.	*qrs.*	*lb.*
Gross -	194	2	18
Tare -	13	1	12
Answer Net	181	1	6

E. 2. In 8 bags of hops, each weighing gross 3 *Cwt.* 2 *qrs.* 15 *lb.* tare 12 *lb.* per bag, what is the net weight?

	lb.
Tare, per bag -	12
Number of bags	8
- Tare - -	3 12

Cwt.	*qrs.*	*lb.*	
3	2	15	
		8	
29	0	8	Gross
	3	12	Tare
Answer Net 28	0	24	

CASE 2. When tare is at so much per *Cwt.* to find the net weight,

RULE. Divide the whole gross, by the said part or parts, that the are is of a hundred weight, and the quotient thence arising will be the tare, which subtracted from the gross, will give the net weight.

E. 3. What is the net weight of 57 *Cwt.* 3 *qrs.* 14 *lb.* gross, tare at 16 *lb.* per *Cwt*?

	Cwt.	*qrs.*	*lb.*	
16 *lb.* = $\frac{1}{7}$)	57	3	14	Gross
	8	1	2	Tare
Answer	49	2	12	Net

CASE 3. When tret is allowed with the tare, to find the net weight,

RULE. Find the tare as before, and subtract it from the gross, the remainder will be the suttle, which divide by 26, and the quotient will be the trett, which subtract from the suttle, the remainder will be the net weight.

E. 4. In 12 *Cwt.* 1 *qr.* 18 *lb.* gross, tare 40 *lb.* trett 4 *lb.* per 104, what is the net weight?

From

		Cwt.	qrs.	lb.	
From the grofs	-	12	1	18	
Deduct 40 *lb.* tare =	-	0	1	12	
	26)	12	0	6	Suttle
			1	$23\frac{3}{4}$	Trett
	Anfwer	11	2	$10\frac{1}{4}$	Net weight

CASE 4. When cloff is allowed with tare, to find the net weight,

RULE. Divide the whole grofs by 168, 2 pounds being the 168th part of 3 hundred weight, or 336 pounds; or divide the number of hundreds by 3, which brings them into 3 hundreds; then 2 pounds being allowed for every 3 hundred, fo as many as it produces, fo many 2 pounds it will allow, which divided by 56 (the double pounds in a hundred weight) the quotient will be the hundreds, and the remainder will be fo many 2 pounds, to which adding what may be allowed for the odd hundreds, quarters, and pounds of the given weight, will make the whole cloff, which fubtract from the grofs, will be the net weight.

E. 5. What will be the net weight of 5647 *Cwt.* 3 *qrs.* 13 *lb.* grofs, allowing for cloff 2 *lb.* for every hundred weight?

	Cwt.	qrs.	lb.	
168)	5647	3	13	Grofs
	33	2	13	Cloff
Anfwer	5614	1	0	Net

CASE 5. When tare, trett, and cloff are allowed with any quantity grofs, to find the net weight,

RULE. For the tare and trett, proceed as in cafe 3, and the remainder, which was called the net there, will be the futtle here, and to find the cloff, proceed as in the laft cafe.

EXAMPLE. What is the net weight of tobacco, weighing

	Cwt.	qr.	lb.	
No. 1,	5	3	10	Grofs
2,	4	1	12	

Tare 7 *lb.* per *Cwt.* trett 4 *lb.* per 104, and cloff 2 *lb.* per *Cwt.*

	Cwt.	qr.	lb.	
Wt. grofs	10	0	22	
7 *lb.* is $\frac{1}{16}$)	10	0	22	Grofs
	0	2	$15\frac{1}{4}$	Tare
26)	9	2	$6\frac{3}{4}$	
Deduct	0	1	13	Trett
	9	0	$21\frac{3}{4}$	2d Suttle
Deduct	0	0	6	Cloff
Net -	9	0	$15\frac{3}{4}$	Anfwer

	Cwt.
	9
	2
3)	18
	6 Cloff

Note. What odd weight remains in finding cloff, is inconfiderable, and need not be noticed.

E. 2.

E. 2. The net proceeds of a hogſhead of ſugar, were 4*l.* 14*s.* 6*d.* the cuſtom and fees 2*l.* 8*s.* 6*d.* freight 22*s.* 8*d.* factorage 4*s.* 9*d.* the groſs weight was 9 *Cwt.* 3 *qrs.* 10 *lb.* tare 1 *lb.* in ten; pray then, how was the ſugar rated in the bill of parcels?

	£.	s.	d.
Nett proceeds	4	14	6
Cuſtom, &c.	2	8	6
Freight	1	2	8
Factorage -	0	4	9
	8	10	5=2045 *d.*

	C.	qrs.	lb.	
$\frac{1}{10}$)	9	3	10	Groſs
	0	3	$26\frac{1}{5}$	Tare
	8	3	$11\frac{4}{5}$	Net

4
35
28
281
71
991
5
4959 Fifths

Cwt. qr. lb.
1 0 0
4
4
28
112
5
560 Fifths

Then, if 4959 : 2045 :: 560
560
122700
10225
4959)1145200(230 12)
9918 19*s.* 2*d.*
15340
14877
4630
4
4959) 18520($\frac{3}{4}$ *qrs.*
14877
3643 Remainder

Anſwer 19*s.* $2\frac{3}{4}$*d.* $\frac{3643}{4959}$

XVIII. SIMPLE INTEREST,

IS the profit allowed for the uſe of any ſum of money for a certain time; the money ſo lent upon intereſt is called the principal; the rate per cent. per annum is the ſum allowed for the uſe of 100*l.* a year, which according to law muſt not exceed 5*l.* the amount is the principal and intereſt added together.

CASE 1. To find the intereſt of any ſum of money for any number of years,

RULE. Multiply the principal by the rate per cent. and cut off two figures towards the right-hand (which is the ſame as dividing by 100) and the figures towards the left are pounds. Then multiply the figures thus cut off to the right-hand by 20, and take in the odd ſhillings (if any) and cut off two figures as before, and the figures on the left-hand are ſhillings; then multiply the remainder by 12, and cut off two figures, and the figures on the left-hand are pence. Again, multiply by 4, and cut off as in the others, and you have the farthings.

Note. The rules for ſimple intereſt ſerve alſo for calculating factorage, brokerage, inſurance, purchaſing of ſtocks, or any thing elſe, that is rated at ſo much per cent.

EXAMPLES.

EXAMPLE 1.

What is the intereſt of 465*l.* for a year, at 5*l.* per cent. per annum?

£.
465 Principal
5 Rate per cent.
23|25
20
5|00
Anſwer 23*l.* 5*s.*

The ſame by practice.

l. *l.*
$5 = \frac{1}{20}$) 465
Anſwer £. 23 5

The above example is worked by two different methods, to ſhew the conciſeneſs of each.

E. 2. What is the intereſt of 212*l.* 9*s.* 1*d.* for a year, at 4 per cent. per annum?

£.	*s.*	*d.*
212	9	1

4
8|48
20
9|69
12
8|29
4
1|16

Anſwer 8*l.* 9*s.* $8\frac{1}{4}$*d.* $\frac{16}{100}$

E. 3. What is the amount of 526*l.* 18*s.* 8*d.* for 6 years, at 4 per cent. per annum?

£.	*s.*	*d.*
526	18	8

4
21|04
20
|98
12
11|94
4
3|76

£.	*s.*	*d.*	
21	0	$11\frac{3}{4}$	Int. for 1 yr.
		6	No. of years
126	5	$10\frac{1}{2}$	Int. for 6 yrs.
526	18	8	Principal
l. 653	4	$6\frac{1}{2}$	Anſwer

CASE 2. When the rate per cent. is $\frac{1}{4}$, $\frac{1}{2}$, or $\frac{3}{4}$, more than the pounds given in the ſaid rate,

RULE. Multiply the principal by the pounds in the rate per cent. then take parts for $\frac{1}{4}$, $\frac{1}{2}$, or $\frac{3}{4}$, from the principal, which add to the product, and the ſum divide by 100 as before.

E. 4. *At ſimple intereſt tell me plain*
What fourteen thouſand pounds will gain,
At 3 pound ten per cent. per annum,
For ſeven years, to pleaſe a grannum?

£.
$\frac{1}{2}$) 14000
$3\frac{1}{2}$
42000 = 3 }
7000 = $\frac{1}{2}$ } $3\frac{1}{2}$ Rate per cent.
490|00 Intereſt for 1 year
7 Number of years
£. 3430 Anſwer

E. 5. What is the amount of 320*l.* for 4 years, at $4\frac{3}{4}$ per cent. per annum?

£.
$\frac{1}{2}$ | $\frac{1}{2}$ | 320
$4\frac{3}{4}$
1280
$\frac{1}{4}$ | $\frac{1}{2}$ | 160
80
15|20
20
4|00

£.	*s.*	
15	4	
	4	
60	16	Intereſt
320	0	Principal
£. 380	16	Amount

COMMISSION

COMMISSION is an allowance generally made from a merchant to his agents or factors abroad, for buying and selling of goods, and is at a certain rate per cent. according to the custom of the country, where the factor or merchant resides in.

E. 6. My factor writes me word, that he has bought goods upon my account, to the value of 649*l.* 10*s.* I desire to know what his commission comes to, at $3\frac{1}{4}$ per cent ?

	£.	s.	d.
$\frac{1}{4}$)	649	10	
		$3\frac{1}{4}$	
	1948	10	
	162	7	6
	21|10	17	6
	20		
	2|17		
	12		
	2|10		

Answer 21*l.* 2*s.* 2*d.* $\frac{10}{100}$

BROKERAGE is an allowance or fee paid to brokers, for assisting others in buying, or disposing of their goods ; and in the city of London they are not to act without a licence from the lord-mayor.

CASE 3. To find the brokage for any sum of money, at any rate under one pound per cent,

RULE. Divide the given sum by 100, and it will give the interest at one pound per cent. which interest you must take parts from, with the rate per cent. and add them together, the sum will be the brokage required.

E. 7. What is the brokage of 682*l.* 10*s.* 6*d.* at 5*s.* 10*d.* per cent?

£.	s.	d.
6|82	10	6
20		
16|50		
12		
6|06		

		£.	s.	d.
5	$\frac{1}{4}$	6	16	6
10	$\frac{1}{6}$	1	14	$1\frac{1}{2}$
			5	$8\frac{1}{4}$
Answer		£. 1	19	$9\frac{3}{4}$

E. 8. Suppose a broker disposes of goods for me to the amount of 864*l.* 12*s.* 4*d.* what does the brokage come to, at 12*s.* 6*d.* per cent ?

£.	s.	d.
8|64	12	4
20		
12|92		
12		
11|08		
4		
|36		

		£.	s.	d.
10	$\frac{1}{2}$	8	12	11
2*s.* 6*d.*	$\frac{1}{4}$	4	6	$5\frac{1}{2}$
		1	1	$7\frac{1}{4}$
Answer		£. 5	8	$0\frac{3}{4}$

INSURANCE is security given by persons who oblige themselves to answer for the loss or damage of ships, houses, goods, &c. by storms, pirates, fire, &c. in consideration of a premium paid by the proprietors of the thing injured.

E. 9.

E. 9. Suppose I make an insurance of goods to the value of 6840*l.* at 2*s.* 6*d.* per cent. per annum, what doth the insurance come to?

£.
68|40
20
8|00

s. d. £. s.
2 6 = $\frac{1}{8}$)68 8

Answer £. 8 11

E. 10. Shipped at Jamaica goods to the value of 2500*l.* upon which I made an insurance at 6$\frac{7}{8}$*l.* per cent. what does it come to?

2500
6
15000
2187 10
171|87 10
20
17|50
12
6|00

2500
7
8)17500
2187—$\frac{4}{8}$=10*s.*

Answer 171*l.* 17*s.* 6*d.*

PURCHASING of STOCKS.

Stocks are the public funds of the nation, the shares of which being transferable from one person to another, occasion that extensive business called stock-jobbing.

RULE. Multiply the sum to be purchased, by the excess of the rate per cent. above 100; the product divide by 100, as before, and the quotient added to the given sum, will give the purchase required.

Note. If under 100 per cent. proceed as in case 2.

E. 11. What is the purchase of 460*l.* South Sea stock, at 116*l.* 4*s.* per cent?

s. £.
4=$\frac{1}{5}$)460
16
7360
92
74|52
20
10|40
12
4|80
4
3|20

£. s. d.
460 0 0 Principal
74 10 4$\frac{3}{4}$ Interest for the excess
534 10 4$\frac{3}{4}$ Answer

E. 12. What is the purchase of 320*l.* bank stock, at 87$\frac{3}{4}$ per cent?

£.
$\frac{1}{2}$|$\frac{1}{2}$ 320
87
2240
2560
27840
$\frac{1}{4}$|$\frac{1}{2}$ 160
80
280|80
20
16|00

Answer 280*l.* 16*s.*

CASE 4. When the interest is for $\frac{3}{4}$, $\frac{1}{2}$, or $\frac{1}{4}$ of a year, or any number of years besides,

RULE. Find the interest for the years, as in case 1; then for $\frac{1}{4}$, $\frac{1}{2}$, or $\frac{3}{4}$, take parts from the interest of 1 year, i. e. for $\frac{1}{4}$, take one-fourth part of the said interest, for $\frac{1}{2}$ take one-half, &c. which, added to the interest for years (if any) the sum will be the required interest.

E. 13. What is the intereſt of 462*l.* for 3 months, at 4 per cent. per annum?

	£.	
	462	
	4	
	18|48	
	20	
	9|60	
	12	
	7|20	

mo.		£.	s.	d.	
3	$\frac{1}{4}$	18	9	7	Int. for 1 year
		4	12	$4\frac{3}{4}$	Anſwer

E. 14. A gentleman dying, left his daughter 604*l.* 17*s.* 6*d.* for her fortune, to be paid her when at age, with intereſt, at 5*l.* per cent. per annum. Now ſhe came of age in 3 years 9 months, after her father's death, what is the amount of her fortune?

5	$\frac{1}{20}$	604	17	6	
6 mo.	$\frac{1}{2}$	30	4	$10\frac{1}{2}$	
				3	
		90	14	$7\frac{1}{2}$=3	years int.
3 mo.	$\frac{1}{2}$	15	2	$5\frac{1}{4}$=6	months
		7	11	$2\frac{1}{2}$=3	months
		604	17	6	Principal
	Anſ. £.	718	5	$9\frac{1}{4}$	Amount

CASE 5. When the intereſt required is for any number of weeks,

RULE. Find the intereſt of the given ſum for a year, and then ſay as 52 weeks are to that intereſt, ſo are the weeks given to the intereſt required.

E. 15. What is the amount of 800*l.* for 13 weeks, at $4\frac{3}{4}$ per cent. per annum?

	£.
$\frac{1}{2}=\frac{1}{2}$	800
	$4\frac{3}{4}$
	3200
$\frac{1}{4}=\frac{1}{2}$	400
	200
	£. 38|00

w.	£.	s.	
13=$\frac{1}{4}$)	38	0	
	9	10	Int. for 13 weeks
	800	0	Principal
Anſwer	809	10	Amount

CASE 6. To find the intereſt of any ſum for any number of days,

RULE. Find the whole year's amount, then ſay, as 365 days are to the year's intereſt, :: ſo are the number of days given : to the intereſt required.

A TABLE

A TABLE of DAYS for any given time less than a year.

Days	January	February	March	April	May	June	July	August	September	October	November	December
1	1	32	60	91	121	152	182	213	244	274	305	335
2	2	33	61	92	122	153	183	214	245	275	306	336
3	3	34	62	93	123	154	184	215	246	276	307	337
4	4	35	63	94	124	155	185	216	247	277	308	338
5	5	36	64	95	125	156	186	217	248	278	309	339
6	6	37	65	96	126	157	187	218	249	279	310	340
7	7	38	66	97	127	158	188	219	250	280	311	341
8	8	39	67	98	128	159	189	220	251	281	312	342
9	9	40	68	99	129	160	190	221	252	282	313	343
10	10	41	69	100	130	161	191	222	253	283	314	344
11	11	42	70	101	131	162	192	223	254	284	315	345
12	12	43	71	102	132	263	193	224	255	285	316	346
13	13	44	72	103	133	164	194	225	256	286	317	347
14	14	45	73	104	134	165	195	226	257	287	318	348
15	15	46	74	105	135	166	196	227	258	288	319	349
16	16	47	75	106	136	167	197	228	259	289	320	350
17	17	48	76	107	137	168	198	229	260	290	321	351
18	18	49	77	108	138	169	199	230	261	291	322	352
19	19	50	78	109	139	170	200	231	262	292	323	353
20	20	51	79	110	140	171	201	232	263	293	324	354
21	21	52	80	111	141	172	202	233	264	294	325	355
22	22	53	81	112	142	173	203	234	265	295	326	356
23	23	54	82	113	143	174	204	235	266	296	327	357
24	24	55	83	114	144	175	205	236	267	297	328	358
25	25	56	84	115	145	176	206	237	268	298	329	359
26	26	57	85	116	146	177	207	238	269	299	330	360
27	27	58	86	117	147	178	208	239	270	300	331	361
28	28	59	87	118	148	179	209	240	271	301	332	362
29	29	60	88	119	149	180	210	241	272	302	333	363
30	30		89	120	150	181	211	242	273	303	334	364
31	31		90		151		212	243		304		365

The USE of the TABLE.

First, to know the number of days from the beginning of the year, to any given day of any month.

This is obtained by inspection only; thus, from January the 1st to July the 14th, is 195 days; to September the 26th is 269 days, &c.

Secondly, to know what is the number of days from any given day of any month, to the end of the year;

Suppose June the 4th; then from	- - - - - -	365 days
Subtract the number answering to June 4th	- - -	155
There remains the number of days sought, viz.	-	210

Thirdly, to find the number of days between the given day of any one month, and any given day of any other month, in the same year.

For instance, to know how many days there are between May the 8th, and September the 4th;

Thus,

Thus, from the number to September the 4th - - 247
Subtract that answering to May 8th - - - - - - 128

The remainder is the number of days sought - - - 119

Fourthly, to find the number of days from any given day of any month in one year, to any given day of any month in the next year.

How many days is it from September the 7th in one year, to April the 19th in the next?

From the days of a whole year - - - 365
Subtract the number to September the 7th - - 250

Remains the number to the end of the year - 115
To which add the number to April 19th - - 109

The sum is the number of days required, viz. - 224

And thus is the number of days readily found for any interval of time given, in the same year, compleatly; or which is part of one, or part of another year.

E. 16. What is the interest of 399*l.* 13*s.* 4*d.* for 4 days, at 5 per cent. per annum?

£.	s.	d.
399	13	4
		5
19|98	6	8
20		
19|66		
12		
8|00		

Days. £. s. d. days.
If 365 : 19 19 8 :: 4
20
399
12
4796
4
12)
365)19184(52
1825
934 4*s.* $4\frac{1}{2}$*d.* Answer
730
204
4
365(816(2 *qrs.*
730
86

E. 17. What is the amount of 340*l.* 10*s.* from January 1st, 1781, to July 18th following, at 5 per cent. per annum?

First, by the table, from January 1st, to July 18th, there are 199 days.

£. s. Days. £. s. d. days.
340 10 If 365 : 17 0 6 :: 199
5 20
17\|02 10 340
20 12
\|50 4086×199=
12 813114
6\|00

12)
365)813114 (2227
730 2\|0)18\|5 7
831
730 9 5 $7\frac{1}{2}$ Interest
1011 340 10 0 Principal
730 £349 15 $7\frac{1}{2}$ Amount
2814
2555
259
4
365)1036(2 *qrs.*
730
306

CASE

CASE 7. When the amount, time, and rate per cent. are given, to find the principal,

RULES. 1. Say, as the amount of 100*l.* at the rate and time given, is to 100*l.* ſo is the amount given to the principal required.

Note. The examples in this, and the two following caſes, may be ſolved by the rule in compound proportion.

E. 18. What principal being put out to intereſt for 8 years, at 5 per cent. per ann. will amount to 429*l*?

5 Rate per cent.
8 Time
40 Intereſt
100 Principal
140 Amount

£. £. £.
If 140 : 100 :: 429
100
140)42900(306
420
900
840
60
20
140) 1200(8
112
80
12
140) 960(6
840
120
4
140)480(3
420
60

Anſwer 306*l.* 8*s.* $6\frac{3}{4}$*d.* $\frac{60}{140}$

E. 19. What principal, being put to intereſt for $9\frac{1}{2}$ years, at $4\frac{1}{2}$ per cent. per annum, will amount to 856*l.* 10*s*?

£. s.
$\frac{1}{2}=\frac{1}{2}$) 4 10 Rate per cent.
$9\frac{1}{2}$
40 10
2 5
42 15 Intereſt
100 0
142 15 Amount

£. s. £. £. s.
If 142 15 : 100 :: 856 10
20 20
2855 17130
100
2855)1713000(600*l.*
17130
000

Anſwer 600*l.*

CASE 8. When the principal, rate per cent. and the amount are given, to find the time,

RULE. Say, as the intereſt of the principal for a year is to one year, :: ſo is the whole intereſt to the time required.

E. 20.

E. 20. In what time will 132*l.* amount to 171*l.* 12*s.* at 5 per cent. per annum ?

132	Principal	£.	s.	
5	Rate per cent.	171	12	Amount
		132	0	Principal
6|60				
20		39	12	Interest
12|00				

	£.	s.		Year.		£.	s.
If	6	12	:	1	::	39	12
	20					20	
	132					132)792(6 Years, the Answer	
						792	

CASE 9. When the principal, amount, and time are given, to find the rate per cent.

RULE. 1. Say, as the principal : is to the interest for the whole time, :: so is 100*l.* to the interest for the same time.

2. Divide that interest by the given time, and the quotient will be the rate per cent. required.

E. 21. At what rate per cent. per annum, will 528*l.* amount to 686*l.* 8*s.* in 6 years ?

£.	s.	
686	8	Amount
528	0	Principal
158	8	Interest

2|0)60|0

Time 6) 30

Answer 5 Rate per cent.

	£.		£.	s.		£.
If	528	:	158	8	::	100
			20			
			3168			
			100			
			528)316800(60|0 Shill.			
			3168			
			000			

PROMISCUOUS QUESTIONS.

Q. 1. Lent at Christmas 1771, the sum of 5000*l.* at $4\frac{1}{2}$ per cent. after which time I lent several sums at the same rate, and drew upon the borrower as business required ; viz. on Lady-day 1772, I drew for 185 guineas ; on Midsummer-day following I lent 500 moidores, and drew for 700*l.* and on Michaelmas-day in the same year, I lent 569*l.* 17*s.*—I demand what cash the borrower owed me at that time ?

First,

First, 5000
$4\frac{1}{2}$

20000
2500

4)225|00 Interest for 1 yr.

£. 56 5*s*. Interest due to Lady-day, which is a quarter; and by proceeding in this manner with each new principal, you will gain the respective interests; as in the work following:

		£.	s.	d.	
1771.	Lent at Christmas —	5000	0	0	at $4\frac{1}{2}$ per cent.
1772.	Interest due at Lady-day	56	5	0	
	Amount — — —	5056	5	0	
	Drew out — — —	194	5	0	= 185 Guineas
	Remains — —	4862	0	0	New principal
	Interest of the same to Midf.	54	13	$11\frac{1}{4}$	
	Amount — — —	4916	13	$11\frac{1}{4}$	
	Paid 500 moidores =	675	0	0	
	Sum — — —	5591	13	$11\frac{1}{4}$	
	Drew out — —	700	0	0	
	Remains — — —	4891	13	$11\frac{1}{4}$	New principal
	Interest to Michaelmas	55	0	$7\frac{1}{2}$	
	Amount — — —	4946	14	$6\frac{3}{4}$	
	Paid in part — — —	569	17	0	
	Answer £.	5516	11	$6\frac{3}{4}$	Cash due to me.

Q. 2. Lent to John Jemeson, per bill, dated 18th of Jan. 1771, payable one day after date, 878*l*. 19*s*. 10*d*. which I received back in the following partial payments, viz. on the 27th of Feb. 57*l*. 15*s*. 7*d*. on the 18th of March 37*l*. 14*s*. on the 29th of April 34*l*. 11*s*. on the 12th of May 136*l*. 15*s*. 7*d*. on the 19th of June 67*l*. 13*s*. 4*d*. on the 15th of July 15 guineas and 6*d*. on the 25th ditto 111*l*. 11*s*. 11*d*. on the 3d of October 78*l*. 7*s*. 4*d*. on the 19th of November 100*l*. on the 23d ditto 100*l*. and on the 30th of December received the ballance of the principal; how much interest ought I to claim, at 5 per cent?

Note. The respective products are found by multiplying each principal by the number of days it was employed; see the following operation.

To

	£.	s.	d.	Days	Products £.	s.	d.
To a bill payable 1 day after date	878	19	10	40	35159	13	4
Feb. 27, Received in part	57	15	7				
Balance	821	4	3	19	15603	0	9
Mar. 18, Received	37	14	0				
Balance	783	10	3	42	32907	10	6
Apr. 29, Received	34	11	0				
Balance	748	19	3	13	9736	10	3
May 12, Received	136	15	7				
Balance	612	3	8	38	23262	19	4
June 12, Received	67	13	4				
Balance	544	10	4	26	14157	8	8
July 15, Received	15	15	6				
Balance	528	14	10	10	5287	8	4
25, Received	111	11	11				
Balance	417	2	11	70	29200	4	2
Oct. 3, Received	78	7	4				
Balance	338	15	7	47	15922	12	5
Nov. 19, Received	100	0	0				
Balance	238	15	7	4	955	2	4
23, Received	100	0	0				
Dec. 30, Rd. in full of the prin.	138	15	7	37	5134	16	7
	138	15	7				
The total sum of the products					187327	6	8

Then,

```
          £.      s. d. l.
73|00)1873|27  6 8(25
      146
      ---
       413
       365
       ---
       4827
         20
       ----
73|00)965|46(13s.
      73
      ---
      235
      219
      ---
      1646
        12
      ----
73|00)197|60(2d.
      146
      ---
      5160
         4
      ----
73|00)206|40(2qrs.
      146
      ---
       604
```

Answer, 25*l.* 13*s.* 2½*d.* $\frac{604}{730}$ the interest required

Note. The reason of dividing by 73 in the above operation, is this; as 100 : 365 :: 5, or any other rate to the fourth term.— Or, as 100 : 73 :: 1, that is $\frac{365}{5} = 73$, and $\frac{5}{5} = 1$. Hence the second and third terms will always admit of the same abbreviations.

Q. 3. June 23d, 1745, bought 900*l.* of New South-sea annuities at 111⅝ per cent. viz. the day before the closing the books, the brokerage whereof is always 2*s.* 6*d.* per cent. on the capital, whether you buy or sell; the Midsummer dividend 2 per cent. became due and payable on the 10th of August following, by which time the rebellion growing considerable in the North, the said annuities were down at 92½ per cent. In the general alarm, sold 400*l.* capital at that price; but continued the remainder, till a second, third, fourth, and fifth dividend,

vidend, as before, came due; and on opening the books on the 10th of August, 1747, sold out at 102⅝ per cent. Now reckoning I might have made 5 per cent. of my money, had I kept it out of the stocks, how stood this article in point of profit and loss?

	£.	s.	d.
First, 900*l*. at 111⅜ per cent. = — —	1002	7	6
Brokerage of ditto, at 2*s*. 6*d*. per cent. = --	1	2	6
	£. 1003	10	0
Midsummer dividend, at 2 per cent. —	18	0	0
	985	10	0
Interest of 1003*l*. 10*s*. for 49 days. at 5 per cent.	6	14	8
Brokerage of 400*l*. at 2*s*. 6*d*. per cent. — —	0	10	0
	992	14	8
Sold 400*l*. at 92½ per cent. — — —	370	0	0
	622	14	8
Interest for half a year, due February 10, 1746	15	11	4¼
	638	6	0¼
Dividend received at that time — — —	10	0	0
	628	6	0¼
Interest due to August 10th —— ——	15	14	1¾
	644	0	2
Dividend received at that time —— —	10	0	0
	634	0	2
Interest due to February, 1747 — —	15	17	0
	649	17	2
Dividend received then —— — —	10	0	0
	639	17	2
Interest to the 10th of August —— — —	15	19	11
	655	17	1
Midsummer dividend received August 10th	10	0	0
	645	17	1
Sold off 500*l*. at 102⅝ per cent. — —	512	2	6
	133	14	7
Brokerage —— — — — — — — —	0	12	6
Answer, Lost in the whole	£. 133	2	1

XIX. COMPOUND INTEREST,

IS that which arises from any principal, and its interest put together, as that interest becomes due but not paid, the same interest is allowed upon that interest unpaid, so it becomes part of the principal, for which reason it is called interest upon interest, or compound interest.

RULE. 1. Find the amount of the given sum by simple interest for the first year, which is the principal for the second year; then find the amount of that principal for the second year, and that is the principal for the third year; and so on for any number of years.

 2. Subtract

2. Subtract the given principal from the last amount, and the remainder is the compound interest required.

EXAMPLES. What is the compound interest of 900*l.* forborne 3 years, at 5 per cent. per annum?

£.	
900	= 1st year's principal
5	
45|00	

£.	
900	Principal
45	Interest
945	= 2d year's principal
5	
47|25	
20	
5|00	

£.	s.	
945	0	Principal
47	5	
992	5	= 3d year's principal
	5	
49|61	5	
20		
12|25		
12		
3|00		

	£.	s.	d.	
	992	5	0	
	49	12	3	
	1041	17	3	Amount
	900	0	0	Principal
Answer	141	17	3	Comp. Int.

The preceding example performed otherwise, thus:

	£.	s.	d.	
$5=\frac{1}{20}$)	900	0	0	= 1st year's pr.
	45	0	0	= Interest
$5=\frac{1}{20}$)	945	0	0	= 2d year's pr.
	47	5	0	= Interest
$5=\frac{1}{20}$)	992	5	0	= 3d year's pr.
	49	12	3	= Interest
	1041	17	3	= Amount
	900	0	0	= Principal
Answer	141	17	3	Comp. interest, as before.

The foregoing methods being rather tedious (though generally taught in schools) I have thought proper to omit giving any more examples, till I come to treat on decimals, where the same may be more conveniently and expeditiously performed.

XX. REBATE or DISCOUNT,

IS the satisfying any sum of money due at some time to come, by paying so much present money as being put to interest would amount to the given sum, in the same space of time.

RULE. 1. Find the interest for 100*l.* for the time given, and rate per cent, which interest add to 100*l.*

2. Then say, as that sum is to the interest of 100*l.* so is the debt, or sum proposed, to the rebate, or present worth required. Or when the present worth is subtracted from the given sum, the remainder is the rebate required.

EXAMPLES.

EXAMPLE 1. What is the rebate of 210*l.* for 7 months 6 days, at 5 per cent. per annum?

mo. da.		£.	s.	
6 0	$\frac{1}{2}$	5	0	Per cent.
1 6	$\frac{1}{5}$	2	10	
		0	10	
		3	0	Intereft
		100	0	Principal
		103	0	

If 103 : 3 :: 210
210
103)630(6
618
12
20
103)240(2
206
34
12
103)408(3
309
99
4
103)396(3
309
87

Anfwer 6*l.* 2*s.* $3\frac{3}{4}$*d.* $\frac{87}{103}$.

E. 2. What is the prefent worth of 100*l.* for 12 months, at 6 per cent? Firft, 100 + 6 = 106*l.* amount of 100*l.* for a year.

Then If 106 £. : 100 £. :: 100 £.
× 100

106)10000(94*l.* 6*s.* $9\frac{1}{2}$*d.* $\frac{2}{53}$ the Anfwer

E. 3. Sold goods to the value of 73*l.* 5*s.* to be paid in a year's time; what muft be difcounted for the prefent payment, if rebate be allowed at $4\frac{1}{2}$ per cent?

£.
$\frac{1}{2}$) 100
$4\frac{1}{2}$
400
50
4|50
20
10|00

£.	s.	
100	0	Prin.
4	10	Int.

If 104 10 : 4 10 :: 73 5
20 20 20
2090 90 1465
90
2|0
209|0)13185|0(6|3
1254
645
627
18
12
209)216(1
209
7

*l.*3 3*s.* 1*d.* $\frac{7}{209}$ Anf.

The

The common method of diſcounting bills is done by the intereſt of the whole ſum for ſo long a time: but ſuch ſum cannot be eſteemed a principal, nor is it in full value, till the time of payment is expired; therefore, leſs intereſt muſt be required according to the true rules of diſcount.

E. 4. What difference is there between the intereſt of 500*l.* at 5 per cent. per annum, for 12 years, and the diſcount of the ſame ſum, at the ſame rate, and for the ſame time?

£. *l.*
$5 = \frac{1}{20}$) 500 Principal

25 Int. of 500*l.* for 1 year
12 Number of years

300 Intereſt for 12 years

Then $5 \times 12 = 60$*l.* int. of 100*l.* for 12 yrs.
If 160 : 60 :: 500
500

16|0)3000|0(187*l.* 10*s.* Diſc.
16
140
128
120
112
8
20
16(160(10
16
0

	£.	*s.*
	300	0 Intereſt
	187	10 Diſcount
Anſwer	112	10 Difference

Note. By the preceding examples it is evident, he who allows intereſt for diſcount wrongs himſelf conſiderably; for ſo much money ought to be paid, as at intereſt would amount to the ſum due in the time propoſed.

E. 5. What ready money will diſcharge a debt of 134*l.* due two years, three quarters, diſcount at $4\frac{3}{8}$ per cent. per annum?

20
3
8|60
7 6 = $\frac{3}{8}$*l.*

mo.	£.	*s.*	*d.*
6 | $\frac{1}{2}$	4	7	6 = $4\frac{3}{8}$*l.*
			2
	8	15	0
3 | $\frac{1}{2}$	2	3	9
	1	1	$10\frac{1}{2}$
Intereſt	12	0	$7\frac{1}{2}$
Prin.	100	0	0
Amount	112	0	$7\frac{1}{2}$

£. *s.* *d.* £. £.
If 112 0 $7\frac{1}{2}$: 100 :: 134
20 20
2240 2680
12 12
26887 32160
4 4
107550 128640
100

10755|0) 1286400|0(119*l.*
12*s.* $2\frac{1}{4}$

Anſwer 119*l.* 12*s.* $2\frac{1}{4}$*d.* $\frac{1525}{10755}$

E. 6.

E. 6. What is the present value of a 10*l.* bill due 4 months hence, discounted at 4 per cent?

```
mo.        £.  s.  d.
4 = 1/3)   4   0   0
           1   6   8 Interest
         100   0   0 Principal
         -------------
         101   6   8

   £.  s. d.         l.      l.
If 101  6  8   :   100  ::  10
    20                      20
   ----                    ----
  2026                     200
    12                      12
 -------                 ------
 24320                    2400
                           100
          2432|0) 24000|0(9
                  21888
                  -----
                   2112
                     20
                  -----
          2432)42240(17
               2432
               -----
               17920
               17024
               -----
                 896
                  12
               -----
          2432)10752(4
                9728
               -----
                1024
                   4
               -----
          2432)4096(1
```

Answer 9*l.* 17*s.* 4¼*d.* $\frac{1664}{2432}$

E. 7. What is the present money and discount of one pound for one year, at 5 per cent. per annum?

```
      £.
     100
       5
     ---
     500
      l.        l.       l.
If   105   :   100  ::   1
                20
              -----
         105) 2000(19
               105
               ---
               950
               945
               ---
                 5
                12
               ---
                60
                 4
               ---
           105)240(2
               210
               ---
  £. s. d.      30
From 1  0  0
Take 0 19  0½ 30/105 present money
     -------------
     0  0 11¼ 75/105 reb. or disc.
```

By the last example it appears that one pound in a year is decreased to 19*s.* 0½*d.* $\frac{30}{105}$ at 5*l.* per cent.

XXI. EQUATION of PAYMENTS,

IS when several debts are payable at different times, but is mutually agreed upon between debtor and creditor, that all those several sums be paid at once, without loss to debtor or creditor.

RULE. Multiply the sum of each particular payment by the time it is to continue in the hands of the debtor; add these products together, and divide the sum by the whole debt, the quotient is the equated time for the payment of the whole debt.

REMARK. The above rule is not exactly true, though it may serve in common business; but to find the just mean or equated time of payment, you must first find out the present payment of every particular sum in the question, payable at a time to come, by rebating at the rate of interest agreed on; then find in what time the sum of those present worths will be augmented to the total of all the particular sums payable

payable at times to come, according to the first agreement; so shall the the time found out be the mean for paying the whole debt.

EXAMPLE 1. A owes B 140*l.* which by agreement was to be paid as follows, viz. 50*l.* at 2 months, and 90*l.* at 6 months; but they agree that the whole should be paid at once; required the equated time of payment?

l.		*mo.*		*l.*
50	×	2	=	100
90	×	6	=	540
140				

14|0) 64|0(4 Mo. 2 weeks, 2 days, Answer
56
8

E. 2. James owes Thomas 80*l.* which is to be paid as follows, viz. 40*l.* at 3 months, and 40*l.* at 7 months, but they agree to reduce the whole to one payment; query, the equated time?

l.		*mo.*		*l.*
40	×	3	=	120
40	×	7	=	280

8|0)40|0

Answer 5 Months

E. 3. C owes D 600*l.* whereof 200*l.* is to be paid at 3 months, 150*l.* at 4 months, and the rest at 6 months; but they agree the whole should be paid at once, required the time?

l.		*mo.*		*prod.*
200	×	3	=	600
150	×	4	=	600
250	×	6	=	1500
6|00				6|00) 27|00

Answer 4 Mo. 15 days.

E. 4. B owes C a certain sum, which is to be discharged thus, viz. $\frac{1}{4}$ present, $\frac{1}{4}$ at 4 months, $\frac{1}{4}$ at 5 months, and the rest at 6 months; what is the equated time for the whole?

In this example the debt is to be paid at 4 equal payments, and $\frac{1}{4}$ being paid down, there remains $\frac{3}{4}$ to be paid at three equal payments; consequently, the sum of the different times that each payment is to be made, being divided by 3, will give the answer, thus:

4
5
6
3)15
Ans. 5 Months.

E. 5. A debt is to be discharged in the following manner, viz. $\frac{1}{2}$ at 3 months, $\frac{1}{3}$ in 4 months, and $\frac{1}{6}$ in 9 months; but they afterwards agree to have but one payment of the whole; the equated time is required?

Suppose 120*l.* to be the sum owed.

Then,	£.		£.		*mo.*		*Prod.*
$\frac{1}{2}$			60	×	3	=	180
$\frac{1}{3}$	120	=	40	×	4	=	160
$\frac{1}{6}$			20	×	9	=	180

12|0)52|0

mo. 4—4

30 Days in one month

Answer, 4 months, 10 days. 12)120

10 Days

In

In examples of the above nature, any number may be taken at pleasure, that is divisable into the proposed parts, without a remainder.

SCHOLIUM. I might introduce various other rules by different authors who have endeavoured to make improvements on this common method, but room will not permit them; and the common method being more adapted to practice, and is near enough the truth in common affairs. Mr. Malcolm's rule is the only true one, which is as follows.

Put d for the first payment, t the distance of its term of payment; D the last payable debt, and T the distance of its term, and r the rate of one year's interest for 1*l.* and $x =$ the distance of the equated time. Then by proceeding according to the principles of simple interest, we have $T+t+\frac{D+d}{dr}$ the first number found.

And $\frac{DT+dt}{ar}+Tt$ the second number found, which two numbers are called a and s, then $ax-x^2=s$, whence $x=\frac{a\pm\sqrt{a^2-4s}}{2}$ the present rule, or equated time for any two payments.

XXII. SINGLE FELLOWSHIP:

OR,

FELLOWSHIP WITHOUT TIME,

IS when two or more persons join their stocks and trade together: to determine how much gain or loss is due to every partner concerned, by having the whole gain or loss, and their particular stocks given.

RULE. As the sum of their several stocks to the gain or loss, so is each person's share in the stock, to his share in the gain or loss.

PROOF. Add all the shares together, and that sum (if right) will be equal to the whole gain or loss.

EXAMPLE 1. Two persons, A and B, join in partnership; A lays in 40*l.* B 80*l.* and they gain 50*l.* what is each man's share of the said gain?

	l.		*l.*		*l.*		*l.*		*l.*	*s.*	*d.*		
A's stock	40	If	120	:	50	::	40	:	16	13	4	A's	Gain
B's ——	80	If	120	:	50	::	80	:	33	6	8	B's	
	120						Proof		£. 50	0	0		

E. 2. Three persons, C, D, and E, trade together, and make a joint stock of 824*l.* and in three years time they gained as much, and 70*l.* over; C's stock was 320*l.* D's 340*l.* I demand E's stock, and what each person gained by trading?

First,

First, $320 + 340 = 660l.$ C and D put in; then from $824l.$ take $660l.$ remains $164l.$ E's stock; and $824 + 70 = 894l.$ their whole gain; then,

	£.	£.		£.		£.	s.	d.	Rem.		
As	824	: 894	::	320	:	347	3	$8\frac{1}{4}$	72	C's	Gain
				340	:	368	17	8	128	D's	
				164	:	177	18	$7\frac{1}{2}$	624	E's	
					Proof	894	0	0			

E. 3. *Some time ago, as people say,*
A debt four men agreed to pay,
Of just one pound each share was fix'd,
One-third, one-fourth, one-fifth, one-sixth.
Then, Tyro, *what was each man's due*
Of cash to pay? Pray tell me true.

First, the fractions in this example, viz. $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, parts of 20 shillings, when added together make just $19s. = 228d.$ and each respective part, viz. $\frac{1}{3} = 80d.$ $\frac{1}{4} = 60d.$ $\frac{1}{5} = 48d.$ $\frac{1}{6} = 40d.$ then say,

```
   d.      s.      d.
As 228  :  20  ::  80
           80
  228)1600(7s.
      1596
         4
        12
        48
         4
       192 Rem.
```

```
   d.      s.      d.
As 228  :  20  ::  60
           60
  228)1200(5s.
      1140
        60
        12
  228)720(3d.
      684
       36
        4
      144 Rem.
```

```
   d.      s.      d.
As 228  :  20  ::  48
                   20
            228)960)4s.
                912
                 48
                 12
            228)576(2d.
                456
                120
                  4
            228)480(1/2 qr.
                456
                 24 Rem.
```

```
   d.      s.      d.
As 228  :  20  ::  40
           40
  228)800(3s.
      684
      116
       12
  228)1392(6d.
      1368
        24
         4
        96 Rem.
```

The remainders in the above operations being 192, 144, 24, and 96, which added together, thus, $192 + 144 + 24 + 96 = 456$, which, divided

divided by the sum of their money, paid the quotient = 2, which is added to the farthings for the proof, thus :

		£.	s.	d.	
Answer	1st Man's share	0	7	0	$\frac{192}{228}$
	2d	0	5	3	$\frac{144}{228}$
	3d	0	4	$2\frac{1}{2}$	$\frac{24}{228}$
	4th	0	3	6	$\frac{96}{228}$
	Proof	1	0	0	

Note. The late Mr. Sadler has expeditiously solved this question by vulgar fractions ; but to shew my readers that questions of this kind are very well adapted to this rule, was the reason of my inserting it in this place.

E. 4. Four merchants, A, B, C, and D, join their stocks and trade together, of which A put in one-half, B one-third, C one-fourth, and D one-fifth ; but at the expiration of twelve months, they had the misfortune to lose 120*l*. what must each person suffer of the said loss ?

Note, you may suppose any sum at pleasure to be their stock ; as, suppose 600*l*.

$\left.\begin{matrix}\frac{1}{2}\\ \frac{1}{3}\\ \frac{1}{4}\\ \frac{1}{5}\end{matrix}\right\}$ of 600*l*. = $\left\{\begin{matrix}300 \text{ A's}\\ 200 \text{ B's}\\ 150 \text{ C's}\\ 120 \text{ D's}\end{matrix}\right\}$ stock

770 Sum, then

			£.	£.	s.	d.	Rem.	
As 770 : 120 ::	300	:	46	15	$0\frac{3}{4}$	9	A's	Loss
	200	:	31	3	$4\frac{1}{2}$	6	B's	
	150	:	23	7	$6\frac{1}{4}$	43	C's	
	120	:	18	14	$0\frac{1}{4}$	19	D's	
		Proof	120	0	0			

E. 5. Four merchants, A, B, C, D, gain 2000*l*. by trade, whereof $\frac{1}{2}$ of A's share is equal to $\frac{3}{4}$ of B's, $\frac{4}{5}$ of C's, and $\frac{5}{6}$ of D's ; what share had each ?

Take any number at pleasure, and divide in proportion to their shares, thus :

A's share 120, then
B's —— 80
C's —— 75
D's —— 72

			£.		£.	s.	d.	Rem.	
Sum 347 :	2000 ::		120	:	691	12	10	328	For A
			80	:	461	1	$10\frac{3}{4}$	103	B
			75	:	432	5	$6\frac{1}{4}$	205	C
			72	:	414	19	$8\frac{1}{2}$	78	D
				Proof	2000	0	0		

E. 6. A and B venturing equal sums of money, clear by joint trade 308*l*.—By agreement, A was to have 8 per cent. because he spent time

in execution of the project, and B was to have only 5; the question is, what was allotted A for his trouble?

$$\begin{array}{r} 8 \\ \underline{5} \\ 13 \end{array} = \text{their gain per cent.}$$

$$\text{Then, as } \overset{£.}{13} : \overset{£.}{308} :: \left\{\begin{array}{l} \overset{£.}{8} \\ 5 \end{array}\right. \begin{array}{l} : \\ : \end{array} \begin{array}{rrl} \overset{£.}{189} & \overset{s.}{10} & \overset{d.}{9}\ \tfrac{12}{13} \\ 118 & 9 & 2\tfrac{3}{4}\ \tfrac{1}{13} \end{array} \begin{array}{l} \text{A's} \\ \text{B's} \end{array} \left.\right\} \text{Gain}$$

$$\text{Answer, A had for his trouble} - 71 \quad 1 \quad 6\tfrac{1}{4}\ \tfrac{11}{13}$$

XXIII. DOUBLE FELLOWSHIP:

OR,

FELLOWSHIP WITH TIME,

IS when each person's stock continues unequal time in company; so that a consideration must be made of the time of continuance, as well as of the stock.

RULE. Multiply the particular stocks of each person by the time of continuance, and the sum of the several products, make the first term in the single rule of three direct; the whole gain or loss the second, and every man's particular stock, multiplied by its time, the third.

PROOF. Add all the parts of the gain or loss together, which must be equal to the whole.

EXAMPLE 1. Two persons, A and B, enter into partnership thus; A puts in 40*l.* for 18 months, and B. 40*l.* for 12 months; they gain 120*l.* what is each man's share of the gain?

$$\left.\begin{array}{l} 40 \times 18 \\ 40 \times 12 \end{array}\right\} = \left\{\begin{array}{r} 720 \\ 480 \end{array}\right\} = \left\{\begin{array}{l} \text{A's} \\ \text{B's} \end{array}\right\} \text{Stock multiplied into his time.}$$

$$1200 \text{ The sum of the products}$$

$$\text{Then, as } 1200 : 120 :: \left\{\begin{array}{r} 720 \\ 480 \end{array}\right. \begin{array}{l} : \\ : \end{array} \begin{array}{r} 72 \text{ A's} \\ 48 \text{ B's} \end{array} \left.\right\} \text{Share of the gain.}$$

$$£.\ 120 \text{ Proof}$$

E. 2. Three merchants, A, B, and C, enter into partnership; A puts in 65*l.* for 8 months; B 78*l.* for 12 months; and C 84*l.* for 4 months, and 90*l.* for 2 months; they gain 166*l.* 12*s.* what is each man's share of the gain?

$$\left.\begin{array}{rl} & 65 \times 8 = 520 \text{ A's} \\ & 78 \times 12 = 936 \text{ B's} \\ \text{Add} \left\{\begin{array}{l} \\ \\ \end{array}\right. & \left.\begin{array}{l} 84 \times 4 = 336 \\ 90 \times 2 = 180 \end{array}\right\} \text{C's} \end{array}\right\} \text{Stock and time}$$

$$1972 \text{ Sum}$$

$$\text{As } 1972 \;:\; \underset{\pounds.\;\;\; s.}{166 \;\; 12} \;::\; \left\{\begin{array}{rcl} 520 & : & 43\;\;18\;\;7\tfrac{1}{2} \text{ A's} \\ 936 & : & 79\;\;\;1\;\;6\tfrac{1}{4} \text{ B's} \\ 516 & : & 43\;\;11\;\;10\tfrac{1}{4} \text{ C's} \end{array}\right\} \text{Gain}$$

$$\begin{array}{r} \pounds.\;\; s.\;\; d. \\ \hline 166\;\;12\;\;0 \text{ Proof} \end{array}$$

E. 3. Two merchants together make up a stock of 600*l*. A's stock continued in company 9 months, and B's 11; they gain 200*l*. which they divide equally; how much did each put in?

First, since the gains are equal, A's stock multiplied by his time 9, is equal to B's stock multiplied by his time 11, ∴ A's stock is to B's stock as 11 to 9.

$$\begin{array}{l} 11 \\ \underline{\;\,9\,} \\ 20 \;:\; 600 \;::\; \left\{\begin{array}{rcl} 11 & : & \overset{\pounds.}{330} \text{ A's stock} \\ 9 & : & 270 \text{ B's stock} \end{array}\right. \end{array}$$

$$\begin{array}{r} \hline \pounds.\; 600 \text{ Proof} \end{array}$$

E. 4. A ship's company take a prize, value 4000*l*. which they agree to divide amongst them according to their pay, and time they have been on board; now the officers and midshipmen have been on board 4 months, and the sailors 3; the officers have 50*s*. a month, the midshipmen 40*s*. and the sailors 28*s*.—moreover, there are 4 officers, 8 midshipmen, and 120 sailors; I desire to know what each person's share is of the said prize?

$$\begin{array}{lrl} \text{First } 4 \times 4 \times 50 = & 800 & \text{Officers' pay and time} \\ 8 \times 4 \times 40 = & 1280 & \text{Midshipmen's ditto} \\ 120 \times 3 \times 28 = & 10080 & \text{Sailors' ditto} \\ \hline & \pounds.\; 12160 & \text{Sum} \end{array}$$

$$\text{As } 12160 \;:\; 4000 \;::\; \left\{\begin{array}{rcrrrrl} & & \pounds. & s. & d. & & \\ 800 & : & 263 & 3 & 1\tfrac{3}{4} & 704 & \text{Officers} \\ 1280 & ; & 421 & 1 & 0\tfrac{1}{2} & 640 & \text{Midshipmen} \\ 10080 & : & 3315 & 15 & 9\tfrac{1}{4} & 1088 & \text{Sailors} \\ \hline & & 4000 & 0 & 0 & & \end{array}\right.$$

Note. The above being the share of each company, each person's share is found as follows, thus:

$$\left.\begin{array}{rrrcrcrrr} \pounds. & s. & d. & & \text{Number} & & \pounds. & s. & d. \\ 263 & 3 & 1\tfrac{3}{4} & \div & 4 & = & 65 & 15 & 9\tfrac{1}{4} \\ 421 & 1 & 0\tfrac{1}{2} & \div & 8 & = & 52 & 12 & 7\tfrac{1}{2} \\ 3315 & 15 & 9\tfrac{1}{4} & \div & 128 & = & 27 & 12 & 7\tfrac{1}{2} \end{array}\right\} = \text{Each person's share.}$$

The fractions, or remainders, are omitted, as inconsiderable.

E. 5. A and B paid equally for a horse, February 7, 1781; A on the 10th took him a journey into the West, and returned on the 10th of June following; B on the 2d of August took him into Scotland, and stayed till Nov. 13, and this concluded his service for this year. From January 17 following A used him ten Days, and in six weeks after his return, employed him till April 30th; B then rode him

him from May-day to Midſummer; A had him from the 14th of July to 14 days after St. James's tide; B, on September 30th, took him into Norfolk, and came back October 19th; he then was ſold for 7*l.* 10*s.* and they would have the money parted equitably between them, viz. in proportion to the uſe each made of their ſteed?

Firſt, from February 10, to June 10	= 122	}		Days.
January 17, to April 30	61	}	= 208	A's time
July 14, to 14 after St. James's 25		}		
Auguſt 2, to November 13	104	}		
May 1, to July 24	55	}	= 179	B's time
September 30, to October 19	20	}		
			387	= the time

the horſe was in uſe. Then,

	£.	s.			£.	s.	d.		
As 387 :	7	10 ::	{ 208 :		4	0	$7\frac{1}{4}$ $\frac{297}{387}$	A's }	Share
			{ 179 :		3	9	$4\frac{1}{2}$ $\frac{90}{387}$	B's }	
					7	10	0 Proof		

XXIV. BARTER,

IS the exchanging wares for wares, or one commodity for another; and informs merchants ſo to proportion their goods, that neither may ſuſtain loſs or diſadvantage by ſuch a barter or exchange.

If the commodities exchanged are not of equal value, the defect is ſupplied with money.

RULE. 1. Find the value of that commodity, whoſe quantity is given; then find what quantity of the other, at the given rate, you can have for the aforeſaid value, which quantity will be the anſwer.

2. When one has goods at a certain price ready money, but in barterage advances it to ſomething more, ſay, as the ready money price of the one : is to its bartering price, :: ſo is the ready money price of the other, to its bartering price; then the quantity of the latter commodity may be found either from the ready money or bartering price.

EXAMPLE 1. How much ſugar, at 1*l.* 10*s.* per hundred weight muſt be given in barter for 4 hundred weight of tea, at 12*s.* per pound?

Cwt.		*s.*	*Cwt.*	*s.*
4	If	30 :	1 ::	5376
112		3|0)	537|6	
448 *lb.* at 12*s.* per *lb.*		*C.*	179—6 *Rem.*	
12			112	
5376 *s.* The value of the tea		3|0)	67|2	
Anſwer 179 *Cwt.* 22 $\frac{12}{30}$ *lb.*		*lb.*	22—12	

E. 2.

E. 2. How much coffee, at 5*s.* per pound, must be given for 367 pounds of tea, at 8*s.* per pound?

	lb.		*s.*		*lb.*		*s.*		*lb.*		*s.*
First, as	1	:	8	::	367	If	5	:	1	::	2936

367
8
2|0)293|6
Value of the tea £. 146 16

5)2936
587—1
16
5)16
3—1
16
5)16
3—1 Rem.

Answer 587 *lb.* 3 *oz.* 3 *drs.* $\frac{1}{5}$

E. 3. A hath tea, at 8*s.* 6*d.* per pound ready money, but in barter will have 10*s.* per pound; B hath tobacco worth 18*d.* per pound ready money; how must B rate his tobacco per pound, that his profit may be equivalent with A's tea?

	s. d.		*s.*		*s. d.*
As	8 6	:	10	::	1 6

12   12
102   18
10
s. d.
102)180(1 $9\frac{3}{17}$ Ans.
102
78
12
102)936(9*d.*
918
18

E. 4. A hath 14 *Cwt.* of raisins, at 6*d.* per pound, for which B gives him 1 *Cwt.* 3 *qrs.* of cinnamon; I demand how B rated the cinnamon per pound?

	C. qrs.		*d.*		*Cwt.*
If	1 3	:	6	::	14

4   4
7   56
6
7)336
12)48
Answer 4 *s.* per *lb.*

E. 5. A, with an intention of clearing 30 guineas on a bargain with B, rates hops at 16*d.* per pound, that stood him in 10*d.*—B, apprized of that, set down malt, which cost 20*s.* a quarter, at an adequate price; how much malt did they contract for?

	d.		*d.*		*d.*		*s.*
If	10	:	16	::	240	=	20

16
1|0)384|0
12)384
32*s.* The advanced value of the malt
20*s.* real value
12*s.* B. gains per quarter

30 Guineas
21
12)630
Answer $52\frac{1}{2}$ Quarters

E. 6.

E. 6. A, in order to put off to B 720 ells of damaged holland, worth 5*s.* an ell, at 6*s.* 8*d.* proposes, in case he has half the value in money, to give B thereon a disc. of 10 per cent. the rest A is to take out in saffron, which B, apprized of the whole management, rates in justice at 36*s.* the pound; pray what was it really worth in ready money, and what quantity of saffron was he to deliver on the change?

First, 5*s.* $= \frac{1}{4}$) 720 Ells

180 Real value of the holland

6*s.* 8*d.* $= \frac{1}{3}$) 720

240 Adv. val. of ditto

$\frac{1}{10}$) 240

£. 24 = Discount at 10*l.* per cent.

£.

Then, 240 Adv. value of the hol.

24 Discount

2) 216

£. 108 = the ready money pd.

	£.		£.		s.
Again, as	216	:	180	::	36
	20				180
	4320				432|0)648|0 (1*l.* 10*s.* = 30*s.* Real value of the saffron per pound

432

2160

20

432|0)4320|0 (10*s.*

432

£.

108

20

3|0)216|0

72*lb.* Quant. of saff. deliv.

E. 7. A has 100 reams of paper, at 8*s.* ready money, which in barter he sets down at 10*s.* B, sensible of this, has pamphlets at 6*d.* a-piece ready money, which he adequately charges, and insists, besides, on $\frac{1}{4}$ of the price of those he parts with in specie; what number of the books is he to deliver in lieu of A's paper, what cash will make good the difference, and how much is B the gainer by this affair?

	s.		d.		s.		d.
If	8	=	96	:	10	::	6
					6		
					60		
					12		
				96)	720 (7½*d.* Barter price of the pamphlets		

100 Reams at 8*s.* = 40*l.* real } Value of the paper
Ditto at 10*s.* = 50*l.* advanced }

$\frac{1}{4}$ = 50*l.* ÷ 4 = 12*l.* 10*s.* B to have in cash

40*l.* Value of B's pamphlets

× 40 Six-pences in a pound

1600 Pamphlets to be delivered

From 40*l.* take 12*l.* 10*s.* remains 27*l.* 10*s.* what they stood him in; so B, in this transaction, gains 12*l.* 10*s.*

E. 8.

E. 8. A and B truck; A has 14 *Cwt.* 2 *qrs.* 25 *lb.* of Farnham hops, at 2*l.* 19*s.* per hundred weight, but in barter insists on three guineas; B has wine worth 6*s.* per gallon, which he raises in proportion to A's demand on the balance; A received but a hogshead and a half of wine: pray what had he in ready money?

First, 14 *Cwt.* 2 *qrs.* 25 *lb.* at 3*l.* 3*s.* per hundred weight, = 46*l.* 7*s.* $6\frac{3}{4}$*d.* the advanced value of A's hops.

$$\begin{array}{lllllll} & £. & s. & s. & & s. & \\ \text{If} & 2 & 19 = 59 & : & & 63 & :: \ 6 \\ & & & & & 6 & \\ \hline \end{array}$$

59)378(6*s.* $4\frac{3}{4}$*d.* $\frac{31}{59}$ the advanced price of B's wine per gallon. Now $1\frac{1}{2}$ *Hhd.* = 94 gallons, at 6*s.* $4\frac{3}{4}$*d.* $\frac{31}{59}$

$$\begin{array}{llll} & 6 & 4\frac{3}{4}\frac{31}{59} & \\ & & 94\frac{1}{2} = 10 \times 9 + 4\frac{1}{2} & \\ \hline £.\ 30 & 5 & 5\frac{1}{4}\frac{9}{59} & \text{Value of B's wine} \\ 46 & 7 & 6\frac{3}{4} & \text{Value of A's hops} \\ \hline 16 & 2 & 1\frac{1}{4}\frac{50}{59} & \text{In ready money, Answer} \end{array}$$

XXV. LOSS and GAIN,

IS a rule by which men of trade and business know what they get by retailing goods; and in case of damage, what they lose by selling it at any given rate; and whether they gain or lose, to know at what rate per cent.

In this rule there are four varieties.

1. To know what is gained or lost per cent.
2. To know what it should be sold for to gain or lose so much per cent.
3. Having gained or lost so much per cent. to know what it cost.
4. There being so much gained per cent. when sold at such a price, to know what is gained per cent. when sold for more, or what is lost per cent. when sold for less;

RULE. When there is gain per cent. add the gain per cent. to 100*l.* but when there is loss per cent. subtract as much as you lose per cent. from 100*l.* the sum or difference is the third number in the rule of three.

E. 1. Bought 240 yards of cloth, at 14*s.* 6*d.* per yard, and sold it again at 18*s.* per yard; what did I gain by the whole?

$$\begin{array}{ll} 6d. = \frac{1}{2})\ 240 \text{ yards, at } 14s.\ 6d. & \qquad 240 \text{ yards, at } 18s. \\ \qquad 14 & \qquad 18 \\ \hline \qquad 3360 & \qquad 2|0)432|0 \\ \qquad 120 & \\ \hline 2|0)348|0 & \qquad 216 \text{ Sold for} \\ & \qquad 174 \text{ Cost} \\ \hline £.\ 174 \text{ Cost} & \text{Answer } 42l. \text{ Gained} \end{array}$$

Again,

Again, anſwered by a practical method at the end of practice; ſee ſection XVI.

Thus, 240 yards at 18s. 0d. = 216l. Sold for
240 ditto at 14s. 6d. = 174l. What coſt

Anſwer £. 42 Gained thereby, as above

Mr. Vyſe's anſwer, in his Key to the Tutor's Guide, is 102l.

E. 2. If 276 fodders of lead, each $19\frac{1}{2}$ hundred weight, be ſold for 256l. at 5 months credit, and I gain 11l. per cent. per annum; the queſtion is, how much the whole coſt ready money?

£. 100
+11

111 Amount

	£.		£.		£.		£.	s.	d.	
Then, if	111	:	256	::	100	:	230	12	$7\frac{1}{4}\ \frac{45}{111}$	Anſwer

E. 3. If by ſelling cloth, at 5s. per ell, I gain 8l. per cent. what ſhall I gain per cent. if I ſell the ell at 6s. 3d?

First, 100 + 8 = 108l. amount; then,

	s.		£.		s.	d.		£.	
As	5	:	108	::	6	3	:	135	From which
					Subtract			100	
					Remains			35	Anſwer

Mr. Webſter, in his Arithmetic, makes the anſwer only 10l.

E. 4. At 5s. per dozen I gain 7l. 10s. per cent. how much ſhall I gain per cent. if I ſell the dozen at 5s. 9d?

Firſt, 100l. + 7l. 10s. = 107l. 10s. amount

	s.		l.	s.		s.	d.		l.	s.	d.	
Then, if	5	:	107	10	::	5	9	:	123	12	6	Amount
			From which deduct						100	0	0	
								Anſwer	23	12	6	

Mr. Stonehouſe's anſwer, in his arithmetic, is only 8l. 12s. 6d.

E. 5. Suppoſe I ſell 500 deals, at 15d. per piece, and 9l. per cent. loſs, what do I loſe by the whole quantity?

Firſt, from 100
Take 9

			l.		l.	s.		l.	s.	d.	
Then, as	91	:	100	::	31	5	:	34	6	$9\frac{3}{4}$	= the
price of the deals, at 15d. each.						Subtract		31	5	0	
						Anſwer		3	1	$9\frac{3}{4}$	

Mr. Dilworth's anſwer to this queſtion in the ſecond edition of his arithmetic, is only 2l. 16s. 3d.

E. 6. A Mancheſter tradeſman going to a fair, ſold fuſtians for 11s. 6d. the end, wherein was gained 15l. per cent. but ſeeing no other tradeſman had ſo good, raiſed them, at the latter end of the fair, to 12s. the end; I demand what he gained per cent. by this laſt ſale?

Firſt,

First, $100 + 15 = 115$*l.* the amount; then,

	s.	*d.*		*l.*		*s.*		*d.*	
As	11	6	:	115	::	12	:	120	The amount per cent.

$\therefore 120 - 100 = 20$*l.* per cent. answer.

Mr. Hill's answer, in his arithm. page 289, is only 15*l.* 13*l.* $0\frac{1}{2}$*d.* $\frac{2}{23}$

E. 7. Suppose I sell 1 hundred weight of hops, for 6*l.* 15*s.* and gain 25*l.* per cent. what would have been the gain per cent. if I had sold them for 8*l.* per hundred weight?

First, $100 + 25 = 125$*l.* amount; then,

	l.	*s.*		*l.*		*l.*		*l.*	*s.*	*d.*	
As	6	15	:	125	::	8	:	148	2	$11\frac{1}{2}$	Amount per cent.

Then, 148*l.* 2*s.* $11\frac{1}{2}$*d.* $- 100 = 48$*l.* 2*s.* $11\frac{1}{2}$*d.* Answer

Mr. Walkingame's answer to this question, in his arithmetic, page 70, 3d edit. is only 29*l.* 12*s.* 7*d.* $\frac{1}{9}$.

Note. The reason of these errors in the above authors, in questions of this sort, is by making the gain or loss of 100*l.* the second term in the stating, instead of its amount (in case of gain) or deduction in case of loss. Some of these questions have been remarked by other authors; but as my readers should not be at a loss to solve questions of this sort, I thought it necessary to give them a place in this treatise.

E. 8. If by sending pewter to Turkey, and parting with it at $25\frac{2}{3}$*d.* per pound, the merchant clears cent. per cent. what does he gain in Holland, where he disposes of the hundred weight for 8*l*?

	s.	*d.*	
	2	$1\frac{2}{3}$	
		$8 \times 7 \times 2 = 112$	
	16	8	
		7	
5	16	8	
		2	
11	13	4	$= 112$
	6	$2\frac{2}{3}$	$= \frac{2}{3}$
2) 11	19	$6\frac{2}{3}$	$= 112\frac{2}{3}$ Sold for at Turkey
£. 5	19	$9\frac{1}{3}$	What cost him

112
2
3) 224
$74\frac{2}{3}$*d.* = 6*s.* $2\frac{2}{3}$*d.*

Then from	8	0	0
Take -	5	19	$9\frac{1}{3}$
Rem. £.	2	0	$2\frac{2}{3}$ his loss, Ans.

E. 9. Sold a repeating-watch for 50 guineas, and by so doing lost 17 per cent. whereas I ought in dealing to have gained 20 per cent. then how much was it sold under the just value?

First, 100
— 17
83

And, 100
+ 20
120

	l.		*l.*		*l.*	*s.*			*l.*	*s.*
Then, if	83	:	100	::	52	10 (=50 guineas)	:		63	$5\frac{5}{83}$

	l.		*l.*		*l.*	*s.*		*l.*	*s.*	*d.*	
Again, as	100	:	120	::	63	$5\frac{5}{83}$	:	75	18	$0\frac{3}{4}$	Worth
								52	10	0	Sold for
							Answer	23	8	$0\frac{3}{4}$	Under value

E. 10. Bought hose in London, at 4*s.* 3*d.* the pair; now taking the charges at an average to be 2*d.* the pair, and considering that I must lose 12 per cent. by remitting the money home again, what do I gain per cent. by this article of trade?

First, 4*s.* 3*d.* + 2*d.* = 4*s.* 5*d.* prime cost and charges.

	s.	*d.*		*s.*		*l.*		*l.*	*s.*	*d.*
Then, as	4	5	:	6	::	100	:	135	16	$11\frac{3}{4}\frac{5}{53}$

Again, 100 — 12 = 88*l.*

	l.		*l.*		*l.*	*s.*	*d.*		*l.*	*s.*	*d.*	
Also, as	100	:	88	::	135	16	$11\frac{3}{4}\frac{5}{53}$	:	119	10	$11\frac{1}{4}$	The amount
								Subtract	100	0	0	
Answer,			Gained per cent.						19	10	$11\frac{1}{4}$	

XXVI. ALLIGATION MEDIAL,

TEACHETH how to mix or unite many simples, or particulars, into one mass or sum, according to any price or sum required.

RULE. Multiply each quantity by its price; then say, as the sum of all the quantities, is to the sum of the said products, so is any part of the mixture, to the mean price of that part.

PROOF. Find the value of the whole mixture, at the mean price or rate, and if it agrees with the total value of the several quantities at their respective prices, the work is right.

EXAMPLE 1. A farmer would mix 24 bushels of wheat at 6*s.* per bushel, with 40 bushels of rye at 3*s.* per bushel, what will a bushel of this mixture be worth?

	Bush.		*s.*	*s.*	
First,	24	multiplied by its price	6 =	144	Value of the wheat
And,	40	——— by ———	3 =	120	Ditto of rye
	64		Sum	264	

Then, if 64 : 264 :: 1
              1

64) 264 (4*s.* $1\frac{1}{2}$ Answer.
     256
       8
      12
64)96(1*d.*
     64
     32
      4
64)128(2 *qrs.*
    128

4*s.* $1\frac{1}{2}$*d.* = 198 *qrs.* which multiplied by 64, the number of bushels = 12672 ÷ 4 = 3168*d.* ÷ 12 = 264*s.* proof.

E. 2.

E. 2. An hostler mixed provender for his horses, viz. 18 bushels of oats at 2*s.* 1*d.* per bushel, with 16 bushels of beans at 4*s.* 9*d.* per bushel, and 13 bushels of malt, at 3*s.* 10*d.* per bushel; I demand what a bushel of this mixture is worth?

$$\text{First,}\begin{cases} \overset{bu.}{18} \times \overset{d.}{25} = \overset{d.}{450} \\ 16 \times 57 = 912 \\ 13 \times 46 = 598 \end{cases}$$

$$\text{As } 47 : 1960 :: 1$$

$$1$$

$$47)1960(41\tfrac{1}{2}d.\ \tfrac{38}{47} = 3s.\ 5\tfrac{1}{2}d.\ \tfrac{38}{47} \text{ Answer}$$

XXVII. ALLIGATION ALTERNATE,

IS that by which the particular quantities of every ingredient in any mixture are found; when the particular rates of every one of the ingredients, and the mean rates, are given.

RULE. 1. Place the rates of the several things one over another, and the proposed price of the composition against them; then link the several rates so together, as that one greater than the mean rate, or price of the composition, may be coupled to a less; then take the differences between the mean rate and the several prices, and place each of them against its yoke-fellow; this being the reverse of alligation medial, may be proved thereby.

EXAMPLE 1. A grocer would mix sugar of 10*d.* 5*d.* and 4*d.* per pound, so that the composition may be worth 6*d.* per pound, what quantity must he take?

$$\overset{d.}{6}\begin{Bmatrix} 10 \\ 5 \\ 4 \end{Bmatrix} \begin{matrix} 2+1 = \overset{lb.}{3} \text{ at } \overset{d.}{10} \\ 4 \text{ at } 5 \\ 4 \text{ at } 4 \end{matrix} = \begin{Bmatrix} \overset{d.}{30} \\ 20 \\ 16 \end{Bmatrix} \text{ Answer}$$

$$11 \qquad 11)66$$

$$\text{Proof } 6$$

Having linked the several rates, agreeable to the rule (whereby it is plain that these rates will admit but of one way of linking) then the difference between 6, the mean price, and 4, viz. 2, is placed against 10, its yoke-fellow: the difference between 6 and 5 is 1, which is also placed against 10 its yoke-fellow; and the difference between 6 and 10 is 4, which, because it has two yoke-fellows, is placed against them both, viz. against 5 and 4; so that as oft as the grocer takes 3*lb.* at 10*d.* he must take 4*lb.* of each of the other two sorts to make up the mixture.

Note. The differences are not only the quantities, which anſwer the queſtion; but any other numbers, in the ſame proportion as they are, will anſwer the queſtion as well.

For ——— ——— ——— ———	3,	4,	4
All multiplied by — — — — — —			3
Produce the proportionals ——— ———	9,	12,	12
Theſe multiplied by ——— ——— ———			4
Produce theſe numbers in the ſame ratio, and ſo on, in infinitum.	36,	48,	48

E. 2. A miller hath four ſorts of meal, viz. one ſort at 6*s*. 8*d*. another at 5*s*. 6*d*. the third at 4*s*. 4*d*. and the fourth at 3*s*. 8*d*. per buſhel; but he is deſirous of mixing ſo much of each ſort together, that he may ſell it at 5*s*. per buſhel; how much of each ſort muſt he take?

	d.				
60	80	16	at	80 = 1280	Anſwer.
	66	8		66 = 528	
	52	6		52 = 312	
	44	20		44 = 880	
		50		5|0)300|0	
				Proof 60	

The ſeveral rates being linked together, and their reſpective differences placed againſt their yoke-fellows, as before, you will find 16 buſhels at 80*d*. 8 at 66*d*. 6 at 52*d*. and 20 at 44*d*. will compoſe the mixture required.

Note. Examples of this nature will admit of as many anſwers as there are different ways of linking together a larger price and a leſſer than the mean rate propoſed.

XXVIII. ALLIGATION PARTIAL,

IS when the particular rates, the mean rate, and the quantity of one ingredient, is given, to find the quantity of all the reſt of the ingredients. This is called alligation partial, becauſe a part of the mixed ingredients only are given.

RULE. 1. Take the difference between each price and the mean rate, as in the laſt rule.

2. As the difference oppoſite to the known quantity, is to the known given quantity; ſo is any other difference, to the quantity of its oppoſite name.

EXAMPLE 1. A farmer being determined to mix 12 buſhels of wheat at 6*s*. per buſhel, with rye at 4*s*. barley at 3*s*. and oats at 2*s*. 6*d*. per buſhel; I demand how much rye, barley, and oats, muſt be mixed with the ſaid 12 buſhels of wheat, ſo that the whole may be ſold for 3*s*. 6*d*. per buſhel?

42*d*.

	d.		
d.	72	6	
42	48	12	Difference
	36	30	
	30	6	

Diff.		*bu.*		*Diff.*		*bu.*	
As 6	:	12	::	12	:	24 of rye	To be mixed with
				30	:	60 of barley	the 12 bushels
				6	:	12 of oats	of wheat

All examples belonging to this and the following rule, may be proved by the rule in alligation medial.

Note. A composition made of 6 bushels of wheat at 72*d.* per bushel, 12 of rye at 48*d.*—30 of barley at 36*d.* and 6 of oats at 30*d.* per bushel, will bear the mean price of 42*d.* or 3*s.* 6*d.* per bushel; you must observe, that in this composition there are only 6 bushels of wheat, but the demand is 12 bushels; therefore the proportion above is found thus:

As the difference annexed to the branch, is to the other particular differences, so is the given quantity to the several quantities required.

To find how much rye, barley and oats must be mixed with the 12 bushels of wheat, say, if 6 bushels of wheat require 12 bushels of rye, what will 12 bushels of wheat require? Answer, 24 bushels of rye. And by proceeding in like manner with the other mixtures, you will find their respective proportions as in the preceding work.

E. 2. A tobacconist has by him 120*lb* of Oroonoko tobacco, worth 2*s.* 6*d.* a pound; to this he would mix York-River ditto at 20*d.* and other inferior tobaccos at 18*d.* and 15*d.* a pound, as will make up a mixture answerable to 2*s.* a pound; what will this parcel weigh?

24	30	4+6+9=19	Differ.
	20	— — 6	
	18	— — 6	
	15	— — 6	

Diff. *lb.* *diff.*
If 19 : 120 :: 6
6
———
19)720(37 $\frac{17}{19}$ *lb.* of each of the other sorts must be mixed with 120*lb.* of the quant. given, Answer.
57
———
150
133
———
17

E. 3. What quantity of gold, at 15, 16, and 18 caracts fine, must be mixed with 80 ounces of pure gold, viz. such as is 24 caracts fine, so that the composition may be 20 caracts fine?

20	24	4+5+2=11	Differ.
	18	— — 4	
	15	— — 4	
	16	— — 4	

oz. *oz.* *oz.*
As 11 : 80 :: 4
4
———
11)320(29 $\frac{1}{11}$ *oz.* of 18, 15, and 16 caracts fine, Ans.

XXIX.

XXIX. ALLIGATION TOTAL,

IS when the price of each ſimple is given, alſo the mean rate and quantity of the compound, to find how much of each ſort will make that quantity.

RULE. Say, as the ſum of the differences, to the quantity given, ſo is every particular difference, to its reſpective quantity.

EXAMPLE 1. A brewer hath three ſorts of beer, viz. at 9*d.* 13*d.* and 18*d.* per gallon, which he would mix together, and the whole mixture to contain 60 gallons ; how much of each ſort muſt be taken that the mixture may be worth 10*d.* per gallon ?

$$\begin{array}{c} d. \\ 10 \end{array}\left\{\begin{array}{r} 9 \\ 13 \\ 18 \end{array}\right\} \begin{array}{r} 8+3=11 \\ 1 \\ 1 \end{array}\Bigg\} \text{ Differences}$$

$$\text{Sum } 13$$

$$\begin{array}{c} \textit{Sum.} \\ \text{As } 13 \end{array} : \begin{array}{c} \textit{gall.} \\ 60 \end{array} :: \left\{\begin{array}{rcl} 11 & : & 50\frac{10}{13} \\ 1 & : & 4\frac{8}{13} \\ 1 & : & 4\frac{8}{13} \end{array}\right\} \text{ at } \left\{\begin{array}{r} 9 \\ 13 \\ 18 \end{array}\right\} \text{ Per gallon, Anſwer}$$

PROOF. As 60 *gall.* : 614 *d.* the value of the whole mixture :: 1 *gall.* : 10 *d.* the mean price given.

E. 2. A mixture of wine is to be made up, conſiſting of 130 quarts, from theſe five ſorts, whoſe prices are 7*d.* 8*d.* 10*d.* 14*d.* and 15*d.* a quart ; and the whole is to be ſold at 12*d.* per quart ; how much of each ſort muſt be taken ?

First way.

$$\begin{array}{c} d. \\ 12 \end{array}\left\{\begin{array}{r} 15 \\ 14 \\ 10 \\ 8 \\ 7 \end{array}\right\} \begin{array}{r} \textit{Differences.} \\ \text{— — } 5 \\ 4+2=6 \\ \text{— — } 2 \\ \text{— — } 2 \\ \text{— — } 3 \\ \hline \text{Sum } 18 \end{array}$$

Second way.

$$\begin{array}{c} d. \\ 12 \end{array}\left\{\begin{array}{r} 15 \\ 14 \\ 10 \\ 8 \\ 7 \end{array}\right\} \begin{array}{r} \textit{Differences.} \\ 4+2=6 \\ \text{— — } 5 \\ \text{— — } 3 \\ \text{— — } 3 \\ \text{— — } 2 \\ \hline \text{Sum } 19 \end{array}$$

Third way.

$$\begin{array}{c} d. \\ 12 \end{array}\left\{\begin{array}{r} 15 \\ 14 \\ 10 \\ 8 \\ 7 \end{array}\right\} \begin{array}{r} 2+4+5=11 \\ 2+4+5=11 \\ \text{— — } 5 \\ \text{— — } 5 \\ \text{— — } 5 \\ \hline \text{Sum } 37 \end{array}$$

Operation

Operation by the laſt way thus:

$$\text{As } 37 : 130 :: \left\{ \begin{array}{l} 11 : 38\tfrac{24}{37} \text{ quarts, at } 15d. \text{ and } 14d. \\ \qquad\qquad\qquad\qquad \text{per quart} \\ 5 : 17\tfrac{21}{37} \text{ qts. at } 10d.\ 8d. \text{ and } 7d. \end{array} \right\} \text{Anſwer}$$

Now as alligation anſwers not queſtions compleatly, that is, does not give all the anſwers ſuch queſtions are capable of; and, perhaps, not always thoſe that ſuit the occaſion; I ſhall ſhew, for the ſatisfaction of my ingenious readers, how this imperfection of common arithmetic is ſupplied by *Algebra*, and all the poſſible anſwers to any queſtion may be clearly and eaſily diſcovered.

E. 3. A tobacconiſt hath three ſorts of tobacco, viz. one at 2*s.* 8*d.* per pound, another at 20*d.* per pound, and a third ſort at 16*d.* per pound; of theſe he would make a mixture to contain 56 pound, that may be ſold for 22*d.* per pound; how much of each ſort muſt he take?

$$\text{Let} \left\{ \begin{array}{l} a = \text{the quantity of that worth } 2s.\ 8d. = 32d. \\ e = \text{that at } 20d. \text{ per pound} \\ y = \text{that at } 16d. \text{ per pound} \end{array} \right.$$

Then	1	$a + e + y = 56$
And	2	$32a + 20e + 16y = 1232$
$1 - a$	3	$e + y = 56 - a$
$2 - 32a$	4	$20e + 16y = 1232 - 32a$
3×16	5	$16e + 16y = 896 - 16a$
$4 - 5$	6	$4e = 336 - 16a$
$6 \div 4$	7	$e = 84 - 4a$
$3 - 7$	8	$y = 3a - 28$

Hence it is evident from the 7th ſtep, that the quantity ſignified by a muſt be leſs than 21, and (by the 8th ſtep) greater than $9\frac{1}{8}$; that is, a may be any number between 21 and $9\frac{1}{3}$; whence 12 anſwers flow from the limits of a only, and by proceeding with each ſingle value of a, all the anſwers in whole numbers may be obtained.

If there be more than three quantities concerned in the queſtion, the work will be more large; becauſe the limits of all the quantities above two muſt be found.

E. 4. A vintner would mix four ſorts of wine together, viz. one worth 7*s.* 4*d.* a ſecond worth 4*s.* 7*d.* a third worth 3*s.* 8*d.* and a fourth worth 2*s.* 9*d.* per gallon; how much of each ſort muſt be taken to make a mixture of 63 gallons, to be ſold for 5*s.* 6*d.* per gallon without loſs?

		s.	*d.*		*d.*
First, let a	= that quantity worth	7	4	=	88
e		4	7		55
y		3	8		44
u		2	9		33
The mean rate - -		5	6		66

Then

Then	1	$a+e+y+u=63$
And	2	$88a+55e+44y+33u=4158$
$1-a$	3	$e+y+u=63-a$
$2-88a$	4	$55e+44y+33u=4158-88a$
3×33	5	$33e+33y+33y=2079-33a$
$4-5$	6	$22e+11y=2079-55a$
$6\div 11$	7	$2e+y=189-5a$
3×55	8	$55e+55y+55u=3465-55a$
$8-4$	9	$11y+22u=33a-693$
$9\div 11$	10	$y+2u=3a-63$
Suppose	11	$a=22$. Then $5a=110$, and $3a=66$
per 7th	12	$2e+y=189-5a=79$
$12-2e$	13	$y=79-2e$
per 3d	14	$e+y+u=63-a=41$
$14-e$	15	$y+u=41-e$
$15-13$	16	$u=e-38$

From the seventh and tenth steps it appears that the quantity denoted by a, must be less than $37\frac{4}{5}$, and greater than 21; whence 16 answers flow from the limits of a. Then if a be put $=22$, by the 13th and 16th steps it appears $e=39$, $y=1$, and $u=1$; and thus proceeding with each single value of a, above 120 answers may be found to this question in whole numbers; in fractions, infinite.

XXX. EXCHANGE,

CONSISTS in finding the true sum or value of one country coin, &c. equivalent to any given sum or value of that of another country.

The par of exchange is fixed, and standard value of foreign coins, &c. expressed in sterling money of our own; it is so called, because in exchange, one equal value for another is given.

The course of exchange is the current price, and is always unsettled, being sometimes above, and sometimes below the par; according to the various circumstances and accidents of trade, and nations.

Money in the bank of other kingdoms, is finer, or purer than that which is current, the difference of value in each is called Agio.

As it would be endless to treat of every kind of exchange, I shall only give a few examples of the exchange of England, with a few of the chief countries in Europe.

First, with FRANCE.

At France, accounts are kept in livres, sols, and deniers, exchange being made by the French crown, whose par is $4s$. $6d$. sterling.

12 Deniers	make one	Sol
20 Sols	make one	Livre
3 Livres	make one	Crown

First,

First, to change French money into sterling,

RULE. As 1 crown is to the given rate, so is the given French sum, to the sterling required.

Second, to change sterling money into French,

RULE. As the rate of exchange is to one crown, so is the sterling sum, to the French required.

Note. The same rule must be observed with most of the following countries.

EXAMPLE 1. What sterling money must a merchant pay in London, to receive in Paris 2000 crowns, exchange at 54*d.* per crown?

As 1 : 54 :: 2000
2000
12)108000
2|0) 900|0
Answer £. 450

E. 2. What number of crowns must be paid in Paris, to receive in London 450*l.* exchange 54*d.* per crown?

d.		*cr.*		*£.*
As 54	:	1	::	450

20
9000
12
54 { 9)108000
6) 12000
Answer 2000 Crowns

E. 3. Change 640 crowns, 12 sols, 8 deniers, at 56*d.* per crown, into sterling?

	c.		*d.*		*c.*	*sols.*	*den.*
If	1	:	56	::	640	12	8

3   3
3   1920
20   20
60   38412
12   12
720   460952
56
2765712
2304760
72|0)2581331|2(3585 1*d.* = 149*l.* 7*s.* 7$\frac{3}{4}$*d.* $\frac{20}{72}$
216
421
360
613
576
373
360
131
72
592
4
72|0)236|8(3
216
20

E. 4. Change 149*l.* 7*s.* 7$\frac{3}{4}$*d.* sterling, into French crowns, exchange at 56*d.* per crown?

	d.		*c.*		*£.*	*s.*	*d.*		*c.*	*sols.*	*den.*	
As	56	:	1	::	149	7	7$\frac{3}{4}$	:	640	12	8	Answer

Second,

Second, with SPAIN.

They keep their accounts in piaſters, reals, and maravedis, and exchange by the piaſtre, whoſe par is 4*s.* 6*d.* ſterling.

4 Maravedis vellon, or $2\frac{1}{8}$ maravedis plate	make one	Quartas
$8\frac{1}{2}$ Quartas, or 34 mar. vellon		Rial vellon
16 Quartas, or 34 maravedis plate		Rial of plate
8 Rials of plate		Piece of eight, or dollar

N. B. A Rial vellon is $\frac{17}{32}$ of a rial of plate, and $\frac{17}{250}$ of a piaſter.

E. 5. Change 630*l.* into Spaniſh money, exchange at 50*d.* per piece of $\frac{8}{8}$?

	d.		*piece.*		*l.*		*pieces.*	
As	50	:	1	::	630	:	3024	the Anſwer

E. 6. Suppoſe Spain draws upon London for 3024 pieces of $\frac{8}{8}$, what ſterling money will this draft amount to, exchange at 50*d.* per piece of eight?

	piece.		*d.*		*pieces.*		*£.*	
As	1	:	50	::	3024	:	630	the Anſwer

E. 7. If I pay in Seville 1426 pieces of $\frac{8}{8}$, 4 rials, 26 maravedis, what may I draw my bill for at London, exchange at $54\frac{1}{4}d.$ per piece of eight?

	p.		*d.*		*p.*	*ri.*	*mar.*		*£.*	*s.*	*d.*	
If	1	:	$54\frac{1}{4}$	::	1426	4	26	:	322	9	$4\frac{3}{4}\frac{66}{272}$	Anſwer

Third, with ITALY.

In Italy they keep their accounts in livres, ſols, and deniers, and exch. by the piece of eight, or dollar, which is equal to 4*s.* 6*d.* at par.

12 Deniers	make one	Sol
20 Sols		Livre
5 Livres		Piece of $\frac{8}{8}$ at Genoa
6 Livres		Piece of $\frac{8}{8}$ at Leghorn

At Florence the exchange is in ducatoons, and at Venice by ducats, divided as follows, viz.

6 Solidi make one groſs, and 24 groſs one ducat.

E. 8. Suppoſe there be owing me, by a correſpondent at Genoa, 640 dollars, how much ſterling does it amount to, exchange at 52*d.* per dollar?

dol.		*d.*		*dol.*
1	:	52	::	640

```
   52
  640
 ----
  208
 312
 ------
12)33280
 ------
2|0) 277|3—4
 ------
Anſwer £. 138  13  4
```

Again, by Practice.

```
s. d.       dol.   s. d.
4 0 | 1/5 | 640 at 4  4
          | ---
0 4 |1/12 | 128
          |   10—13—4
          ---------------
Anſwer  £. 138  13   4  as before
```

E. 9.

E. 9. A merchant remits 138*l.* 13*s.* 4*d.* fterling to Genoa; how many dollars muft he receive there, exchange at 52*d.* per dollar?

	d.		*dollar.*		*£.*	*s.*	*d.*
As	52	:	1	::	138	13	4
					20		
					2773		
					12		

52)33280(640 Dollars, Anfwer

Note. In St. George's bank at Genoa, accounts are kept in piafters or pezzoes, which are divided into folidi and denarii, as the pound fterling.—Some merchants keep their accounts in lires or liras, folide, and denare, divided as before; this money is only one-fifth in value of the bank money.

To change current money into bank, and bank into current, they muft be proportioned thus; As 100 with the agio (that is the difference) added to it, is to 100 bank, fo is any given fum current, to its value in bank: and as 100 is to 100 with the agio added to it, fo is the bank money given to its value current.

E. 10. Change 110 guilders 12 ftivers current, into bank florins, agio $\frac{1}{5} = 4$ per cent?

	guil.		*guil.*		*guil.*	*ft.*		*guil.*	*ft.*	*gr.*	*pen.*	
As	104	:	100	::	110	12	:	106	6	1	6	the anfwer

E. 11. London is indebted to Genoa in 1710*l.* 16*s.* 4*d.* for how many pezzoes may Genoa value on London, exch. at $74\frac{1}{2}$*d.* per pezzoe?

	d.		*pez.*		*£.*	*s.*	*d.*		*pez.*	
As	$47\frac{1}{2}$	:	1	::	1710	16	4	:	8644	the anfwer

E. 12. Change 8644 *pez.* 2*s.* 6*d.* into fterling money, exchange at $47\frac{1}{2}$*d.* per pezzoe?

	pez.		*d.*		*pez.*		*£.*	*s.*	*d.*	
As	1	:	$47\frac{1}{2}$	::	8644	:	1710	16	4	Anfwer

Fourth, with PORTUGAL.

Accounts are kept in Portugal in milreas and reas, and they exchange by the milrea, which London gives from 5*s.* to 6*s.* 9*d.* for the fame.

400 Reas make one crufadoe, and 1000 reas one milrea.

E. 13. A merchant at Lifbon remits to his correfpondent in London 500 milreas, exch. at 5*s.* 6*d.* how much fterling muft he receive?

5*s.*	$\frac{1}{4}$	500 at 5*s.* 6*d.*	
6*d.*	$\frac{1}{10}$	125	
		12—10	
Anfwer		£. 137	10

m.		*s.*	*d.*		*m.*
1	:	5	6	::	500
		12			
		66			
		500			

Or by the rule of three.

12)33000

2|0) 275|0

£. 137 10 Anfwer

E. 14. How many milreas will 1566*l.* 6*s.* 6*d.* amount to, exchange at 64*d.* per milrea?

	d.		*mil.*		*£.* *s.* *d.*	
As	64	:	1	::	1566 6 6	
					20	
					31326	
					12	
	64 {		8)		375918	
			8)		46989—6 }	= 46
					5873—5 }	1000
					64 { 8)	46000
					8)	5750
Answer 5873 milr. 718 $\frac{6}{8}$ reas						718—6

Fifth, with HOLLAND, FLANDERS, and GERMANY.

In these countries their accounts are kept, sometimes in pounds, shillings, and pence, as in England, and sometimes in guilders, stivers, and pennings. In Holland and Flanders the money is distinguished by the name of Flemish; exchange being made with London from 30*s.* to 38*s.* Flemish per pound sterling?

8 Pennings	make one	Groat
2 Groats		Stiver
6 Stivers		Shilling
20 Stivers		Flor. or guilder
2$\frac{1}{2}$ Florins		Rix dollar
6 Florins		Pound Flemish
5 Guilders		Ducat

To change Flemish money into sterling, and on the contrary, sterling into Flemish, is the same with that of France, only what was French there will be Flemish here.

E. 15. A merchant in Rotterdam remits 282*l.* 5*s.* 3*d.* Flemish, to be paid in London, how much sterling money must he draw for, exchange at 34*s.* 4*d.* per pound sterling?

	s. *d.*		*£.*		*£.* *s.* *d.*
As	34 4	:	1	::	282 5 3
	12				20
	412				5645
					12
					£. *s.* *d.*
					412)67743(164 8 5$\frac{3}{4}$ $\frac{316}{412}$ Answer

E. 16. Suppose a merchant delivered in London 164*l.* 8*s.* 5$\frac{3}{4}$*d.* to receive the value at Amsterdam in Flemish money; how many pounds must he receive there, exchange at 34*s.* 4*d.* Flemish per pound sterling?

	£.		*s.* *d.*		*£.* *s.* *d.*		*£.* *s.* *d.*	
As	1	:	34 4	::	164 8 5$\frac{3}{4}$	:	282 5 3	the answer

To reduce Flemish pounds, shillings and pence, into guilders.

RULE. Divide the whole sum when reduced into pence Flemish by 40 (the number of pence in one guilder) and the quotient will be guilders;

guilders ; the remainder (if any) divide by 2 (the pence in one ſtiver) and the quotient will be ſtivers.

E. 17. In 423*l.* 8*s.* Flemiſh, how many guilders ?

£.	s.
423	8
20	

8468
12

4|0)10161|6

guil. *ſliv.*

$2540 \frac{16}{40}$ = 2540 8 Anſwer

E. 18. In 2540 guild. 8 ſtivers, how many Flemiſh pounds ?

guil.	ſliv.	d.
2540	8 =	16
40		

12)101616

2|0) 846|8

£. 423 8*s.* Anſwer

Sixth, with VENICE.

Money of exchange here is always underſtood to be ducats in bank, which is imaginary, 100 whereof make 120 ducats current money ; ſo that the difference betwixt bank and current money is an agio of 20*l.* per cent. though the brokers have invented another agio to be added, which is more or leſs, according to bargain.

The courſe of exchange of a ducat of the bank of Venice is from 45 to 50*d.* ſterling.

E. 19. Venice draws on London for 2350 ducats banco, exchange at 47*d.* per ducat, how much ſterling money will pay the draught ?

	du.		*d.*		*du.*		*£.*	*s.*	*d.*	
As	1	:	47	::	2350	:	460	4	2	Anſwer

Seventh, with POLAND and PRUSSIA.

Dantzic and Koningſberg, exchange with London by way of Amſterdam and Hamburgh ; 270 Poliſh groſch being equal to 1*l.* groſs banco in Holland, 110 Poliſh groſch being equal to 1 rix-dollar banco of Hamburgh.

18 Penningen	make one	Groſch
3 Groſch	make one	Ditkin
2 Ditkins	make one	Sixer
3 Sixers	make one	Tymph
$7\frac{1}{2}$ Groſch	make one	Arch de Halber
4 Arch de Halbers	make one	Florin or guilder
3 Florins or guilders	make one	Current Dollar
4 Gilders	make one	Specie Dollar

E. 20. Change 2342 florins into ſterling money, 270 groſchi Poli, per pound Flemiſh, and 34*s.* 4*d.* Flemiſh per pound ſterling ?

	G. P.		*£.*		*Flor.*
As	270	:	1	::	2342
					30

	£.	*s.*	*d.*	
27|0) 7026|0(	260	4	$5\frac{3}{4}$	Flemiſh

	s.	*d.*		*l.*		*l.*	*s.*	*d.*
Again, as	34	4	:	1	::	260	4	$5\frac{3}{4}$

Or, as 1648 : 1 :: 249815 : 151*l.* 11*s.* $8\frac{3}{4}$*d.* Anſwer

Eighth,

Eighth, with RUSSIA.

3 Copecs	make 1	Altine	2 Polpolitons	make 1	Poltin
10 Copecs		Grievener	2 Poltins		Rubble
25 Copecs		Polpoliton	2 Rubbles		Ducat

The Ruſſian rubbles are converted into florens current money of Amſterdam, and the current into bank money, according to agio of three or five per cent. and bank money into ſterling, according to agio of three or five per cent. and bank money into ſterling according to the courſe of exchange between England and Amſterdam.

E. 21. In 6420 rubbles, 42 copecs, exchange 122 copecs per rix-dollar current, agio 3 per cent. and 34*s.* 6*d.* Flemiſh per pound ſterling, how much ſterling money?

$$\begin{array}{l} 6420 \quad 42 \\ \quad \times 100 \\ \hline 122)\ 642042(5262\tfrac{78}{122} \text{ Rix-dollars} \\ \quad 610 \\ \hline \quad\ 320 \\ \quad\ 244 \\ \hline \quad\ \ 764 \\ \quad\ \ 732 \\ \hline \quad\ \ \ 322 \\ \quad\ \ \ 244 \\ \hline \quad\ \ \ \ 78 \end{array}$$

$$\begin{array}{l} 5262\tfrac{78}{122} \text{ Rix-dollars} \\ \quad \times 2\tfrac{1}{2} \\ \hline \left.\begin{array}{l} 10525\tfrac{34}{122} \\ 2631\tfrac{39}{122} \end{array}\right\} \text{Florins current} \\ \hline 13156\tfrac{73}{122} \end{array}$$

	Fl. cur.		Flor. ba.				
	As 103	:	100	::	13156 $\frac{73}{122}$		
Or, as	12566	:	100	::	1605105	:	12773 $\frac{2421}{6283}$ Flor. ba.

Now $12773\frac{2421}{6283} \times 40 = 510935\frac{5325}{6283}$ Pence

And 34*s.* 6*d.* = 414*d.* Then,

d.		l.		d.		l.	s.	d.
As 414	:	1	::	510935 $\frac{5325}{6283}$	:	1234	2	10$\frac{1}{2}$ Anſwer

Ninth, with IRELAND.

In Ireland they keep their accounts in pounds, ſhillings, and pence Iriſh, divided as in England: but having no coins of their own, they are ſupplied by the different countries with which they traffic.

The par of exchange between England and Ireland is 100*l.* ſterling for 108*l.* 6*s.* 8*d.* Iriſh; or 1*s.* Engliſh for 13*d.* Iriſh.

The courſe of exchange is from 5 to 12 per cent, according to the balance of trade.

E. 22. Dublin draws upon London for 370*l.* 7*s.* 3*d.* Iriſh exchange at 12 per cent. how much ſterling muſt London pay Dublin to diſcharge this bill?

	l.		l.		l.	s.	d.	
As	112	:	100	::	370	7	3	

d.		l.		d.		l.	s.	d.
Or, as 26880	:	100	::	8888700	:	330	13	7$\frac{1}{4}$ $\frac{1516}{2688}$ Anſ.

E. 23. London remits to Ireland 330*l.* 13*s.* $7\frac{1}{4}$*d.* ſterling; how much Iriſh muſt London be credited with, exchange at 12 per cent?

	l.		*l.*		*l.* *s.* *d.*		*l.* *s.* *d.*	
As	100	:	112	::	330 13 $7\frac{1}{4}$	:	370 7 3	the Anſwer

Tenth, with AMERICA and the WEST-INDIES.

Accounts are kept, and the money divided, as in England; their money is called currency.

The ſcarcity of caſh obliges them to ſubſtitute a paper currency for carrying on their trade; which being ſubject to caſualties, ſuffer a very great diſcount for ſterling in the purchaſe of bills of exchange.

E. 24. Philadelphia is indebted to London 4168*l.* 16*s.* $10\frac{1}{2}$*d.* currency, what ſterling may London reckon to be remitted, when the exchange is 150 per cent?

	l.		*l.*		*l.* *s.* *d.*		*l.* *s.* *d.*	
As	150	:	100	::	4168 16 $10\frac{1}{2}$	:	2779 4 7	Anſwer

E. 25. A, at Paris, draws on B, of London, 1200 crowns, at 55*d.* ſterling per crown; for the value whereof B draws again on A 56*d.* ſterling per crown, beſides commiſſion $\frac{1}{2}$ per cent. Did A gain or loſe by this tranſaction, and what?

	l.		*s.*		*l.*		*s.* *d.*	
If	100	:	10	::	275	:	27 6	the commiſſion

Then 275*l.* + 1*l.* 7*s.* 6*d.* = 276*l.* 7*s.* 6*d.* = 66330*d.*

Therefore, as 56*d.* : 1 cr. :: 66330*d.* : $1184\frac{13}{28}$ crowns

Conſequently, 1200 — $1184\frac{13}{28}$ = $15\frac{15}{28}$ crowns, A's gain, Anſwer

E. 26. V, of Amſterdam, draws on X, of Hamburgh, at 67*d.* Flemiſh per dollar, of 32 ſols Lubeck; and on Y of Nurembergh, at 70*d.* Flemiſh per florin, of 63 crutzers current. If V has orders to draw on X, in order to remit to Y at the ſaid prices, how would run the exchange between Hamburgh and Nuremberg?

	d.		*ſols.*		*d.*		*ſols.*
As	67	:	32	::	70	:	$33\frac{29}{67}$ Lubeck, per florin, anſwer

E. 27. M, of Amſterdam, orders N, of London, to remit O, of Paris, at 54*d.* ſterling per crown, and to draw on P, of Antwerp, for the value, $33\frac{1}{2}$*s.* Flemiſh per pound ſterling; but as ſoon as N received the commiſſion, the exchange was on Paris at $54\frac{1}{2}$*d.* per crown; pray at what rate of exchange ought N to draw on P to execute his orders, and be no loſer?

	d.		*s.* *d.*		*d.*		*d.*	*s.* *d.*	
If	54	:	33 6	::	$54\frac{1}{2}$	:	$398\frac{34}{109}$ =	33 $2\frac{34}{109}$	Flem. anſ.

At

	d.		s. d.		d.
At length thus, as	54	:	33 6	::	54½
	2		12		2
	108		402		109

402
108
3216
402
109) 43416 (398*d.* $\frac{34}{109}$ Anfwer
327
1071
981
906
872
34

XXXI. Comparifon of WEIGHTS and MEASURES,

IS when the weights or meafures of different countries are compared together; and is a very neceffary rule (of great importance to the merchant) to be acquainted with.

RULE. Place the numbers alternately under each other, in two perpendicular columns, fo that there may not be found in either column two terms of one kind; then the numbers in the leffer column muft be multiplied together for a divifor; and the numbers in the greater column, where the odd term is, for a dividend; the quotient will be the anfwer. The work may often be abridged by throwing out numbers that are alike in both columns.

EXAMPLE 1. If 6 pounds of fugar be equal in value to 7 pounds of raifins; 5 pounds of raifins to 4 yards of ribbon; 10 yards of ribbon to 40 nutmegs, and 7 nutmegs to 18 pence; what is 3 pounds of fugar worth?

6 Sugar	7 Raifins
5 Raifins	4 Ribbon
10 Ribbon	40 Nutmegs
7 Nutmegs	18 Pence
	3 Sugar

And 6
5
30
10
300
7
2100

Then, per rule, 7
4
28
40
1120
18
20160
3
21|00) 604|80 (28
42
184
168
16

Anfwer 28 $\frac{16}{21}$ pence

E. 2.

E. 2. If 100 *lb.* at Copenhagen be equal to 80 *lb.* at Rome, and 100 *lb.* at Rome be equal to 114 *lb.* at Madrid; how many pounds at Madrid are equal to 180 *lb.* at Copenhagen?

l.	*lb.*	
100 =	80	Then, 80 × 114 × 180 = 1641600
100 =	114	And 100 × 100 = 10000
	180	

Also 10000) 1641600 (164 $\frac{4}{25}$ pounds, the answer

E. 3. Suppose 100 *lb.* of Portugal be equal to 92 *lb.* of Antwerp, and 100 *lb.* of Antwerp be equal to 110 *lb.* at Lyons; how many pounds at Lyons are equal to 60 *lb.* of Portugal?

100	92	Then 92 × 110 × 60 = 607200
100	110	And 100 × 100 = 10000
	60	

Also 10000) 607200 (60 $\frac{72}{100}$ pounds, the answer

XXXII. POSITION;

OR,

THE RULE OF FALSE,

IS so called, because we suppose, or make a position of some uncertain numbers, in order that by reasoning from them we may gain the true number sought; and because those positions are altogether at randon, or adventure, the rule is also called false.

The use of this rule, before the common knowledge of algebra, was much more considerable than since; because that art supplies theorems for resolving all kinds of questions in this rule in a better and more curious manner than here.

Some authors have entirely discarded it, and others postpone it, as obsolete, and of little use since algebra; but, in my opinion, it is a very good approximation, and in exponental equations, as well as many other things, succeeds better than any other method, and is very useful in solving many intricate problems, not only in arithmetic and algebra, but in the more abstruse parts of the mathematics (as Mr. Emerson remarks in his Cyclomathesis, page 151) where he says, in many difficult problems, there is hardly any other way to come at a solution, but by this method of trial and error.

Questions in this are performed by one or two suppositions; if by one, the rule is said to be of single position; if two suppositions are necessary, it is called double position.

SINGLE POSITION.

RULE. Make choice of some fit number, and proceed with this, according to the nature of the question, as if it were the true number, and if you find the result either too much, or too little, you may then find the answer by the rule of three, viz.

As the result of this position is to the position, so is the given number to the number required.

EXAMPLE 1. What sum is that, of which the half, third, and fourth, makes 520*l*?

Suppose the sum to be 96
Then the $\frac{1}{2}$ is 48
The $\frac{1}{3}$ is 32
The $\frac{1}{4}$ is 24

Result 104

Then, if 104 : 96 :: 520

$$\begin{array}{r} 520 \\ \hline 192 \\ 480 \\ \hline 104)49920(480 \text{ Ans.} \\ 416 \\ \hline 832 \\ 832 \\ \hline 0 \end{array}$$

For the half of 480 = 240
The third - - = 160
And the fourth = 120

Sum = 520 Proof

E. 2. A, B, and C, buy a parcel of timber, which cost 48*l.* and it is agreed that B should pay a third part more than A, and C a fourth part more than B; what sum must each pay?

Suppose A pays 3
Then B's part is 4
And C's - - 5

Result - 12

$$\text{Then, as } 12 : 48 :: \begin{cases} 3 : 12 \text{ A's share} \\ 4 : 16 \text{ B's share} \\ 5 : 20 \text{ C's share} \end{cases}$$

£. 48 Proof

E. 3 A schoolmaster being asked how many scholars he had, answered, if I had as many, half as many, and one-fourth as many, I should have 198; how many had he?

Suppose he had 16
Then, as many - 16
Half as many - - 8
One-fourth ditto - 4

Sum 44

If 44 : 16 :: 198

$$\begin{array}{r} 16 \\ \hline 1188 \\ 198 \\ \hline 44)3168(72 \text{ Scholars} \end{array}$$

Answer

E. 4. *An old woman of above threescore and ten,*
Has buried four husbands, and married again
To Jerry the mugman, a bagpiper rare!
And none can with him for his music compare;
The music he play'd pleas'd the old woman much,
Till she hopp'd, and she caper'd about without crutch.
Though wrinkled and wither'd—no tooth in her head,
Yet money she had, and she got married:
To his bagpipes she mov'd, with one foot in the grave,
For all her delight was a husband to have!
The sum of both ages one hundred years are,
Wanting five;—and one-fourth of her age I declare,
Is the age of the husband;—now Tyro you'll find
The bagpiper's age, with his spouse's so kind.

Suppose

Suppose the wife's age to be - - 60
Then the husband's will be - - 15

Sum 75

Then, as 75 : 60 :: 95 The sum of both their ages
60

75) 5700(76 The wife's age
Then, per question, 76 ÷ 4 = 19 The husband's age
Proof 95

E. 5. A man overtaking a maid driving a flock of geese, said to her, How do you do, sweetheart? where are you going with these 80 geese? No, Sir, said she, I have not 80; but if I had as many more, half as many more, and 20 geese besides, I should have 80; how many geese had she?

Suppose she had 20	Then 80	As 50 : 20 :: 60	
Then as many 20	— 20	60	
One-half as many 10	—	5|0) 120|0	
	60		
Sum 50		24	Her flock, Answer

XXXIII. DOUBLE POSITION,

IS when two suppositions are used, because here the numbers cannot be parted to find the answer as before; therefore, when we make two suppositions, and miss in both, observe the nature of the errors, whether they be greater or less than the number proposed; and accordingly mark them with the signs + or —; and place them against their proper suppositions; but if with either of the suppositions we find the number that answers the question, the work is done.

When the errors are equal, and have unlike signs, half the sum of the suppositions is the number sought.

RULE. As the difference of the errors, if alike, or their sum, if unlike, is to the difference of the suppositions; so is either of the errors to a fourth number, which added to the supposition over-against it, if less, or subtracted from it, if more, gives the number sought.

EXAMPLE 1. A man agreed to thrash 60 bushels of corn, part of it wheat, and part oats, at the rate of 2*d.* per bushel for the wheat, and $1\frac{1}{2}$*d.* for the oats; at last he received 8*s.* for his labour; how much of each did he thrash?

Suppose there were 30 bushels of wheat, price 60 pence
Then there are 30 bushels of oats, price 45 pence

Too much - - 105
There should only be - - - - - - - - - 96

First error - + 9

Again, suppose 20 bushels of wheat, price = 40 pence
Then there will be 40 bushels of oats, price = 60 pence

Too much - - - 100
96

Second error - - + 4

Sup. *er.*
30 × 9
20 × 4

180 120

Then 180
— 120

60 Diff. of prod.

Also 9
— 4

5 Difference of the errors

Therefore 5) 60

12 *bu.* of wheat
Consequently there is 48 Ditto oats

Total 60

PROOF.

bu.	*d.*	*d.*
12 at	2	= 24
And 48 at	$1\frac{1}{2}$	= 72
	Sum	= 96 = 8*s*.

E. 2. A gentleman finding several beggars at his door, gave each of them 3*d*. a-piece, and had 5*d*. remaining; he would have given them 4*d*. a-piece, but wanted 7*d*. to do it; how many beggars were there?

Suppose the No. of beggars 14
3
42
+ 5
His money 47

And 14
4
56
— 7
49 His money also
47
The first error + 2

Again, suppose the number to be 10
3
30
+ 5
35
33
Second error — 2

And 10
4
40
— 7
33

Sup. *er.*
14 × 2
10 × 2
20 28

2
2
4

28
20
) 48
12 Beggars, the Answer

E. 3. Double my money for me, said A to B, and I will give thee 6*d*. out of the stock, with the remainder; he applied in like manner to C, with equal success, and gave him also 6*d*. he repeated this proposal to D, and then 6*d*. was all he had to give. Pray what sum had he to begin with?

Suppose

Suppose he had 10*d*.	Again, suppose he had 7*d*.
Then $10 + 10 = 20$	Then $7 + 7 = 14$
Also $20 - 6 = 14$	$14 - 6 = 8$
$14 + 14 = 28$	$8 + 8 = 16$
$28 - 6 = 22$	$16 - 6 = 10$
$22 + 22 = 44$	$10 + 10 = 20$
$44 - 6 = 38$	$20 - 6 = 14$
Too much	Too much also

Sup. *er.*

$$\begin{array}{cc} 10 & 38 \\ 7 & 14 \\ \hline 266 & 140 \end{array}$$

Then

$$\begin{array}{rr} 38 & 266 \\ 14 & 140 \\ \hline 24\,) & 126\,(5\tfrac{1}{4}d. \text{ Answer} \\ & 120 \\ \hline & 6 \\ & 4 \\ \hline 24) & 24(\tfrac{1}{4} \end{array}$$

E. 4. *When first the marriage knot was tied*
Betwixt my wife and me,
My age did her's as far exceed,
As three times three doth three ;
But when ten years, and half ten years,
We man and wife had been,
Her age came up as near to mine
As eight is to sixteen.
What both our ages was, I pray,
Now tell me, on the wedding-day?

Suppose her age to be - 13
Then (per quest.) he will be $13 \times 3 =$ 39
$13 + 10 + 5 =$ 28
$39 + 10 + 5 =$ 54 } Subt.
$28 \times 2 =$ 56 }
First error -2

Again, suppose her age to be - - 17
Then (per quest.) he will be $17 \times 3 =$ 51
$17 + 10 + 5 =$ - - 32
$51 + 10 + 5 =$ - - 66 } Subt.
$32 \times 2 =$ - - - 64 }
Second error $+2$

Sup. *er.*

$$\begin{array}{cc} 13 & 2 \\ 17 & 2 \\ \hline 34 & 26 \end{array}$$

$$\begin{array}{rr} 2 & 34 \\ 2 & 26 \\ \hline 4\,) & 60 \\ & 15 \end{array}$$

15 Years, her age when married

Then $15 \times 3 = 45$ years, his age when married
$15 + 10 + 5 = 30$ her age } At the time proposed
$45 + 10 + 5 = 60$ his age }

For, as $8 : 16 :: 30$

$$\begin{array}{r} 30 \\ \hline 8)480 \\ \hline 60 \text{ Proof} \end{array}$$

Note. You may observe, that when the errors happen to be alike in quantity, but unlike in quality (as in the above solution) the answer may be more easily obtained than by proceeding as above (as I hinted

at

at the beginning of this section) for in such case, half the sum of the suppositions will be the number sought, as in this solution the sum of the suppositions $13 + 17 = 30$, half whereof is 15, the same as above.

EXAMPLE 5.

A farmer with a plowman doth agree,
That 30 days his servant he should be;
Each day he wrought the farmer is to pay
Him 16 pence; but when he was away,
Five groats he is for each day to abate.
The time expired, they their accompts do state,
Whereby the master nothing is to give,
Nor has the servant any to receive.
How many days he wrought I do demand,
And how many he play'd I'd understand.

	d.	
Suppose he wrought 15 days, his wages at 16*d.* per day =	240	} subt.
Then (per q.) he was idle 15 da. which at 20*d.* per day =	300	
First error, too little	60	
Again, sup. he wr. 20 days, his wages at 16*d.* per day =	320	
Then (per q.) he was idle 10 days, which at 20*d.* per d. =	200	
Second error, too much +	120	

Sup.	*er.*		
15	60	Then 60	1800
20	120	120	1200
1200	1800	18|0) 300|0(16 *days.*	8 *hours.* the Answer

E. 6. A thief breaking into an orchard, stole from thence a certain number of apples, and at his coming forth he met with three men, one after another, who threatened to accuse him of theft; and, in order to appease them, he gave unto the first man half the apples he had stolen, who returned him back 12 of them; then he gave unto the second half of those he had remaining, who returned him back 7 of them; and unto the third person he gave half the residue, who returned him back 4; at last getting safe away, he finds he has 20 left. How many had he at first?

Suppose he had - - -	60	Again, suppose he had - -	92
Gave the first 30 — 12 =	18	Then, 46 — 12 =	34
Remains -	42	Remains - - -	58
Gave the second 21 — 7 =	14	29 — 7 =	22
Remains -	28	Remains - - -	36
Gave the third 14 — 4 =	10	18 — 4 =	14
Too little	18	Too much - -	22
Should be	20	Should be - - -	20
First error	— 2	Second error	+ 2

Sup.

Sup.	*er.*		
60 ×	2	Then 2	184
92	2	+ 2	+ 120
184	120	4)	304

Anfwer 76 what he had at firft

Or, thus 60 + 92 = 152, which ÷ 2 = 76, the Anfwer as before

E. 7. *A man that was idle, and minded to ſpend*
Both money and time went to drink with a friend;
He ſaid to his hoſt, if you'll now to me lend
As much coin as I have, then my ſix-pence I'll ſpend.
His hoſt lent the money, his ſix-pence he ſpent,
And having ſo done, to another houſe went,
Where the ſame he requeſted, and the ſame ſum he ſpent:
He went to a third houſe, where, Landlord, cries he,
Lend me as much money as here you ſee,*
Which having received, his ſix-pence he ſpent,
So all being gone home, the fuddle-cap went
To caſt up his reck'nings; but his head aching ſore,
He beg's you to do it, and he'll do ſo no more;
What had he at firſt, and how much on ſcore?

	d.		d.	*Sup.*	*er.*
Suppoſe he had	8	Again, ſuppoſe he had	7	8 ×	22
Then 8 + 8 =	16	Then 7 + 7 =	14	7	14
Alſo 16 — 6 =	10	14 — 6 =	8		
10 + 10 =	20	8 + 8 =	16	154	112
20 — 6 =	14	16 — 6 =	10		
14 + 14 =	28	10 + 10 =	20		
28 — 6 =	22	20 — 6 =	14		
Too much		Too much			

Then, 22 — 14 = 8, and 154 — 112 = 42 ∴ 42 ÷ 8 = $5\frac{1}{4}$ Anſwer

E. 8. *Gentlemen, of you I now muſt enquire,*
How the poll ſtood for the knights of our ſhire?
The number of votes, as I have ſeen,
Was five thouſand two hundred and nineteen;
Which amongſt four was juſt ſo divided,
As one the ſecond and third exceeded
By twenty-two and four-ſcore, bating ſeven,
The fourth by no more than ſix-ſcore and ten:
Then how many votes had each candidate?
You cannot in finding much trouble your pate.

* Shewing what he had left.

Suppoſe

		Votes.			Votes.
Suppose the first had	-	2000	Again, suppose the first had		1600
The 2d	$2000 - 22 =$	1978	The 2d	$1600 - 22 =$	1578
The 3d	$2000 - 73 =$	1927	The 3d	$1600 - 73 =$	1527
The 4th	$2000 - 130 =$	1870	The 4th	$1600 - 130 =$	1470
	Too much -	7775		Too much -	6175
	Should be -	5219		Should be - -	5219
	First error	+ 2556		Second error	+ 956

Sup.		er.
2000	×	2556
1600		956
4089600		1912000

Then $2556 - 956 = 1600$, and $4089600 - 1912000 = 2177600$ $\because$ $2177600 \div 1600 = 1361$, the number of votes the first candidate had.

The second had $1361 - 22 = 1339$; the third had $1361 - 73 = 1288$; and the fourth had $1361 - 130 = 1231$ votes, the answer. For $1361 + 1339 + 1288 + 1231 = 5219$ proof.

E. 9. There is a fish, whose head is 9 inches long; the tail as long as his head and half his body, and his body is as long as both his head and tail; I demand the whole length of the fish?

Suppose the body to be 12 inches

Then $\overline{12 \div 2} + 9 = 15$ tail, also $15 + 9 = 24$, too much by 12

Again, suppose the body to be 14 inches

Then $\overline{14 \div 2} + 9 = 16$ tail; also $16 + 9 = 25$, too much by 11

Sup.		er.
12	×	12
14		11
168		132

Then 12 168
— 12 132
0) 36 (
36 Length of the body

And $\overline{36 \div 2} + 9 = 27$ length of the tail; therefore $36 + 27 + 9 = 72$, length of the whole fish, answer

E. 10. *A painter of skill and much fame in the town,*
Had procur'd himself work for more hands than his own;
He employ'd an assistant, to help him in part,
A proficient in every branch of his art.
O'er a glass of good wine upon terms they debate,
And the bottle was drained while they state and unstate,
For as plenty of Bacchus' enlivening juice,
Does most commonly projects and whimsies produce;
So when that their spirits grew warm with the liquor,
Fresh maggots were started, and fancies grew quicker;

The

They were long in contriving what both sides could please,
And at length the proposals agreed on were these:
For a single year's service the man should be ty'd;
And for every day that he was full employ'd
Seven shillings per day should his wages be paid;
But for all such as those when he rested or play'd,
He should forfeit three shillings; the year was compleat,
Neither master nor man was in each other's debt.
Now, what time he neglected, ye artists, is sought,
And how much for his master in painting he wrought?

First, suppose he wrought 100 days; then $365 - 100 = 265$ days he was idle

Therefore, $100 \times 7 = 700$
And, $265 \times 3 = 771$ } Subtract $= 95$, too little

Again, sup. he wr. 108 days, then $365 - 108 = 257$ days he played

Therefore, $108 \times 7 = 756$
And $265 \times 3 = 771$ } Subtract $= 15$, too little

Sup.	*er.*	
100	95	Then $10260 - 1500 = 8760$.
108 ×	15	And $95 - 15 = 80$ ∴ $8760 \div 80 = 109\frac{1}{2}$ days
		he wrought. And $365 - 109\frac{1}{2} = 255\frac{1}{2}$ days
10260	1500	idle, the answer

For $109\frac{1}{2} \times 7 = 255\frac{1}{2} \times 3 = 766\frac{1}{2}$ Proof

E. 11. If $\frac{2}{3}$ of my age be added to $\frac{4}{6}$ thereof, and that sum multiplied by 4, that product divided by 8, and that quotient made less by 8, the remainder will be 14; what was my age at the time of making this question?

Suppose my age to be 21

$$\begin{array}{lr} \text{Then} & 21 \\ & 2 \\ \hline & 3)42 \\ \hline & 14 = \frac{2}{3} \\ & + 14 = \frac{4}{6} \\ \hline & 28 \\ & \times 4 \\ \hline & 8)112 \\ \hline & 14 \\ & 8 \\ \hline \text{Too little} & - 6 \\ \text{Should be} & - 14 \\ \hline \text{First error} & -8 \end{array} \quad \Big\} \text{ Subt.}$$

Again, suppose my age to be 27

$$\begin{array}{lr} \text{Then} & 27 \\ & 2 \\ \hline & 3)54 \\ \hline & 18 = \frac{2}{3} \\ & + 18 = \frac{4}{6} \\ \hline & 36 \\ & \times 4 \\ \hline & 8)144 \\ \hline & 18 \\ & 8 \\ \hline \text{Too little} & 10 \\ \text{Should be} & 14 \\ \hline \text{Sec. er.} & -4 \end{array}$$

Sup. *er.*

$$\begin{array}{rr} 21 \times & 8 \\ 27 & 4 \\ \hline 216 & 84 \end{array}$$

$$\begin{array}{rr} 8 & 216 \\ 4 & 84 \\ \hline 4) & 132 \\ \hline \text{Ans.} & 33 \text{ Years} \end{array}$$

Note, it will sometimes shorten the work by making a cypher and unit the two suppositions.

E. 12. What number is that, which being multiplied by 6, the product increased by 18, and the sum div. by 9, the quot. will be 20?

First, suppose 30 to be the number sought; then $\frac{30 \times 6 + 18}{9}$ $= 10 \times 2 + 2 = 22$; but ought to have been 20; therefore the error is 2 in excess.

Again, suppose 18 the number sought; then $\frac{18 \times 6 + 18}{9} = 2 \times 6 + 2 = 14$; but ought to be 20; therefore the error is 6 in defect, and the errors are of different kinds or affections.

$$\begin{array}{cc|l} \textit{Sup.} & \textit{er.} & \\ 30 & 2 & \text{Then } 180 + 36 = 216, \text{ and } 6 + 2 = 8 \;\therefore 216 \div 8 \\ 18 & 6 & = 27, \text{ the number sought} \\ \hline 36 & 180 & \end{array}$$

But to work this by the note in the preceding page, suppose first 0; then $\frac{0 \times 6 + 18}{9} = \frac{18}{9} = 2$; but ought to be 20; therefore the error is 18 in defect.

Again, suppose 1; then $\frac{1 \times 6 + 18}{9} = \frac{2 + 6}{3} = \frac{8}{3} = 2\frac{2}{3}$; but should have been 20; therefore the error is $17\frac{1}{3}$ in defect also, and the errors are of the same kind.

Whence per rule, $\frac{0 \times 17\frac{1}{3} \backsim 18 \times 1}{18 - 17\frac{1}{3}} = \frac{18}{\frac{2}{3}} = 9 \times 3 = 27$, the number sought.

XXXIV. ARITHMETICAL PROGRESSION,

IS when a rank or series of numbers increase or decrease by a common difference, or by a continual adding or subtracting some equal numbers,

As, $\left\{ \begin{array}{l} 1, 2, 3, 4, 5, 6 \\ 6, 5, 4, 3, 2, 1 \end{array} \right\}$ Here the common difference is 1.

Or 1, 3, 5, 7, 9, 11; here the common difference is 2.

Also 30, 25, 20, 15, 10, 5; here the common difference is 5.

The numbers that compose a rank or series of progressionals, are called its terms, whereof the first and last are called extremes, and any two equally distant from them, means. Now when the number of terms are even, as 1, 3, 5, 7, 9, 11, the sum of the two extremes will be equal to the sum of any two means that are equally distant from the extremes, viz. 1, 3, 5, 7, 9, 11,

$$1 + 11 = 5 + 7 = 3 + 9 = 12.$$

When the numbers of terms are odd, as 4, 10, 16, 22, 28, the double of the middle figure or term will be equal to the sum of the extremes, or to any two means equally distant from the middle term.

Viz. 4, 10, 16, 22, 28,

$$16 + 16 = 32, \quad 22 + 10 = 32.$$

In this rule there are five things to be considered, viz.

1. The first term, commonly the least.

2. The

2. The laſt term, commonly the greateſt.
3. The number of terms.
4. The common exceſs, or difference.
5. The aggregate, or ſum of all the terms.

Any three of which being given, the other two may be eaſily found.

PROPOSITION 1. When the two extremes, and the number of terms are given, to find the ſum of all the ſeries or terms,

RULE. Multiply the ſum of the two extremes by half the number of terms, or multiply the ſum of the two extremes into the number of terms, and divide the product by 2; the quotient will be the ſum of all the ſeries.

E. 1. How many ſtrokes does the hammer of a clock ſtrike in 12 hours?

$1+12=13$ the ſum of the extremes
6 half the No. of terms

Anſwer 78 Strokes

Or thus:

$1+12=13$ ſum of the extremes
12 the numb. of terms

2)156

Anſwer 78 Strokes, as before

E. 2. Suppoſe 100 ſtones were placed in a right line, a yard diſtant from one another, and the firſt ſtone was a yard from the baſket; I demand how many miles he muſt travel that gathers them ſingly into the baſket?

$2+200=202$ The ſum of the extremes
$\times$ 50 Half the number of term
Yds. ——— *Miles.*
A mile $=$ 1760)10100 (5 $\frac{3}{4}$ wanting 20 yards, the diſtance ran,
8800 Anſwer

1300 Yards

E. 3. A butcher buys 100 ſheep, and gave for the firſt ſheep 1*s.* and for the laſt 9*l.* 19*s.* I demand what he gave for the 100 ſheep?

First 9*l.* 19*s.* $=$ 199 Shillings
Then $1+199$ $=$ 200 The ſum of the extremes
And $100\div2$ $=$ 50 Half the number of terms

2|0) 1000|0

Anſwer £. 500

PROPOSITION 2. When the two extremes, and number of terms are given, to find the common difference,

RULE. The difference of the two extremes divided by the number of terms leſs 1, the quotient will be the common difference.

E. 4. One who had 12 children, that differed alike in their ages; the youngeſt was 5 years old, the eldeſt 27; what was the difference of their ages, and the age of each?

$27-5=22$ The difference of the extremes
And $12-1=11$ The number of terms leſs 1
$\therefore$ $22\div11=2$ The common difference

Which added to the age of the youngeſt, and ſo on to the reſt, will give their ſeveral ages, viz. $5+2=7$, the age of the ſecond, and ſo for the reſt.

E. 5. A debt is to be discharged at 10 different payments in arithmetical progression; the first payment is to be 5*l.* and the last 50*l.* what is the whole debt, and what must each payment be?

First $50 - 5 = 45$ Difference of the extremes
Then $10 - 1 = 9$ Number of terms less 1
$\because$ $45 \div 9 = 5$ The common difference
Consequently $5 + 10 + 15 + 20 + 25 + 30 + 35 + 40 + 45 + 50 = 275$*l.* the whole debt.

Prop. 3. When the two extremes and common difference are given, to find the number of terms,

RULE. Divide the difference of the two extremes by the common excess; add unity or 1, to the quotient, and the sum will be the number of terms.

E. 6. A man being asked how many children he had, answered, my youngest child is five years old, and the eldest 27, and that he had increased one in his family every two years; how many children had he?

First $27 - 5 = 22$ Difference of the extremes
Then $22 \div 2 = 11$ Number of terms less 1
$\therefore$ $11 + 1 = 12$ Children, the answer

E. 7. A person was to go a journey, and his first day's travel is to be 6 miles, and the last 60, every day increasing his journey three miles? how many days would he be in compleating the same?

First $60 - 6 = 54$ Difference of the extremes
And $54 \div 3 = 18$ $\therefore$ $18 + 1 = 19$ Days, the answer

Prop. 4. When the last term, the common difference, and the number of terms are given, to find the first term,

RULE. Multiply the number of terms less 1, by the common difference; the product subtracted from the last term leaves the first.

E. 8. A man in 19 days went from Birmingham to a certain place; every day's journey was greater than the preceding one by three miles, his last day's journey was 60 miles, what was the first?

First $19 - 1 = 18$ Number of terms less 1
Then 18×3 (the common difference) $= 54$
And $60 - 54 = 6$ The first day's journey, answer

Prop. 5. When the number of terms, common difference, and sum of all the terms are given, to find the first term,

RULE. Divide the sum of all the series by the number of terms, and from that quotient subtract half the product of the common difference, multiplied by the number of terms less 1, gives the first term.

E. 9. An amiable lady being in company with a very agreeable young gentleman, told him that in two years and a half she should receive the whole of her fortune, which was 1000*l.* that next quarter-day she should receive the first payment, and each payment after

would

would exceed the former by 20*l*. "Now, Sir, (says she) I am free to give you the first payment, provided you will tell me what it is from the given data." But the young gentleman being unskilled in numbers, could not comply with her proposal, but leaves it to the study of the ingenious arithmetician to resolve the question, and tell him the first payment?

First $1000 \div 10$ (the quarters in $2\frac{1}{2}$ years) $= 100$

And $10 - 1 = 9$, which multiplied by 20 (the common difference) the product is 180: then $180 \div 2 = 90$, which deducted from 100, leaves 10*l*. the answer.

Or thus: From $1000 \times 2 = 2000$

Take $10 \times 20 \times 9 = 1800$

Divisor $10 \times 2 = 2|0)\ 20|0$

£. 10 The answer as above

Prop. 6. When the first term, number of terms, and the common difference, are given, to find the last term,

RULE. Subtract the common difference from the product of the number of terms, multiplied by the common difference; the remainder added to the first term will give the last.

E. 10. What is the last term of an arithmetical progression, beginning at 6, and continuing by the increase of 3 to 21 places?

First $21 \times 3 = 63$, and $63 - 3 = 60$

Then $60 + 6 = 66$, the last term required.

E. 11. What is the last term of an arithmetical progression, beginning at 1, and continuing by the increase of 2 to 50 places?

First $50 \times 20 = 100$; then $100 - 2 = 98$

And $98 + 1 = 99$, the last term required.

Prop. 7. The first term, common difference, and number of terms given, to find the sum of all the terms,

RULE. From the product of the number of terms in the common difference, subtract the common difference, and to the remainder add the double of the first term; half the product of that sum multiplied by the number of terms, gives the sum of all the terms or series.

E. 12. Suppose I agree with a pump-maker to sink a well 30 yards deep, upon these terms, viz. to pay him three shillings for the first yard, five for the second, seven for the third, &c. raising two shillings for every yard; what will the whole amount to?

First $30 \times 2 = 60$; also $60 - 2 = 58$

Again $58 + 9 = 64$; and $64 \times 30 = 1920$

$\because$ $1920 \div 2 = 960$*s*. $= 48$*l*. the answer.

Prop. 8. The first term, the number of terms, and sum of all the terms given, to find the common difference,

RULE. Divide the double sum of all the series by the number of terms, and from the quotient subtract double the first term; divide the remainder

remainder by the number of terms leſſened by unity, the quotient will be the common difference.

E. 13. A perſon travelled from London to York, being 180 miles, in 6 days, and every day travelled equally further than the preceding day; it is known that the firſt day he travelled 6 miles, how many miles did he travel each of the other days?

Firſt $360 \div 6 = 60$, and $60 - 12 = 48$; alſo $6 - 1 = 5$

Then $48 \div 5 = 9\frac{3}{5}$ miles, the common difference required

$\because$ $9\frac{3}{5}$ Added to 6, and every other term reſpectively gives as follows, viz.

$$\left.\begin{array}{rl} 6 & \text{for the firſt} \\ 15\frac{3}{5} & \text{ſecond} \\ 25\frac{1}{5} & \text{third} \\ 34\frac{4}{5} & \text{fourth} \\ 44\frac{2}{5} & \text{fifth} \\ 54 & \text{ſixth} \end{array}\right\} \text{Days journey. Q. E. F.}$$

Proof 180 Miles

Prop. 9. When one perſon or thing moves with an equal, and another the ſame way by a progreſſive motion, to find in what time the firſt will be overtaken.

RULE. To double the ſpace gone each day by the purſued, add the common difference of the purſuer's day's journey; from that ſum ſubtract double the ſpace he travelled the firſt day, and divide the remainder by the common difference, the quotient will give the number of days, in which the purſued will be overtaken by the purſuer.

E. 14. A noted highwayman having committed a robbery, and ſuſpecting a purſuit, rode off at the rate of 40 miles a day; a thief-taker (one of Sir John Fielding's men) upon the ſcent, follows him in a progreſſive motion only 30 miles the firſt day, 34 the ſecond, 38 the third, and ſo on, increaſing every day's journey 4 miles; in how many days will the highwayman be overtaken?

Firſt $80 + 4 = 84$, and $84 - 60 = 24$

Then $24 \div 4 = 6$ days, the anſwer

For $6 \times 40 = 240$ miles, the ſpace travelled by the robber

Then by prop. 7. $6 \times 4 = 24$, alſo $24 - 4 = 20$, and $20 + 60 = 80$ $\because$ $80 \div 2 \times 6 = 240$ miles when the thief-taker comes up with the highwayman.

XXXV. GEOMETRICAL PROGRESSION,

IS, when any rank or ſeries of numbers increaſe by one common multiplier, or decreaſe by one common diviſor.

As 4, 8, 16, 32, 64, 128; here the common multiplier or ratio is 2.

Alſo, 729, 243, 81, 27, 9, 3; here the com. diviſor or ratio is 3.

In any ſeries of numbers in geometrical progreſſion, the product of the two extremes are equal to the product of any two means, that are equally diſtant from the extremes.

As

As 3, 9, 27, 81, 243, 729.

Here $3 \times 729 = 9 \times 243 = 27 \times 81 = 2187$.

When the number of terms are odd, the middle term multiplied into itself will be equal to the product of the two extremes, or any two means equally distant from the said mean or middle term.

As 3, 6, 12, 24, 48. $12 \times 12 = 6 \times 24 = 48 \times 3 = 144$.

In geometrical progression, five things are to be observed, as in arithmetical progression, viz.

1. The first term. 2. The last term. 3. The number of terms. 4. The ratio 5. The sum of the terms.

Any three of these being known, the rest may be easily found.

If over any rank of geometrical numbers, you place a series of arithmetical ones beginning with 0, the addition and subtraction of the indices, answer to the multiplication and division of the numbers they stand over.

Thus $\begin{cases} 0, 1, 2, 3, 4, 5, 6, 7 & \text{Indices} \\ 1, 2, 4, 8, 16, 32, 64, 128 & \text{Numbers in geomet. progress} \end{cases}$

That is $\begin{cases} \text{As } 2 + 3 = 5 \text{ which is the indice of } 32 \\ \text{So } 4 \times 8 = 32 \text{ the 5th term in geometrical progression} \end{cases}$

Again $\begin{cases} \text{As } 2 + 4 = 6 \\ \text{So } 4 \times 16 = 64 \text{ the 6th term.} \end{cases}$

Now by these indices and a few of the first terms, the last term, or any distant one, may be speedily found, without producing the whole series.

PROPOSITION 1. When the first term is unity, the ratio and number of terms being known, to find the last of any remote term.

RULE. Find a few of the leading terms, over which place their indices, as before directed; then find what figures of the indices, when added together, will give the index of the term wanted; multiply the numbers standing under such indices into each other, the last product will be the term required.

Note. When the indices begin with a cypher, the sum of the indices made choice of must be always one less than the number of terms given in the question, because 1 in the indices stands over the second term.

EXAMPLE 1. A boy agrees for 14 oranges, to pay only the price of the last, reckoning a farthing for the first, a half-penny for second, &c. doubling the price to the last; how much did he give for them?

First $\begin{cases} 0, 1, 2, 3, 4, 5, 6 & \text{Indices} \\ 1, 2, 4, 8, 16, 32, 64 & \text{Terms} \end{cases}$

Then $\begin{cases} 2 + 5 = 7 \\ 4 \times 32 = 128 \end{cases}$ Also $\begin{cases} 7 + 6 = 13 \\ 128 \times 64 = 8192 \text{ } qrs. \end{cases}$

Which is the 14th term, because the indices are less than the terms by one. And 8192 *qrs.* = 8*l.* 10*s.* 8*d.* Answer.

E. 2. A man bought a horse, and by agreement was to give what the last nail would come to, at a farthing for the first nail, two for the second,

second, four for the third, &c. There were four shoes, and 8 nails in each shoe; I demand the price of the horse?

First $\begin{cases} 0, 1, 2, 3, 4, 5, 6, 7, 8 \text{ Indices} \\ 1, 2, 4, 8, 16, 32, 64, 128, 256 \text{ Terms} \end{cases}$

Then $\begin{cases} 7 + 7 = 14 \\ 128 \times 128 = 16384 = 14\text{th term} \end{cases}$

$14 + 14 = 28$

$16384 \times 16384 = 268435456 = 28\text{th term}$

$\times 8 = 3\text{d term}$

$= 2147483648$ Farthings

Or 32d term, which reduced to pounds, will give 2236962*l.* 2*s.* 8*d.* the price of the horse, answer.

Prop. 2. In any series, not proceeding from unity, the ratio and first term being given, to find any remote term, without producing all the intermediate terms.

RULE. Proceed as in the last proposition, only observe to divide every product by the first term, and the quot. will be the term required.

E. 3. A person dying left 11 children, to whom, and to his executor, he bequeathed in manner following, viz. to his executor, for seeing his will performed, 10*l.* the youngest child to have 30*l.* and so on, every child to exceed the next younger in triple proportion; what will be the share of the eldest?

First $\begin{cases} 0, 1, 2, 3, 4, 5, 6 \text{ Indices} \\ 10, 30, 90, 270, 810, 2430, 7290 \text{ Terms} \end{cases}$

$4 + 6 = 10$ Number of terms less 1

Then $810 \times 7290 = 5904900$, which $\div 10$, the first term, gives 590490*l.* the eldest child's fortune.

Prop. 3. When the first term, ratio, and number of terms are given, to find the sum of all the terms,

RULE. Find the last term as before, from which take the first, divide the remainder by the ratio, less one, and to that quotient add the last term, and you have the sum required.

E. 4. A gentleman married, and received of his father-in-law one guinea, on condition that he was to have a present every month for the first year, which should be double still to what he had the month before; what was the young lady's portion?

First $\begin{cases} 0, 1, 2, 3, 4, 5, 6 \text{ Indices} \\ 1, 2, 4, 8, 16, 32, 64 \text{ Terms} \end{cases}$

Then $\begin{cases} 6 + 5 = 11 \text{ The number of terms less one} \\ 64 \times 32 = 2048 \text{ The last term} \end{cases}$

And $\overline{2048 - 1} \div \overline{2 - 1} = 2047$

Also $2047 + 2048 = 4095$ Guineas

$1s. = \frac{1}{20})\ 4095\ .\ 0$

$204\ \ 15$

Answer £. 4299 15*s.* The young lady's portion

Note.

Note. If the ratio of any rank or ſeries of proportionals be double, the difference of the greateſt and leaſt terms are equal to the ſum of all except the greateſt ; if the ratio be triple, the exceſs or difference is double the ſum of all except as aforeſaid ; if quadruple, triple ; if quintuple, quadruple, and ſo on.

E. 5. A laceman well verſed in numbers, agreed with a gentleman to ſell him 20 yards of rich gold-brocaded lace, for 2 pins the firſt yard, 6 for the ſecond, 18 for the third, and ſo on in triple proportion ; I demand how much the lace produced? The pins afterwards ſold at a farthing per 100 ; alſo whether the laceman gained or loſt by the ſale thereof, ſuppoſing the ſaid lace to have been bought at 10*l.* 1*s.* per yard?

First $\begin{cases} 0, 1, 2, 3, 4, 5 \text{ Indices} \\ 2, 6, 18, 54, 162, 486 \text{ Terms} \end{cases}$

Then $5 + 5 = 10$

$486 \times 486 = 236196$ the 10th term

And $5 + 4 = 9$

$486 \times 162 = 78732$ the 9th term

$$\begin{array}{r} \therefore\ 236196 \\ \times\ 78732 \\ \hline 472392 \\ 708588 \\ 1653372 \\ 1889568 \\ 1653372 \\ \hline 18596183472 \text{ laſt term} \\ -\ 2 \text{ firſt ditto} \\ \hline \end{array}$$

Ratio $3 - 1 = 2)18596183470$

This added to the laſt term - 9298091735 gives

The ſum of all - - 1|00) 278942752|07 the terms

Value of the pins $278942752\frac{7}{100}$ *qrs.* $= 290565\ 7\ 4$

Lace comes to 10*l.* 1*s.* $\times 5 \times 4\ (=20) =$ 201 0 0.

Anſwer, The laceman gained - - £. 290364 7 4

Prop. 4. When the firſt term and ratio of any infinite decreaſing geometrical ſeries, or infinite ſeries of decreaſing proportionals are given, to find the ſum of the ſeries,

RULE. Divide the ſquare of the firſt term by the difference between the ſaid firſt term and the ſecond term in the ſeries ; the quotient will be the ſum of the ſeries.

Note. A geometrical ſeries that decreaſeth ad infinitum, or in other words, an infinite ſeries of decreaſing proportionals, are ſuch whoſe laſt or leaſt term is a cypher, or leſs than any thing aſſignable, and its number of terms inexpreſſible.

E. 6. A great ſhip purſues a ſmall one, ſteering the ſame way, at the diſtance of ſix leagues from it, and ſails twice as faſt as the ſmall ſhip ; how far muſt the great ſhip ſail before it overtakes the leſſer?

Firſt 6, 3, $1\frac{1}{2}$, $\frac{3}{4}$, &c. ad infinitum

Then $6 \times 6 = 36$, ſquare of the firſt term

And $36 \div 3$ the ſecond term $= 12$ leagues, the anſwer.

E. 7. *Suppose a round ball for to move in the air,*
In a certain proportion which I shall declare;
Let the first hour be 12 *miles, the next to move* 10,
And so in proportion from whence it began,
As 12 *is to* 10. *Now try if you can*
Tell the miles it will move, suppose it to be
Continued in motion to ETERNITY?

First $12 \times 12 = 144$ square of the first term
And $12 - 10 = 2$ difference of the first and second term
Then $144 \div 2 = 72$ miles, the answer.

Otherwise thus; As 2 : 10 :: 12 : 60 miles, the sum of all the terms except the first or greatest; to which add 12 the first term, and the sum will be 72 miles, the answer as above.

XXXVI. PERMUTATION, OR, VARIATION,

IS the changing or varying the order of things, in respect of their places.

RULE. Multiply all the given terms into one another continually, whose first term, or common difference, is unity or 1, and the last product will be the number of changes or variations required.

E. 1. A young scholar coming into town for the conveniency of a good library, demands of a gentleman with whom he lodged, what his diet would cost for a year? who told him 10*l.* but the scholar not being certain what time he should stay, asked him what he must give him for so long as he could place his family (consisting of seven persons besides himself) in different positions every day at dinner; the gentleman thinking it would not be long, tells him 5*l.* to which the scholar agrees; what time did he stay with the gentleman?

First $1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8 = 40320$ days, answer.

Days in a year = 365)40320(110 years 170 days, the time the scholar (was to stay
365
382
365
170

E. 2. *At Birmingham we've a church that's nearly new,*
Which beauteous pile can be outvied by few:
Here sacred grandeur captivates the eye,
Trav'lers admire the same as they pass by;
To grace this structure, there's a lofty tower,
With ten fine bells, which harmonize each bower.

How

How many changes may be rung declare,
On these ten bells, and likewise tell me fair
How long they would be ringing them once o'er,
Allowing six seconds per change, not more?

First $1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8 \times 9 \times 10 = 3628800$, number of changes

Then $3628800 \times 6 = 21772800$ seconds

Seconds.

In a day there are 864|00)217728|00(252 days
1728
4492
4320
1728
1728
0

Answer 252 days

E. 3. An accomptant told a gentleman, who had constantly eight persons at his table, that he would gladly make a ninth, and was willing to give 20 guineas for his board, so long as he could place the said company at dinner, differently from any one day before; this being accepted, what did his entertainment cost him per year?

First $1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8 \times 9 = 362880$ days

Then, as 362880 : 20 :: 365
21
20
40
420
12
5040
365
25200
30240
15120

362880)1839600 ($5\frac{25200}{362880}d.$ Answer
1814400
25200

Answer $5\frac{5}{72}d.$ in its lowest terms

E. 4. *A famous gen'ral having serv'd his king,*
Who always from the wars did victory bring,
For his good service (with a pleasant smile)
Ask'd of his king one farthing for each file
Of ten men in a file, which he could then
Make with a body of one hundred men.
The king considering his brave actions past,
And seeming modesty of his request,
Gave his consent; to what will it amount
In sterling money? Take your pen and count.

Note. To solve questions of this nature, you must place the given quantity by itself, decreasing it gradually by unity, so often as there are quantities in the combinations; placing them one after another with a sign of multiplication between them, which numbers must be multiplied into one another for a dividend; then placing an unit with the

the like number of places, increasing by unity till you arrive at the number to be combined; which multiply continually for a divisor, and the quotient will be the number of combinations sought, thus:

$$\frac{100}{1}\times\frac{99}{2}\times\frac{98}{3}\times\frac{97}{4}\times\frac{96}{5}\times\frac{95}{6}\times\frac{94}{7}\times\frac{93}{8}\times\frac{92}{9}\times\frac{91}{10}=$$

$$\frac{62815650955529472000}{3628800}=17310309456440\ qrs.$$ which are equal to 18031572350 *l.* 9*s.* 2*d.* Answer

E. 5. *Two gamesters one day, at dice they did play,*
And being full merry with wine;
Says B *unto* A, *what odds will you lay,*
I cast not the six aces this time?
Says A *then to* B, *ten to one I'll lay thee,*
With six dice, the six aces you cast not:
Pray youths shew, and here let me know,
For the odds on the cast, Sirs, they know not.

First $6\times6\times6\times6\times6\times6=46656=6$ different combinations
And $1\times2\times3\times4\times5\times6=720$ variations
Then $46656-720=45936$ chances against A
But as A laid 10 to 1, therefore 7200 chances to B
Therefore A's chance to that of B's is as 45936 : 7200; or as 6.38 : 1, the Answer.

PRACTICAL ARITHMETIC.

PART II.

VULGAR FRACTIONS.

A FRACTION is some part or parts of an integer, or whole thing, represented by 1; as $\frac{3}{4}$ is a fraction, denoting three-fourth parts of an integer or 1. Every fraction consists of two numbers, placed one above the other, with a line between them, as in this fraction $\frac{2}{3}$; the lower number 3 is called the denominator, and shews how many parts the integer is divided into; the upper number 2, is called the numerator, and expresses how many of these parts the fraction consists of. And both numerator and denominator are called terms of the fraction.

A Vulgar Fraction is either proper, improper, single compound, or mixed.

A Proper

A Proper Fraction is that wherein the numerator is less than the denominator, as $\frac{3}{4}$; and is called proper with respect to the relative integer, because it expresses a quantity less than it.

An Improper Fraction is such whose numerator is equal to, or greater than its denominator, as $\frac{3}{3}$, $\frac{4}{3}$, and is called improper with respect to the relative integer, because it expresses a quantity greater than it.

A Single Fraction is that which consists of but one numerator, and one denominator, and is referred immediately to some integer, as $\frac{3}{4}$, or $\frac{1}{2}$ of any thing.

A Compound Fraction is the fraction of a fraction, consisting of two or more simple fractions referred to one another in order, and the last referred to some integer, as $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{4}{8}$, &c.

A Mixed Number is composed of a whole number and fraction, as $6\frac{1}{2}$, $34\frac{2}{7}$, $152\frac{6}{8}$, &c.

XXXVII. REDUCTION of Vulgar Fractions.

CASE 1.

To reduce a fraction to another of equal value,

RULE. Multiply or divide both terms of the fraction by the same number, and you will have a new fraction equivalent to that given.

EXAMPLE. Let the fraction be $\frac{4}{6}$; now (per rule) multiply both terms of the given fraction by 5, thus:

$$\begin{array}{rr} 4 & 6 \\ 5 & 5 \\ \hline 20 & 30 \end{array}$$

Whence the new fraction $\frac{20}{30} = \frac{4}{6}$. Again,

Divide both terms of the fraction by 5, thus:

$5\,)\,\frac{20}{30}\,(\,\frac{4}{6}$ The fraction given.

CASE 2. To reduce a whole number to the form of a fraction,

RULE. Set 1 under it for a denominator.

EXAMPLE. Suppose 8, 6, 4, 36, were numbers tobe reduced to fractions. Then (per rule) they become $\frac{8}{1}$, $\frac{6}{1}$, $\frac{4}{1}$, $\frac{36}{1}$, the fractional quantity required.

CASE 3. To reduce a whole number to a fraction of a given denomination,

RULE. Multiply the whole number by the given denominator, and under the product write the same denominator.

EXAMP. Reduce 9 into a fraction whose denominator shall be 6?

$$\begin{array}{r} 9 \\ 6 \\ \hline 54 \end{array}$$

Numerator; then $\frac{54}{6}$ is the fraction required.

CASE

CASE 4. To reduce a compound fraction to a single one of the same value,

RULE. Multiply all the numerators together for a new numerator, and all the denominators for a new denominator.

EXAMPLE. Reduce $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{4}{5}$ to a single fraction ?

$$\begin{array}{ll} 2 & 3 \\ \underline{3} & \underline{4} \\ 6 & 12 \\ \underline{4} & \underline{5} \\ \text{N. } 24 & 60 \text{ D.} \end{array}$$

then $\frac{24}{60}$ is the single fraction.

Note. N. stands for numerator, and D. for denominator.

CASE 5. To reduce any mixed number to an improper fraction,

RULE. Multiply the whole number by the denominator of the fraction, and to the product add the numerator for a new numerator, which place over the denominator.

EXAMPLE. Reduce $26\frac{3}{8}$ to an improper fraction ?

$$\begin{array}{l} 26\frac{3}{8} \\ \underline{\quad 8} \\ 211 \text{ N.} \end{array}$$

$\therefore$ $\frac{211}{8}$ is the fraction required.

CASE 6. To reduce an improper fraction to its equivalent whole or mixed number,

RULE. Divide the numerator by the denominator, and the quotient is the whole number. Then what remainder there is, place it over the denom. and annex this fraction to the quotient before found.

EXAMPLE. Reduce $\frac{211}{8}$ to its equivalent whole or mixednumber?

$$\begin{array}{rl} 8) & \underline{211} \\ \text{Answer} & 26\frac{3}{8} \end{array}$$

Reduce $\frac{631}{16}$ to its equivalent whole or mixed number?

$$\begin{array}{l} 16)631(39\frac{7}{16} \text{ Answer} \\ \quad\;\; \underline{48} \\ \quad\;\; 151 \\ \quad\;\; \underline{144} \\ \quad\quad\; 7 \end{array}$$

Note. The preceding six cases being so exceeding easy, I thought more than one example in each case would be quite unnecessary.

CASE 7. To find the greatest common measure or divisor for the numerator and denom. of any given fraction, or for any two numbers,

RULE. Divide the greater term by the lesser, and the last divisor by the remainder, and so on continually till nothing remain; the last divisor is the common measure required.

EXAMPLE

EXAMPLE 1. What is the greatest common measure of $\frac{252}{364}$?

```
252)364(1
    252
    ---
    112)252(2
        224
        ---
         28)112(4
            112
            ---
              0
```

Answer 28 is the last divisor, which is the greatest number that will divide both numerator and denominator without a remainder.

When there are mixed numbers given, they must be reduced to a common denominator; then proceed with the two new numerators to find their greatest common measure, make that the numerator, and under put the common denominator, which fraction will be the greatest common measure required.

E. 2. What is the greatest common measure of $14\frac{4}{5}$ and 32?

```
14 4/5     32        74)160(2
    5       5            148
 -----    -----          ---
 74 N.    160 N.         12)74(6
                            72
                            --
                            2)12(6
                              12
                              --
```

Answer $\frac{2}{5}$ is the greatest common measure of $14\frac{4}{5}$ and 32.

CASE 8. To reduce a fraction to its least, or lowest terms,

RULE. Find the greatest common measure, by which divide both terms of the fraction; the quotients will give the fraction required.

EXAMPLE 1. Reduce $\frac{2832}{12848}$ to its lowest terms?

```
2832)12848(4
     11328
     -----
     1520)2832(1
          1520
          ----
          1312)1520(1         Then 16)2832/12848(=177/803 Answer
               1312
               ----
                208)1312(6
                    1248
                    ----
                      64)208(3
                         192
                         ---
                          16)64(4
                             64
                             --
```

Note. If the last remainder is 1, the fraction is already in its lowest terms.

E. 2. Reduce $\frac{204}{228}$ to its lowest terms?

```
204)228(1
    204
    ---
     24)204(8
        192
        ---
         12)24(2
```

Then $12)\frac{204}{228}(=\frac{17}{19}$ Answer

When the terms of the fraction are even numbers they may be divided by 2 continually, thus: $\frac{24}{136}$, being continually halved, is $\frac{24}{136} = \frac{12}{68} = \frac{6}{34} = \frac{3}{17} =$ in value to the given fraction $\frac{24}{136}$.

When both terms end with 5; or one with 5, and the other with a cypher, divide both by 5, thus: $5)\frac{65}{80}(=\frac{13}{16}$ Answer.

When both terms end with cyphers, cut of an equal number in both. Thus $\frac{1|00}{24|00}$ becomes $\frac{1}{24}$ the terms required.

If you can discern any number that will divide both terms, divide by that number.

Reduce $\frac{24}{112}$ to its lowest terms? $8)\frac{24}{112}(=\frac{3}{14}$ Answer

Likewise $7)\frac{119}{168}(=\frac{17}{24}$ Answer

CASE 9. To reduce fractions of different denominations to fractions of equal value, that shall have one common denominator,

RULE. Multiply each numerator by all the denominators except its own, for a new numerator; then multiply all the denominators together for a new denominator.

EXAMPLE 1. Reduce $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{4}{5}$, to a common denominator?

2	3	4	3
4	3	4	4
8	9	16	12
5	5	3	5
40 N.	45 N.	48 N.	60 D.

Therefore $\frac{2}{3} = \frac{40}{60}$, $\frac{3}{4} = \frac{45}{60}$, and $\frac{4}{5} = \frac{48}{60}$ Answer.

E. 2. Reduce $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, to a common denominator?

1	1	1	1	3
4	3	3	3	4
4	3	3	3	12
5	5	4	4	5
20	15	12	12	60
6	6	6	5	6
120 N.	90 N.	72 N.	60 N.	360 Common denominator

Answer $\frac{120}{360}$, $\frac{90}{360}$, $\frac{72}{360}$, $\frac{60}{360}$, are all of the same value with the respective original ones, and have one common denominator.

E. 3.

E. 3. Reduce $14\frac{2}{3}$, 7, and $\frac{2}{3}$ of $\frac{5}{8}$ of $\frac{3}{9}$, and $\frac{4}{7}$ to one common denominator?

First $14\frac{2}{3} = \frac{44}{3}$, and $\frac{2}{3}$ of $\frac{5}{8}$ of $\frac{3}{9} = \frac{30}{216} = \frac{5}{36}$ in its lowest terms. Then we have these fractions $\frac{44}{3}$, $\frac{7}{1}$, $\frac{5}{36}$ and $\frac{4}{7}$ to be reduced.

44	7	5	4	3
7	3	1	36	1
308	21	5	144	3
1	36	3	1	36
308	126	15	144	108
36	63	7	3	7
1848	756	105 N.	432 N.	756 Com. denom.
924	7			
11088 N.	5292 N.			

Answer $14\frac{2}{3} = \frac{11088}{756} = \frac{44}{3}$; $\frac{7}{1} = \frac{5292}{756}$; $\frac{5}{36} = \frac{105}{756}$; and $\frac{4}{7} = \frac{432}{756}$.

Note. If one fraction be equivalent to another, it will hold as the numerator of the one is to its denominator, so is the numerator of the other to its denominator; or as one numerator to the other, so is one denom. to the other. For in the above example it is said $\frac{4}{7} = \frac{432}{756}$.

To prove which say, if 432 : 756 :: 4 : 7 Proof

$$\begin{array}{r} 756 \\ \underline{4} \\ 432)3024(7 \\ \underline{3024} \\ 0 \end{array}$$

CASE 10. Several fractions being given; to find as many whole numbers, in the same proportion.

RULE. Reduce the fractions to a common denominator, then the several numerators will be to one another as the fractions given.

EXAMPLE. Suppose $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{6}$, were given, to find whole numbers, in the same proportion?

1	1	1	2
4	2	4	4
4	2	4	8
6	6	2	6
24 N.	12 N.	8 N.	48 Denominator

Then the fractions given are reduced to $\frac{24}{48}$, $\frac{12}{48}$, $\frac{8}{48}$

$\therefore$ $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{6}$, are in the same proportion to one another as 24, 12 and 8.

CASE 11. To reduce coins, weights, measures, &c. into fractions.

RULE. Reduce the given quantity to the lowest denominations mentioned, and make it the numerator; then reduce the whole of the integer, which the given numbers are parts of, and make it the denominator, and you have the fraction required.

EXAMPLE 1. Let it be required to reduce 4*s.* 2*d.* to the fraction of a pound sterling?

s. *d.*	*s.*	
4 2	20	Answer $\frac{50}{240}$
12	12	
50 N.	240 D.	

E. 2. Reduce $8\frac{1}{2}$*d.* to the fraction of a shilling?

d.	*d.*	
$8\frac{1}{2}$	12	Answer $\frac{17}{24}$
2	2	
17 N.	24 D.	

E. 3. Reduce 3 roods, 6 poles to the fraction of an acre?

R. *P.*	*R.*	
3 6	4	
40	40	
126 N.	160 D.	Answer $\frac{126}{160}$

E. 4. Reduce 2 *Cwt.* 1 *qr.* 4 *lb.* to the fraction of 1 *Cwt*?

C. *qr.* *lb.*	*C.*	
2 1 4	1	
4	4	
9	4	
28	28	
256 N.	112 D.	Answer $\frac{256}{112}$

Note. After the above manner may any other weights, measures, &c. be reduced to fractions.

CASE 12. To reduce fractions of one denomination to another, retaining the same value,

RULE. 1. If the fraction given is to be brought from a less to a greater denomination, multiply the denominator by all the denominations, from that given to that sought.

2. If the fraction given is to be brought from a greater to a less denomination, multiply the numerator by all the denominations, from that given to that sought.

EXAMPLE 1. Reduce $\frac{2}{8}$*l.* to the fraction of a penny?

$$\begin{array}{r} 2 \\ 20 \\ \hline 40 \\ 12 \\ \hline 480 \text{ N.} \end{array}$$

Ans. $\frac{480}{8} = \frac{60}{1}$ in its lowest terms.

E. 2. Reduce $\frac{3}{8}$ of a penny to the fraction of a pound?

$$\begin{array}{r} 8 \\ 12 \\ \hline 96 \\ 20 \\ \hline 1920 \end{array}$$

Ans. $\frac{3}{1920} = \frac{1}{640}$ in its lowest ter

E. 3.

E 3. Reduce $\frac{3}{7}$ *lb.* troy, to the fraction of a penny-weight?

3
12

36
20
---
720 N.

Answer $\frac{720}{7}$.

E. 5. Reduce $\frac{4}{12}$ of a pound to the fraction of 1 hundred wt?

12
28

336
4

1344 D.

Anſ. $\frac{1}{1344}=\frac{1}{336}$, in its loweſt ter.

E. 6. Reduce $\frac{2}{3}$ of a pound to the fraction of a guinea?

2   3
20   21
---
40 N.   63 D.

Anſwer $\frac{40}{63}$ the fraction required.

E. 4. Reduce $\frac{9}{12}$ of a ſhilling to the fraction of a farthing?

9
12

108
4
---
432 N.

Anſ. $\frac{432}{12}=\frac{36}{1}$, in its loweſt terms.

E. 7. Reduce $\frac{3}{4}$ of a dram to the fraction of a hundred weight?

4
16

64
16

384
64

1024
28

8192
2048

28672
4

114688 D.

Anſwer $\frac{3}{114688}$.

CASE 13. To find the proper quantity of a fraction in the known parts of an integer,

RULE. Multiply the numerator by the number of parts contained in the integer, and divide the product by the denominator, the quotient ſhews the known parts. If there be any remainder, multiply it by the next inferior denomination, and divide by the denominator as before; continue this work till you come at the loweſt denomination.

EXAMPLE 1. What is the value of $\frac{7}{9}$ of a guinea?

21
7
---
9)147
---
16   3
12
---
9)36

4

Anſ. 16*s*. 4*d*.

E. 2. What is the value of $\frac{5}{8}$ of 4*s*. 5*d*?

s.   *d*.
4   5
5
---
8)1   2   1

£. 0   2 $9\frac{1}{8}$ Anſwer

E. 3. Required the value of $\frac{135}{480}$ of a pound sterling?

$$\begin{array}{r} 135 \\ 20 \\ \hline 480)2700(5s. \\ 2400 \\ \hline 300 \\ 12 \\ \hline 480)3600(7d. \\ 3360 \\ \hline 240 \\ 4 \\ \hline 480)960(2qrs. \\ 960 \\ \hline 0 \end{array}$$

Answer 5*s.* 7½*d.*

E. 4. What is the value of $\frac{484}{1394}$ of a moidore?

$$\begin{array}{r} 484 \\ 9 \times 3 = 27 \\ \hline 4356 \\ 3 \\ \hline 1394)13068(9s. \\ 12546 \\ \hline 522 \\ \times 12 \\ \hline 1394)6264(4d. \\ 5576 \\ \hline 688 \\ 4 \\ \hline 1394)2752(1 qr. \\ 1394 \\ \hline 1358 \end{array}$$

Answer 9*s.* 4¼*d.* $\frac{1358}{1394}$

E. 5. What is the value of $\frac{4}{5}$ of a three-pound-twelve?

$$\begin{array}{rr} £. & s. \\ 3 & 12 \\ & 4 \\ \hline 5)14 & 8 \\ \hline \end{array}$$

Answer £. 2 17 $7\frac{1}{5}$

E. 6. What is the quantity of $\frac{6}{8}$ of an acre?

$$\begin{array}{r} 6 \\ 4 \\ \hline 8)24 \\ \hline \end{array}$$

Answer 3 Roods

E. 7. What is the value of $\frac{5}{8}$ of a day?

$$\begin{array}{r} 5 \\ 24 \\ \hline 8)120 \\ \hline \end{array}$$

Answer 15 Hours

Note. After the same manner the value of any fraction may be found.

XXXVIII. ADDITION of Vulgar Fractions.

R U L E.

REDUCE all the given fractions to simple fractions of the same integer and denominator, if not so already; then the sum of the numerators being made a numerator to the common denominator, makes the fractional sum sought, which may be further reduced, as seems most expedient, or the case will admit.

E. 1. What is the sum of $\frac{2}{8}$ and $\frac{3}{8}$?

$$\begin{array}{r} 2 \\ 3 \\ \hline 5 \end{array}$$

Answer $\frac{5}{8}$

E. 2. What is the sum of $\frac{5}{10}$ and $\frac{8}{10}$?

$$\begin{array}{r} 5 \\ 8 \\ \hline 13 \end{array}$$

Answer $\frac{13}{10}$.

E. 3.

E. 3. What is the sum of $\frac{3}{4}$ of $\frac{5}{9}$ and $2\frac{1}{2}$?

First $\frac{3}{4}$ of $\frac{5}{9} = \frac{15}{36}$, and $2\frac{1}{2} = \frac{5}{2}$; these reduced to a common denominator are equal to $\frac{30}{72}$, $\frac{180}{72}$, whose sum is $\frac{210}{72} = \frac{35}{12}$ in its lowest terms.

E. 4. What is the sum of $\frac{1}{3}$ of $\frac{1}{4}$, and $\frac{3}{8}$, and $1\frac{1}{4}$?

First $\frac{1}{3}$ of $\frac{1}{4} = \frac{1}{12}$, also $1\frac{1}{4} = \frac{5}{4}$, then $\frac{1}{12}$, $\frac{3}{8}$ and $\frac{5}{4}$ reduced to a common denominator, are $\frac{2}{24}$, $\frac{9}{24}$, and $\frac{30}{24}$, the sum whereof is $\frac{41}{24}$, answer.

E. 5. What is the sum of $\frac{3}{5}$ of a pound, $\frac{5}{10}$ of a shilling, and $\frac{7}{8}$ of a penny?

First $\frac{5}{10}$ of a shilling $= \frac{5}{200}$ of a pound, and $\frac{7}{8}$ of a penny $= \frac{7}{1920}$ of a pound. Then $\frac{3}{5}$, $\frac{5}{200}$, and $\frac{7}{1920}$ are reduced to $\frac{5760}{9600}$, $\frac{240}{9600}$, $\frac{35}{9600}$, whose sum is $\frac{6035}{9600}$ of a pound $= \frac{1207}{1920}$ in its lowest terms. $=$ 12s. $6\frac{3}{4}d.$ $\frac{960}{1920}$ Answer

E. 6. *Admit I sail where billows roar, and plough the raging sea,*
And steering to a foreign shore, a prize falls in my way;
When having chang'd a full broadside, as Rodney us'd to do,
Or Hood, and many more beside, who boldly dar'd the foe:
Suppose this prize ten thousand pound, three-fiftieth is my share,
Another sailor's share is found, his part two-eightieths are,
I purchase this, then what's to me, the total worth define?
And you shall with Minerva be and in her temple shine?

First find the value of $\frac{3}{50}$ and $\frac{1}{40}$ of 10000*l.* and add them together, thus:

$$
\begin{array}{rl}
4|0) & 1000|0 \\
 & \quad 3 \\ \hline
5|0) & 3000|0 \\ \hline
\text{Add} \left\{ \begin{array}{l} 600 \\ 250 \end{array} \right. & \begin{array}{l} = \frac{3}{50} = \text{the sailor's own share} \\ = \frac{1}{40} = \text{the purchased share} \end{array} \\ \hline
\text{Answer} & £.\ 850
\end{array}
$$

Note. Reduction of fractions being well understood, addition will be very easy; the reason of which will be obvious, if we consider that the given fractions being such, or reduced to such a state, that all the numerators represent things of the same denomination, both absolute and relative; their sum must therefore be a number of such parts as the common denominator expresses of the same common integer.

XXXIX. SUBTRACTION of Vulgar Fractions.

RULE.

PREPARE the fractions as directed in addition; then subtract one numerator from the other, and their difference will be a new numerator, under which subscribe the common denominator.

EXAMPLE 1. From $\frac{4}{5}$ take $\frac{2}{5}$? E. 2. From $\frac{18}{44}$ take $\frac{12}{44}$?

From	4	18
Take	2	— 12
	2 The remainder is $\frac{2}{5}$ Ans.	6 The rem. is $\frac{6}{44}$ Answer

E. 3.

E. 3. From	$389\frac{6}{8}$	E. 4. From	$8968\frac{8}{12}$
Take - - -	$142\frac{2}{8}$	Take - -	$1442\frac{9}{12}$
Remains -	$247\frac{4}{8}$	Remains -	$7525\frac{11}{12}$
Proof - -	$389\frac{6}{8}$	Proof - -	$8968\frac{8}{12}$

To work the 4th example, say, 9 from 8 I cannot, but 12, the parts the integer is divided into, I borrow to 8, is 20; then 9 from 20, there remains 11, which set down as a numerator to the denominator 12, and carry 1 to the 2, and proceed as in common subtraction, the answer will be $7525\frac{11}{12}$.

E. 5. From $12\frac{3}{4}$ take $9\frac{1}{3}$?

First $12\frac{3}{4} = \frac{51}{4}$, $9\frac{1}{3} = \frac{28}{3}$, then subtract $\frac{28}{3}$ from $\frac{51}{4}$.

51	28	4	∵ From 153
3	4	3	Take 112
153 N.	112 N.	12 D.	Remains 41 Ans. $\frac{41}{12} = 3\frac{5}{12}$.

	£.	s.	d.		£.	s.	d.
E. 6. From	82	$9\frac{3}{7}$	0	=	82	$9\frac{9}{21}$	0
Take - - -	60	$14\frac{2}{3}$	$8\frac{5}{6}$	=	60	$14\frac{14}{21}$	$8\frac{5}{6}$
Difference -					21	$14\frac{15}{21}$	$3\frac{1}{6}$

Note. Such examples as the last, perhaps, may never happen in business; but as such are useful exercises for learners, I thought an example of this sort might be agreeable to my ingenious readers.

XL. MULTIPLICATION of Vulgar Fractions.

RULE.

PREPARE the given numbers (if they require it) by the rules of reduction; then multiply the numerators together for a new numerator, and the denominators for a new denominator.

Note. Multiplication of fractions decrease the value, in the same proportion as whole numbers increase it, which seems to contradict the definition of multiplication; but this difficulty will vanish, if we consider that the more any integral number is increased, the farther is the figure in the highest place removed from unity; and the more any part of an integer is decreased, the farther will its value also be removed from its relative unit; consequently, as it is the nature of integers to increase, and of fractions to decrease, the purpose of multiplication is equally answered in both cases; the reason of which will more plainly appear by the following examples.

E. 1.

E. 1. Multiply 5*s*. by 5*s*. as the fraction of a pound sterling?

First 5*s*. $=\frac{1}{4}$ of a pound; therefore $\frac{1}{4}\times\frac{1}{4}=\frac{1}{16}$ of a pound, which, by case 13, sect. XXXVII. will be found equal to 1*s*. 3*d*.

E. 2. Multiply $\frac{4}{9}$ by $\frac{6}{5}$?

$$\begin{array}{r} 4 \\ 6 \\ \hline 24 \text{ N.} \end{array} \qquad \begin{array}{r} 9 \\ 5 \\ \hline 45 \text{ D.} \end{array}$$

Answer $\frac{24}{45}$.

E. 3. Multiply $\frac{8}{32}$ by $\frac{24}{84}$?

First $\frac{8}{32}=\frac{1}{4}$, and $\frac{24}{84}=\frac{2}{7}$; then the fractions to be multiplied are $\frac{1}{4}$ and $\frac{2}{7}$.

$$\begin{array}{r} 1 \\ 2 \\ \hline 2 \text{ N.} \end{array} \qquad \begin{array}{r} 4 \\ 7 \\ \hline 28 \text{ D.} \end{array}$$

Answer $\frac{2}{28}=\frac{1}{14}$.

If you have a mixed number or fraction to multiply by a whole number; multiply the whole number by the whole number, and then multiply the numerator by the said whole number, and divide by the denominator, and add this quotient to the former product.

E. 4. Multiply $\frac{3}{4}$ by 8?

First $\frac{3\times 8}{4}=\frac{24}{4}$ the product, then

$$\begin{array}{r} 3 \\ 8 \\ \hline 4)24 \\ 6 \end{array}$$

Answer 6 the product

E. 5. Multiply $3\frac{4}{7}$ by 13?

$$\begin{array}{r} 3 \\ 13 \\ \hline 39 \end{array} \qquad \begin{array}{r} 13 \\ 4 \\ \hline 7)52 \\ \hline 7-\frac{3}{7} \end{array}$$

$$\begin{array}{r} 39 \\ +7\frac{3}{7} \\ \hline 46\frac{3}{7} \end{array}$$

$46\frac{3}{7}$ the product

E. 6. Multiply 14*l*. 10*s*. $10\frac{2}{5}$*d*. by 4?

$$\begin{array}{rrr} £. & s. & d. \\ 14 & 10 & 10\frac{2}{5} \\ & & 4 \\ \hline 58 & 3 & 5\frac{3}{5} \end{array}$$ Product.

To work this example, say, 4 times 2 is $8=\frac{8}{5}=1\frac{3}{5}$, which I carry to the product of pence; and proceed as in common multiplication. The operations in this rule are so easy, that more examples would be unnecessary.

XLI. DIVISION of Vulgar Fractions.

RULE.

PREPARE the fractions (as before directed) by the rules of reduction, then multiply the denominator of the divisor by the numerator of the dividend, for a new numerator, and the numerator of the divisor into the denom. of the dividend for a new denominator.

EXAMPLE 1. Divide $\frac{5}{8}$ by $\frac{3}{5}$? $\frac{3}{5})\frac{5}{8}(\frac{25}{24}=1\frac{1}{24}$ Answer.

To work this example, I multiply 5, the numerator of the dividend, into 5, the denominator of the divisor, the product is 25, the numerator for the quotient; then I multiply 8, the denominator of the dividend, into 3, the numerator of the divisor, the product is 24, the denominator of the quotient. ∴ $\frac{25}{24}$ is the quotient required.

E. 2.

E. 2. Divide $\frac{4}{6}$ by $\frac{2}{3}$?
$\frac{2}{3})\frac{4}{6}(\frac{12}{12} = 1$ Anſwer

E. 3. Divide $\frac{8}{12}$ by $\frac{4}{9}$?
$\frac{4}{9})\frac{8}{12}(\frac{72}{48}$ Anſwer

E. 4. Divide $\frac{8}{9}$ of 1*s*. by $\frac{3}{5}$ of 1*l*?
Firſt $\frac{8}{9}$ of a ſhilling $= \frac{8}{180}$ of 1*l*.
Then $\frac{8}{180})\frac{8}{9}(\frac{72}{1440} = 20$ Anſwer

E. 5. Divide $\frac{3}{4}$ of $\frac{2}{5}$ by $\frac{4}{8}$?
Firſt $\frac{3}{4}$ of $\frac{2}{5} = \frac{6}{20} = \frac{3}{10}$
Then $\frac{4}{8})\frac{3}{10}(\frac{24}{40} = \frac{3}{5}$ Anſwer

E. 6. Divide $6\frac{2}{3}$ by $2\frac{3}{8}$?
Firſt $6\frac{2}{3} = \frac{20}{3}$, and $2\frac{3}{8} = \frac{19}{8}$
Then $\frac{19}{8})\frac{20}{3}(\frac{160}{57} = 2\frac{46}{57}$ Anſwer

E. 7. Divide 8 by $\frac{3}{5}$?
$\frac{3}{5})\frac{8}{1}(\frac{40}{3}$ Anſwer

E. 8. Divide $\frac{3}{4}$ of 1*l*. by $\frac{2}{3}$ of 1*s*?
Firſt $\frac{2}{3}$ of 1*s* $= \frac{1}{30}$ of a pound
Then $\frac{1}{30})\frac{3}{4}(\frac{90}{4} = \frac{45}{2} = 22$*l*. 10*s*. Anſ.

E. 9. Divide $\frac{2}{3}$ of 1*s*. by $\frac{3}{4}$ of 1*l*?
Firſt $\frac{2}{3}$ of 1*s*. $= \frac{1}{30}$ of 1*l*.
Then $\frac{3}{4})\frac{1}{30}(\frac{4}{90} = \frac{2}{45} = 10\frac{1}{2}$*d*. $\frac{10}{15}$ A.

1. If it can be done, divide the numerator of the dividend by the numerator of the diviſor, and the denominator by the denominator, for the quotient.

E. 10. Divide $\frac{8}{15}$ by $\frac{2}{3}$? $\frac{2}{3})\frac{8}{15}(\frac{4}{5}$ the quotient

2. If the two numerators, or the two denominators, can be divided by any number, take the quotients inſtead thereof.

E. 11. Divide $\frac{3}{39}$ by $\frac{6}{13}$? $\frac{3}{39})\frac{6}{13}(= 6$
For $6 \div 3 = 2$, and $39 \div 13 = 3$ $\because 3 \times 2 = 6$ the Anſwer

3. A fraction is divided by a whole number, by multiplying the denominator of the fraction by the whole number.

E. 12. Divide $\frac{12}{16}$ by 8?

$$\begin{array}{r} 16 \\ 8 \\ \hline 128 \end{array}$$ D.

Anſwer $\frac{12}{128}$

4. If the denominators are equal, place the numerator of the dividend over the numerator of the diviſor for the quotient.

E. 13. Divide $\frac{6}{18}$ by $\frac{3}{18}$? The quotient is $\frac{6}{3} = \frac{2}{1} = 2$ Anſwer

Note. If you divide by a proper fraction, the quotient is always a greater number than the dividend; contrary to whole numbers, and likewiſe contrary to the ſtrict ſenſe of the word diviſion, which imports the leſſening of a thing. Conſequently, if the diviſor is a proper fraction, multiplication prevails; but diviſion prevails if the diviſor is an improper fraction, as may be ſeen in Ex. 8 and 9.

XLII. RULE of THREE DIRECT,

IN VULGAR FRACTIONS.

RULE 1.

PREPARE the fractions as before directed, and then ſtate and work your ſum as in whole numbers; only multiplying and dividing fraction-wiſe, viz. according to the directions in multiplication and diviſion of fractions.

RULE 2. Multiply the denominator of your firſt number into the numerators of the ſecond and third for a new numerator; then multiply

tiply the numerator of the first number into the denominator of the second and third, for a new denominator, and place it under the new numerator, for the answer, which reduce to its proper quantity.

EXAMPLE 1. If $\frac{2}{3}$ of a pound of hops cost $8\frac{3}{4}d.$ how many pounds may be bought for $25l$?

First $8\frac{3}{4}d. = \frac{35}{4}$ of $\frac{1}{12} = \frac{35}{48}s.$ and $25l. = \frac{25}{1}$ of $\frac{20}{1} = \frac{500}{1}s.$

Then, per rule 1, as $\overset{s.}{\frac{35}{48}} : \overset{lb.}{\frac{2}{3}} :: \overset{s.}{\frac{500}{1}}$

500   3
2   1

1000 N.   3 D. $\frac{35}{48})\frac{1000}{3}(\frac{48000}{105} = \frac{3200}{7} = 457\frac{1}{7}$ Answer

Or thus, $8\frac{3}{4}d. = \frac{35}{4}$ of $\frac{1}{12}$ of $\frac{1}{20} = \frac{35}{960} = \frac{7}{192}l.$ and $25l. = \frac{25}{1}$

Then, by rule 1, as $\overset{£.}{\frac{7}{192}} : \overset{lb.}{\frac{2}{3}} :: \overset{£.}{\frac{25}{1}}$

25   3
2   1
---
50 N.   3 D. $\frac{7}{192})\frac{50}{3}(\frac{9600}{21} = \frac{3200}{7} = 457\frac{1}{7}lb.$ = as before.

Again for variety, by rule 2, thus:

192   7
2   3
---
384   21
25   1

1920   21 D.
768

9600 N.

Answer $\frac{9600}{21} = \frac{3200}{7} = 457\frac{1}{7}lb.$ the same as before.

Note. This is the most expeditious, and easiest method, as appears by the work.

E. 2. If $2\frac{2}{5}$ yards of cloth cost $3\frac{3}{4}l.$ what will $4\frac{4}{5}$ yards cost, at the same rate?

First, $2\frac{2}{5} = \frac{12}{5}$; $3\frac{3}{4} = \frac{15}{4}$; and $4\frac{4}{5} = \frac{24}{5}$.

Then, as $\frac{12}{5} : \frac{15}{4} :: \frac{24}{5}$

5   12
15   4
---
75   48
24   5
---
300   240 D.
150

1800 N.

Answer $\frac{1800}{240} = \frac{15}{2} = 7l.\ 10s.$

E. 3. If $\frac{1}{4}$ of a yard cost $\frac{2}{3}$ of a pound, what will $\frac{3}{5}$ of an English ell cost, at that rate?

First $\frac{1}{4}$ of a yard $= \frac{1}{4}$ of $\frac{4}{5}$ of an ell $= \frac{4}{20}$

Then, if $\overset{Ell.}{\frac{4}{20}} : \overset{£.}{\frac{2}{3}} :: \overset{Ell.}{\frac{3}{5}}$

20   4
2   3
---
40   12
3   5
---
120 N.   60 D.

Answer $\frac{120}{60} = \frac{2}{1} = 2l.$

E. 4. If $1\frac{1}{2}$ herring cost $1\frac{1}{2}d.$ how many may be bought for $11d$?

First $1\frac{1}{2} = \frac{3}{2}$; $1\frac{1}{2}d. = \frac{3}{2}$, and $11 = \frac{11}{1}$; then

If $\overset{H.}{\frac{3}{2}} : \overset{d.}{\frac{3}{2}} :: \overset{H.}{\frac{11}{1}}$

2   3
3   2
---
6   6
11   1
---
66 N.   6 D.

Answer $\frac{66}{6} = \frac{11}{1} = 11$ herrings

E. 5. Suppose a merchant makes an assurance upon a ship and cargo, bound to the West Indies, value 2700*l.* 10*s.* and agrees to pay 10 guineas per cent. what comes the charges of the assurance to?

First $10\frac{1}{2} = \frac{21}{2}l.$ and $2700\frac{1}{2} = \frac{5401}{2}$; likewise $100 = \frac{100}{1}$, then

	£.		£.		£.
If	$\frac{100}{1}$	:	$\frac{21}{2}$	::	$\frac{5401}{2}$
			1		100
			21		2
			21		200
			5401		2
			5401		400 D.
			10802		
			113421 N.		

Ans. $\frac{113421}{400}l. = 283l.\ 11s.\ 0\frac{1}{2}d.$

Or thus, by rule 1:

5401	2
21	2
5401	4
10802	
113421 N.	

$\frac{100}{1})\frac{113421}{4}(\frac{113421}{400} = 283l.\ 11s.\ 0\frac{1}{2}d.$ the answer as before

E. 6. Three workmen can do a piece of work in certain times, viz. A can do it in 3 weeks, B can do thrice the work in 8 weeks, and C five times in 12 weeks; in what time can they finish it jointly?

First it may be easily found that A can do $\frac{1}{3}$, B $\frac{3}{8}$, and C $\frac{5}{12}$ of the work in one week, which fractions being reduced to a common denominator, make $\frac{8}{24}$, $\frac{9}{24}$, and $\frac{10}{24}$, whose sum $= \frac{27}{24} = \frac{8}{9}$, being the work they all can do when working together in one week; then

	Work.		*Days.*		*Work.*
If	$\frac{9}{8}$	:	$\frac{6}{1}$	::	$\frac{1}{1}$

8	9
6	1
48	9
1	1
48 N.	9 D.

Answer $\frac{48}{9} = \frac{16}{3} = 5\frac{1}{3}$ Days

XLIII. The RULE of THREE INVERSE,

IN VULGAR FRACTIONS.

RULE 1.

PREPARE the fractions as before directed, and then proceed as in section XIII.

RULE 2. Multiply the denominator of the third number into the numerator of the first and second for a new numerator; then multiply the numerator of the third number into the denominator of the first and second, for a denominator, which place under the numerator for the answer, and find the proper quantity as before directed.

EXAMPLE

EXAMPLE 1. What quantity of shalloon, that is $\frac{3}{4}$ yard wide, will line $7\frac{1}{2}$ yards of cloth, that is $1\frac{1}{2}$ yard wide?

First by reduction $1\frac{1}{2} = \frac{3}{2}$, and $7\frac{1}{2} = \frac{15}{2}$; then, if $\frac{3}{2} : \frac{15}{2} :: \frac{3}{4}$

$$\begin{array}{rr} 15 & 2 \\ 3 & 2 \\ \hline 45\ \text{N.} & 4\ \text{D.} \end{array}$$

$\frac{3}{4})\frac{45}{4}(\frac{180}{12} = \frac{15}{1} = 15$ yards, Ans.

E. 2. If 8 men can do a piece of work in $16\frac{3}{4}$ days, in how many days will 24 men do the same? As $\frac{8}{1} : 16\frac{3}{4} = \frac{67}{4} :: \frac{24}{1}$

$$\begin{array}{rr} 67 & 4 \\ 8 & 1 \\ \hline 536\ \text{N.} & 4\ \text{D.} \end{array}$$

$\frac{24}{1})\frac{536}{4}(\frac{536}{96} = 5\frac{53}{96}$ Days, Answ.

E. 3. How many yards of cloth, at 8*s.* 6*d.* per yard, must be given for $26\frac{5}{8}$ yards, at 5*s.* 7*d.* per yard?

First 8*s.* 6*d.* $= \frac{17}{2}$, and $26\frac{5}{8} = \frac{213}{8}$ yards; also 5*s.* 7*d.* $= \frac{67}{12}$; then, As $\frac{67}{12} : \frac{213}{8} :: \frac{17}{2}$

$$\begin{array}{rr} 213 & 12 \\ 67 & 8 \\ \hline 1491 & 96\ \text{D.} \\ 1278 & \\ \hline 14271\ \text{N.} & \end{array}$$

$\frac{17}{2})\frac{14271}{96}(\frac{28542}{1632} = \frac{14271}{816} = 17\frac{133}{272}$ yards, Answer

E. 4. Suppose B lends C, $100\frac{2}{3}$*l.* for $6\frac{2}{3}$ months, what sum must C lend B, for $3\frac{5}{6}$ years, to requite him?

First $100\frac{2}{3} = \frac{302}{3}$*l.* and $6\frac{2}{3} = \frac{20}{3}$ months; or $\frac{20}{3}$ of $\frac{1}{12} = \frac{20}{36} = \frac{5}{9}$ years; also $3\frac{5}{6} = \frac{23}{6}$ years.

Then as $\frac{5}{9} : \frac{302}{3} :: \frac{23}{6}$

$$\begin{array}{rr} 302 & 23 \\ 6 & 3 \\ \hline 1812 & 69 \\ 5 & 9 \\ \hline 9060\ \text{N.} & 621\ \text{D.} \end{array}$$

$\frac{9060}{621} = \frac{3020}{207} = 14$*l.* 11*s.* $9\frac{1}{4}$*d.* $\frac{165}{207}$ Answer

E. 5. Suppose 12 men mow down a field of grass in $5\frac{3}{4}$ days, how many men, at the same rate of working, will mow down the same in 3 days?

First $12 = \frac{12}{1}$, and $5\frac{3}{4} = \frac{23}{4}$; also $3 = \frac{3}{1}$; then as $\frac{23}{4} : \frac{12}{1} :: \frac{3}{1}$

$$\begin{array}{rr} 23 & 3 \\ 12 & 4 \\ \hline 276 & 12 \\ 1 & 1 \\ \hline 276\ \text{N.} & 12\ \text{D.} \end{array}$$

$\frac{276}{12} = \frac{23}{1} = 23$ men, the Answer

E. 6. How many yards of matting, of $\frac{1}{2}$ yard wide, will be sufficient to cover a floor that is 16 feet wide, and 28 feet long?

First $\frac{1}{2}$ yard $= \frac{9}{2}$ feet, and $16 = \frac{16}{1}$; also $28 = \frac{28}{1}$; then As $\frac{16}{1} : \frac{28}{1} :: \frac{9}{2}$

$$\begin{array}{rr} 28 & 9 \\ 16 & 1 \\ \hline 448 & 9 \\ 2 & 1 \\ \hline 896\ \text{N.} & 9\ \text{D.} \end{array}$$

Answer $\frac{896}{9} = 99\frac{5}{9}$ yards

XLIV. The DOUBLE RULE of THREE, IN VULGAR FRACTIONS.

RULE.

PREPARE the fractions by reduction, as before directed, and then proceed as in whole numbers. See section XIV.

EXAMPLE 1. If 12 men are hired to do a piece of work in 8 days, at 2*s.* 2*d.* per day, what will be the wages of 9 men for $20\frac{1}{2}$ days?

Firſt 12 men at 2*s.* 2*d.* per day, = 1*l.* 6*s.* = $1\frac{6}{20}l. = 1\frac{3}{10}l. = \frac{13}{10}$; and $20\frac{1}{2}$ days $= \frac{41}{2}$; then

$$\frac{12}{1} : \frac{13}{10} :: \frac{9}{1}$$
$$\frac{8}{1} : 0 :: \frac{41}{2}$$

Now $\frac{41}{2} \times \frac{13}{10} \times \frac{9}{1} = \frac{4797}{20}$ dividend. And $\frac{12}{1} \times \frac{8}{1} = \frac{96}{1}$ the diviſor.

Then $\frac{96}{1})\frac{4797}{20}(\frac{4797}{1920} = 2l.\ 9s.\ 11\frac{1}{2}d.\ \frac{96}{192}$ the Anſwer.

Or by neglecting the denominators, to find the numerator, and multiplying them into the other numerators, thus:

41	2
9	10
369	20
13	12
4797 N.	240
	8
	1920 D.

Anſ. $\frac{4797}{1920} = 2l.\ 9s.\ 11\frac{1}{2}d.\ \frac{96}{192}$ as before.

E. 2. What principal, put to intereſt, will gain 40*l.* in 8 months, at 5 per cent per annuin? Firſt 8 months $= \frac{2}{3}$ of a year

$$\text{Then as } \frac{5}{1} : \frac{100}{1} :: \frac{2}{3}$$
$$\frac{1}{1} : 0 :: \frac{40}{1}$$

Now $\frac{100}{1} \times \frac{40}{1} \times \frac{2}{3} = \frac{1200}{2}$ the divid. And $\frac{5}{1} \times \frac{1}{1} = \frac{5}{1}$ the diviſor.

$\frac{5}{1})\frac{12000}{2}(\frac{12000}{10} = 1200l.$ the Anſwer

Again by two ſtatings,
Firſt as $\frac{5}{1} : \frac{100}{1} :: \frac{40}{1}$
Then $\frac{100}{1} \times \frac{40}{1} = \frac{4000}{1}$
And $\frac{5}{1})\frac{4000}{1}(\frac{4000}{5} = 800l.$

Again as $\frac{1}{1} : \frac{800}{1} :: \frac{2}{3}$
Then $\frac{800}{1} \times \frac{1}{1} = \frac{800}{1}$
And $\frac{2}{3})\frac{800}{1}(\frac{2400}{2} = 1200l.$ the Anſwer, as before.

Note. This laſt ſtating is in inverſe proportion.

E. 3. Six men with their wives, upon calculation, found that their expences for three months paſt amounted to 26*l.* 19*s.* 4*d.* I demand what time 14*l.* 15*s.* may be ſpent by 36 men in the like proportion?

Firſt 26*l.* 9*s.* 4*d.* $= 26\frac{29}{30} = \frac{809}{30}l.$ and 14*l.* 15*s.* $= 14\frac{3}{4} = \frac{59}{4}l.$

$$\frac{12}{1} : \frac{3}{1} :: \frac{809}{30}$$
$$\frac{30}{1} : 0 :: \frac{59}{4}$$

Now $\frac{809}{30} \times \frac{36}{1} = \frac{4854}{5}$ the diviſor
And $\frac{59}{4} \times \frac{12}{1} \times \frac{3}{1} = \frac{531}{1}$ the dividend
Then $\frac{4854}{5})\frac{531}{1}(\frac{2655}{4854} = 16\frac{331}{809}$ Days, the anſwer.

Note. The numerator is multiplied by 30, the days in a month, and then valued as taught in reduction.

QUESTIONS

QUESTIONS IN FRACTIONS.

Qu. 1. Four figures of 6 may be so placed and disposed of, as to denote and read for 67, neither more nor less; pray how is that to be done? First $\frac{6}{6} = 1$, then 66

$$\begin{array}{r} 66 \\ 1 \\ \hline 67 \end{array}$$

Answer 67

Qu. 2. A lad having got 4000 nuts, in his return home was met by Mad Tom, who took from him $\frac{5}{8}$ of $\frac{2}{3}$ of his whole stock. Raving Ned lights on him afterwards, and forced $\frac{2}{5}$ of $\frac{5}{8}$ of the remainder from him; unluckly, Positive Jack found him, and required $\frac{7}{10}$ of $\frac{17}{20}$ of what he had left. Smiling Dolly was by promise to have $\frac{3}{4}$ of a quarter of what nuts he brought home; how many then had the boy left? First his whole stock = 4000

Then $\frac{5}{8}$ of $\frac{2}{3}$ of $4000 =$ $1666\frac{2}{3}$ Mad Tom took

$2333\frac{1}{3} = \frac{7000}{3}$ left

$\frac{2}{5}$ of $\frac{5}{8}$ of $\frac{7000}{3} =$ $583\frac{1}{3}$ Raving Ned took

1750 Left

$\frac{7}{10}$ of $\frac{17}{20}$ of $1750 =$ $1041\frac{1}{4}$ Positive Jack took

$708\frac{3}{4} = 708\frac{48}{64}$ Left

$\frac{3}{4}$ of $\frac{1}{4}$ of $\frac{2835}{4} =$ $132\frac{57}{64}$ Dolly had

Answer - $575\frac{55}{64}$ Left.

Qu. 3. If the scavengers' rate at $1\frac{1}{2}d.$ in the pound, comes to $6s.$ $7\frac{1}{2}d.$ where they ordinarily assess $\frac{4}{5}$ of the rent; what will the king's tax for that house be, at $4s.$ in the pound, rated at the full rent?

First $1\frac{1}{2}d. = \frac{1}{160}$, $6s.\ 7\frac{1}{2}d = \frac{159}{480} = \frac{53}{160}l.$ and $4s. = \frac{1}{5}l.$

Then as $\frac{1}{160} : \frac{1}{1} :: \frac{53}{160}$

$\frac{1}{160})\frac{53}{160}(\frac{8480}{160} = 53l. = \frac{4}{5}$ of the rent

$53 \div 4 = 13\frac{1}{4} = \frac{1}{5}$ ditto

£. $66\frac{1}{4}$ = the whole rent

Again, as $\frac{1}{1} : \frac{5}{1} :: 66\frac{1}{4} = \frac{265}{4}$

$$\begin{array}{r} 265 \\ 1 \\ \hline 265\ \text{N.} \end{array} \qquad \begin{array}{r} 5 \\ 4 \\ \hline 20\ \text{D.} \end{array}$$

Answer $\frac{265}{20} = 13l.\ 5s.$

Qu. 4. X, Y, and Z, can, working together, complete a stair-case in 12 days; Z is man enough to do it alone in 24 days, and X in 34; in what time then could Y get it done himself?

First $\frac{1}{34} = \frac{12}{408}$ X, $\frac{1}{24} = \frac{17}{408}$ Z;

Then $\frac{12}{408} + \frac{17}{408} = \frac{29}{408}$, the work performed in one day by X and Z; and

and $\frac{1}{12} = \frac{34}{408}$ performed in one day by all three working together;

Therefore $\frac{34}{408} - \frac{29}{408} = \frac{5}{408}$ performed in one day by Y.

Work. Day. Work.

Then as $\frac{5}{408}$: 1 :: 1 $\quad \frac{5}{408})\frac{1}{1}(\frac{408}{5} = 81\frac{3}{5}$ the Anſwer

Qu. 5. Miſs Kitty told her brother George, that though her fortune on her marriage took 19312*l.* out of the family, it was but $\frac{3}{5}$ of 2 years rent; heaven be praiſed for this yearly income! pray what was it? As $\frac{3}{5}$: $\frac{19312}{1}$ :: $\frac{5}{5}$

$\frac{3}{5})\frac{19312}{1}(\frac{96560}{3} = 32168$*l.* 13*s.* 4*d.* Two years rent, which $\div$ 2 = 16093*l.* 6*s.* 8*d.* the yearly income required.

Qu. 6. A politician having about him a certain number of crowns, ſaid, if $\frac{1}{4} + \frac{1}{3} + \frac{1}{6}$ of what he had, were added together, they would make juſt Wilkes's number (45); how many crowns had he about him?

First $\frac{1}{4} + \frac{1}{3} + \frac{1}{6} = \frac{18}{72} + \frac{24}{72} + \frac{12}{72} = \frac{54}{72} = \frac{24}{36}$

Then as $\frac{27}{36}$: $\frac{45}{1}$ :: $\frac{36}{36}$

$$\begin{array}{r} 36 \\ 45 \\ \hline 180 \\ 144 \\ \hline 1620\ \text{N.} \end{array} \qquad \begin{array}{r} 36 \\ 1 \\ \hline 36\ \text{D.} \end{array}$$

$\frac{27}{36})\frac{1620}{36}(\frac{58320}{972} = \frac{6}{1} = 60$ Crowns, the anſwer.

PRACTICAL ARITHMETIC.

PART III.

XLV. DECIMAL FRACTIONS.

DECIMALS are different from whole numbers; for whole numbers increaſe from the right-hand towards the left in a ten-fold proportion from unity or one; and decimals decreaſe from unity in the ſame proportion from the left-hand towards the right; the following table makes this evident.

Units	1,	Unit or integer
Primes	,1	One tenth part of the integer
Seconds	,01	One hundredth part
Thirds	,001	One thouſandth part
Fourths	,0001	One ten thouſandth part
Fifths	,00001	One hundred thouſandth part
Sixths	,000001	One millionth part
Sevenths	,0000001	One ten millionth part
Eighths	,00000001	One hundred millionth part
Ninths	,000000001	One thouſand millionth part

So

So that decimal fractions are of several denominations or names, as primes, seconds, thirds, &c. and because the denominator is always 1, with as many cyphers annexed as there are decimal places; for this reason the numerator or decimal is always wrote alone, without the denom. so if I would express the twenty-five hundredth parts of any thing, which vulgarly stands thus $\frac{25}{100}$, because the denominator is 1, with as many cyphers prefixed as there are decimals or places in the numerator, it is always expressed thus ,25; and $\frac{123}{1000}$ thus ,123; and $\frac{6848}{10000}$ thus ,6842, &c. And because vulgar fractions are the foundation of decimals, I shall shew (in its proper place) the manner of reducing them to decimals, by which means all those computations hitherto deemed so intricate, may be performed with the utmost ease and pleasure.

A finite decimal is that which ends at a certain number of places; but an infinite, is that which no where ends.

A circulating or recurring decimal is that wherein one or more figures are continually repeated.

Thus 84,56666, &c. is called a single circulate or recurring decimal.

And 147,642642, &c. is called a compound recurring decimal.

In all operations, if the result consists of several nines, reject them, and make the next superior place a unit more. Thus for 12,2999 write 12,3; and for 32,99 write 33, &c.

XLVI. ADDITION of DECIMALS.

R U L E.

PLACE primes under primes, seconds under seconds, &c. whether they be cyphers or significant figures; when the work is done, make a point or dot with your pen between the whole numbers (if there be any) and decimals; this is known by cutting off so many places to the right hand as your greatest decimal fraction contains.

E X A M P L E S.

21,42	3,121	,31214	61,2182
1,0	2,14	,0214	4,1041
23,4	34,11	,36212	,342
561,21	410,2	,514	6,13
3,424	34,13	,231	78,41
1,212	4,521	,41642	3,4265
611,666	588,222	1,85708	153,6308

£.	s.	d.		£.
59	7	$7\frac{3}{4}$	=	59,3822916
57	17	5	=	57,8708333
57	13	4	=	57,6666666
25	6	8	=	25,3333333
45	13	4	=	45,6666666
245	18	$4\frac{3}{4}$	=	245,9197916
				20
	Shillings			18,3958320
				12
	Pence			4,7499840
				4
	Farthings			2,9999360

Note. When all or any of the decimals repeat a single digit, make the repetends conterminous, and add 1 to the sum of the first, or right-hand column, for every nine that is contained in it.

Agreeing with the above nearly.

XLVII. SUBTRACTION of DECIMALS.

RULE.

PLACE the greater number uppermost, the points under the points, tenths under tenths, &c. then subtract as in whole numbers, placing the points of separation under the other points.

EXAMPLES.

From	,864213	36,1214
Take	,128191	,81642
Remains	,736022	35,30498
Proof	,864213	36,1214

In subtracting integers and decimals, observe the following order:

	£.		£.	s.	d.	q.
Lent	1730,027	=	1730	0	6	1,92
Received	1681,8352	=	1681	16	8	1,792
Remains	48,1918	=	48	3	10	0,128
	20					
Shillings	3,8360					
	12					
Pence	10,0320					
	4					
Farthings	128					

Agreeing exactly with that on the right hand; for the decimal ,1918 of a pound is equal to 3*s.* 10*d.* 0*q.* ,128.

If a single digit is repeated, borrow 9 in the first repeating place when necessary.

From

	£.	s.	d.		£.
From	7849	6	8	=	7849,333
Take	6979	13	4	=	6979,666
Remains	869	13	4	=	869,666

XLVIII. MULTIPLICATION of DECIMALS.

RULE.

MULTIPLY the decimals, as if they were whole numbers, and from the product cut off as many decimal places, as there are in both numbers. If there be not so many places, make them out with cyphers on the left-hand of the product.

EXAMPLES.

,3042	,3042	,3042
,2015	20,15	2015,
15210	15210	15210
3042	3042	3042
6084	6084	6084
,06129630	6,129630	612,9630

Note. I have made use of the same figures throughout each of these examples; yet the reader will find the values of the products are very different.

CONTRACTIONS.

It frequently happens in business, that one or both the factors consist of many decimal places; so that to work them all would be very troublesome, and when done, but little to the purpose, because a less number of places may do the business as well; therefore use the following

RULE. 1. Transpose all the figures of the multiplier in a contrary order to the common way, viz. let the units place stand to the left-hand.

2. The units place of the multiplier must stand under that place of the multiplicand whose decimal place you intend to retain in the product.

3. Begin as in common multiplication, always having regard to the increase of that figure on the right-hand, the figure that stands over your multiplier; making use of no more places of your multiplier than those which stand even with your multiplicand to the left-hand.

E. 1. Let it be required to multiply 3,14159 by 24,8253, and to retain 4 decimal places in the product?

3,14159	Multiplicand
3528,42	Multiplier inverted
628318	
125663	
25132	
628	
157	
9	
77,9907	Product

The operation at large :

3,14	159
24,8	253
9	42477
157	0795
628	368
25132	72
125663	6
628318	
77,9909	19227

Note. As the allowance for what may be carried from the columns neglected is altogether a guess, we may very often make the product less than it ought to be by 1 or 2, as appears by the above example; to avoid which, make one or two columns more than the number of decimal places you would have in the product, and cut them off at pleasure.

E. 2. Multiply 75,4678 by 6,05408, so as to retain only three places of decimals in the product ?

75,4678
80450,6
452806
3773
301
456,880

E. 3. Multiply ,68479 by ,0785 to have 5 decimal places in the product ?

,68479
5870,0
4793
547
34
,05374

If the multiplier is a decimal fraction, put a cypher in the units place, and set the other figures in order from that on the left-hand.

From these examples it is manifest how advantageous these contractions are to shorten the work of long calculations and computations, which the experienced practitioner finds too often occur, in arithmetic, algebra, and geometry.

To multiply by 10, 100, 1000, &c. remove the decimal point so many steps further to the right-hand, as there are cyphers in the multiplier. As, $86,564 \times 100 = 8656,4$; and $45 \times 1000 = 450$, &c.

XLIX. DIVISION of DECIMALS.

R U L E.

DIVIDE as if they were whole numbers; then cut off as many decimal places in the quotient, as the number of decimal places in the dividend exceeds the number in the divisor; if there are not so many in the divisor, prefix so many cyphers.

In dividing a whole number by a whole number, if any thing remains, annex cyphers to the remainder, and continue the division as far

far as you pleaſe; ſo you will have a decimal in the quotient of as many places as you annexed cyphers, and the whole quotient thus found will be a mixed number.

There are nine caſes, which take in the following order, by which the learner will eaſily acquire a true notion of the ground and nature of decimals.

CASE 1. A whole number given to be divided by a whole number.

579268,)314159265,00000(542,33837,
2896340
2452526
2317072
1354545
1158536
1960090
1737804
2222860
1737804
4850560
4634144
2164160
1737804
4263560
4054876
Remains 208684

In this example here are five cyphers added to the remainder, which produce five decimal places in the quotient.

CASE 2. A whole number given, to be divided by a mixed number.

579,268)314159265,0000(542338,3
2896340
2452526
2317072
1354545
1158536
1960090
1737804
2222860
1737804
4850560
4634144
2164160
1737804
Remains 426356

In the laſt example three cyphers are added to the right-hand of the whole number in the dividend, which makes the quotient a whole number; and becauſe there is a remainder, you may go on again, by adding cyphers at pleaſure; ſo the quotient will be a mixed number.

CASE 3. A mixed number given, to be div. by a whole No.

579268,)3,14159265(,00000542
2896340
2452526
2317072
1354545
1158536
Remains 196009

In this example here are five cyphers prefixed to the quotient, that they might be equal to the decimal places of the dividend.

CASE 4. A mixed-number given to be divided by a mixed number.

57,9268)31,4159265(,542
2896340
2452526
2317072
1354545
1158536
Remains 196009

In this example the decimal places in the dividend exceed thoſe in the diviſor by three, therefore the quotient is a decimal.

CASE 5. A whole number given, to be divided by a decimal fraction.

```
,579268)314159265,000000(542338373
        2896340
        -------
         2452526
         2317072
         -------
          1354545
          1158536
          -------
          1960090
          1737804
          -------
           2222860
           1737804
           -------
            4850560
            4634144
            -------
            2164160
            1737804
            -------
             4263560
             4054876
             -------
              2086840
              1737804
              -------
     Remains   349036
```

In this example here are six cyphers annexed to the dividend to answer the decimal places of the divisor, that the quotient might be a whole number.

CASE 6. A mixed number given, to be divided by a decimal fraction.

```
,579268(3,14159265(5,42
          2896340
          -------
           2452526
           2317072
           -------
            1354545
            1158536
            -------
Remainder    196009
```

CASE 7. A decimal fraction given, to be divided by a whole number.

```
579268),314159265(,000000542
         2896340
         -------
          2452526
          2317072
          -------
           1354545
           1158536
           -------
Remainder   196009
```

CASE 8. A decimal fraction given, to be divided by a mixed number.

```
5,79268),314159265(,0542
          2896340
          -------
           2452526
           2317072
           -------
            1354545
            1158536
            -------
Remainder    196009
```

CASE 9. A decimal fraction given, to be divided by a decimal fraction.

```
,579268),314159265(,542
          2896340
          -------
           2452526
           2317072
           -------
            1354545
            1158536
            -------
Remainder    196009
```

If any whole, mixed, or decimal number, is given to be divided by 10, 100, 1000, &c. you only remove the separating point towards the left-hand so many places as there are cyphers in the divisor, contrary to what was taught in multiplication.

Thus, $1523 \div 10 = 152{,}3$; and $1523 \div 1000 = 1{,}523$, &c.

To work any case of division by multiplication, and on the contrary, any case of multiplication by division; and this in many instances will be found very useful,

RULE.

RULE. Divide a unit with cyphers annexed by the given multiplier, and the quotient is the divisor sought.

EXAMPLE. Suppose I have 7315 to multiply by any other number, as 125; but have a desire to divide the said number, and to have a quotient equal to the product of those two numbers; query, the divisor?

Given 125)1,000(,008 the divisor sought
```
           1000
           ----
              0
```

```
Then      7315        And ,008)7315,000(914375 Quotient,
        × 125                  72              equal to the
        ------                 --              product
         36575                 11
        14630                   8
        7315                   --
        ------                 35
Product 914375                 32
                               --
                               30
                               24
                               --
                               60
                               56
                               --
                               40
                               40
                               --
```

Suppose I have 7315 given, to be divided by any other number ,008; but would multiply the said number, and have a product equal to the quotient of the same number divided by ,008; query the multiplier?

RULE. Divide an unit with cyphers annexed by the given divisor, and the quotient will be the multiplier sought.

Thus ,008)1,000(125

The remainder of the work is only the reverse of the former, and therefore need not be repeated.

From the foregoing examples relating to division it may be observed, that the first figure of every quotient must possess the same place (with respect to its value) as that figure of the dividend doth, which stands over the units place of the first figure's product, which is an excellent rule to value quotients, obtained by the following

CONTRACTION. When the divisor consists of many places of decimal parts, the work may be much abbreviated by the following

RULE. Consider in what place the first figure of the quotient ought to stand, and find its value or denomination; taking as many of the left-hand figures as you intend to have figures in the quotient for the first divisor; then take as many figures of the dividend as will answer them. In dividing, omit, or point off one figure at each operation; at the same time, have a due regard to the increase, which would arise from the figure or figures so omitted.

EXAMPLE

EXAMPLE 1.

76,84375)630,92878(8,210541
• ····· 61475000
1617878
1536875
81003
76843
4160
3842
318
307
11
7
4

In this example, 8 is multiplied into 76,84375; then 2 is multiplied into 76,8437, carrying 1 from the last figure pricked off, and so you must proceed with the remainder of the figures in the divisor until they are all pricked off.

If the dividend contains many places of decimals, there is no occasion for using but a few of the first.

E. 2. 57,92|68)3,1415|9265(,0542
28963
2452
2317
135
115
20

The common method.

57,9268)3,1415|9265(,0542

28963	40
2452	526
2317	072
135	4545
115	8536
19	6009

Note. Though much labour may be saved by this method, yet it is only useful when the decimals in the dividend contain many places, and then take all the divisor.

E. 3. 24,324)842,31415216342(,00034629

Note. As these contractions, and those taught in multiplication, answer the same end in almost all operations as the method of circulating or recurring decimals; therefore, to have treated on them, would be swelling this treatise to answer no purpose but curiosity only.

L. REDUCTION of DECIMALS.

CASE I.

TO reduce a vulgar fraction to a decimal,

RULE. Add cyphers to the numerator, representing so many places of decimals, and divide by the denominator; the quotient will be the decimal fraction required.

EXAMPLE 1. Reduce $\frac{1}{4}$, $\frac{1}{2}$, and $\frac{3}{4}$, to decimals?

4)1,00 — ,25 2)1,0 — ,5 4)3,00 — ,75

Answer ,25 $= \frac{1}{4}$; ,5 $= \frac{1}{2}$, and ,75 $= \frac{3}{4}$

E. 2.

E. 2. Reduce $\frac{1}{3}$ to a decimal?

$$3)1,0000$$
$$,3333, \text{ \&c. ad infinitum}$$

E. 3. Reduce $\frac{5}{16}$ to a decimal?

$$16\left\{\begin{array}{l}2)5,0 \\ 8)2,5\end{array}\right.$$

Answer ,3125

Or thus, 16)5,0000(3125

$$\begin{array}{r}48 \\ \hline 20 \\ 16 \\ \hline 40 \\ 32 \\ \hline 80 \\ 80 \\ \hline 0\end{array}$$

E. 4. Reduce $13\frac{4}{7}$ to a decimal, or mixed number?

First $13\frac{4}{7} = \frac{95}{7}$; then

$$7)95$$
$$13,571428$$

Answer $13\frac{4}{7} = 13,571428$

E. 5. Reduce $\frac{1}{2}$ of $\frac{3}{4}$ of $\frac{2}{5}$ to a decimal? First $\frac{1}{2}$ of $\frac{3}{4}$ of $\frac{2}{5} = \frac{6}{40}$

Then 40)6,00(,15 Answer

$$\begin{array}{r}40 \\ \hline 200 \\ 200 \\ \hline 0\end{array}$$

E. 6. Reduce $\frac{164}{395}$ to a decimal?

395)164,000(,415

$$\begin{array}{r}1580 \\ \hline 600 \\ 395 \\ \hline 2050 \\ 1975 \\ \hline 75\end{array}$$

Note. If the decimals will not terminate, but there will still be a remainder, it will be exact enough in most cases, and the remainder may be rejected after the decimal has been carried on to 4 or 5 places.

CASE 2. To reduce coins, weights, measures, &c. into decimals,

RULE 1. Reduce the given money, weights, &c. into the lowest denomination or name mentioned, for a dividend; then reduce the integer into the same denomination for a divisor, the quotient will be the decimal required.

RULE 2. Place the numbers of the several denominations under each other, beginning with the least, and divide each by such a number that will raise it to the next superior name, placing each quotient as a decimal part of the next dividend before it be divided, and the final quotient will be the answer.

EXAMPLE

EXAMPLE 1. Reduce 18*s.* $6\frac{3}{4}$*d.* to the decimal of a pound?

```
                 s.  d.
By rule 1, thus  18  6¾
                 12
                 ---
                 222
                   4
£.      qrs.     ---
1 = 960 )891,000000(,928125 The decimal required
         864
         ---
          270
          192
          ---
           780
           768
           ---
            120
             96
            ---
             240
             192
             ---
              480
              480
              ---
                0
```

By rule 2, thus:

```
 4 | 3,00
12 | 6,75
2|0| 18,5625
   -------
   ,928125 The decimal as
           (before
```

Note. By rule 2, the three farthings are reduced to the decimal of a penny (which = ,75) and set on the right of 6*d.* then 6,75 pence to the decimal of a shilling (= ,5625) then 18,5625 shillings to the decimal of a pound.

E. 2. Reduce 15*s.* 9*d.* to the decimal of a pound?

```
              s.  d.
              15  9
              12
£.    d.     ----
1 = 24|0)189,0000(,7875 the
           (decimal required
```

By rule 2, thus:

```
 12 | 9,00
2|0 | 15,75
    -------
Answer ,7875 same as before
```

E. 3. Reduce $\frac{3}{4}$ of a penny to the decimal of a pound?

First $\frac{3}{4}$ of $\frac{1}{12}$ of $\frac{1}{20} = \frac{3}{960} = \frac{1}{320}$*l.* then,

32|0)1,000000(,003125 the de-
(cimal required

By rule 2, thus;

```
 4 | 3,00
12 | ,75
2|0| ,0625
   -------
Answer ,003125 as before
```

E. 4. Reduce 11 *dwts.* to the decimal of a pound troy?

First 1 *lb.* = 240 *dwts.* then,

dwts.
24|0)11,0000(,4583 the decimal
(required

E. 5. Reduce 10 drams to the decimal of a pound avoirdupoise?

First 1 *lb.* = 256 *drs.* then,

256)10,00000(,03906 the decimal
(required

E. 6.

E. 6. Reduce 9 inches to the decimal of a yard?
First 1 yard = 36 inches; then, 36)9,00(,25 the decimal required.

E. 7. Reduce $3\frac{1}{4}$ inches to the decimal of a foot?

$3\frac{1}{4}$
4
Foot. *qrs.* ———
1 = 48)13,0000(,2708 $\frac{16}{48}$ the decimal required.

E. 8. Reduce 6 furlongs to the decimal of a league?
First 1 league is 3 miles = 24 furl. then 24)6,00(,25 the decimal req.

E. 9. Reduce 12 gallons 2 quarts of wine, to the decimal of a hogshead?

12 2
4
Hhd. *qts.* ———
1 = 252) 50,00000(,1984 the decimal.

E. 10. Reduce 3 quarts 1 pint of ale, to the decimal of a barrel?

3 1
pts. 2
1 Barrel = 256)7,00000(,02734 $\frac{96}{256}$ Answer.

E. 11. Reduce 4 inches to the decimal of a foot?

12)4,000
———
,333 &c. the decimal required.

E. 12. Reduce 36 poles to the decimal of an acre?
First 1 acre = 160 poles; then 16|0)36,000(,225 the answer.

E. 13. Reduce 4 bushels 2 pecks to the decimal of a chaldron?
First 4 bushels 2 pecks = 18 pecks, and a chal. = 144 pecks;
Then 144)18,000(,125 the decimal required.

E. 14. Reduce 12 minutes to the decimal of an hour?

Min.
1 Hour = 60)12,0(,2 the decimal required.

E. 15. Reduce 2 qrs. 25 pounds, to the decimal of a hundred?

2 25
28
Cwt. *lb.* ———
1 = 112)81,0000(,7232 $\frac{16}{112}$ Answer.

E. 16. Reduce 12 days to the decimal of a Julian year?
First 12 days = 288 hours, and 365 days 6 hours = 8766 hours;
Then 8766)288,00(,0328 $\frac{4752}{8766}$ the decimal required.

E. 17. Reduce 440 yards to the decimal of a mile?
First 1 mile = 1760 yards; then 1760)440,00(,25 Answer.

In order to the application of decimals, we ought to have ready calculated the decimal of any integer of money, weight, meafure, &c. anfwering to every number, fimple or mixed, of inferior denomination, and of lefs value than that integer; which decimals being orderly collected and difpofed, make what we call decimal tables, by which any decimal required may be readily found, or alfo the value of any given decimal in known inferior fpecies.

To find every decimal by a feparate application of the foregoing rule, would be a very tedious work (though it is a general and complete one).

DECIMAL TABLES of Coin, Weight, and Meafure.

TABLE I.
COIN.
1 £. fter. the integer

Sh.	*dec.*	*Sh.*	*dec.*
19	,95	9	,45
18	,9	8	,4
17	,85	7	,35
16	,8	6	,3
15	,75	5	,25
14	,7	4	,2
13	,65	3	,15
12	,6	2	,1
11	,55	1	,05
10	,5		

Pence	*Decimals*
11	,045833
10	,041666
9	,0375
8	,033333
7	,029166
6	,025
5	,020833
4	,016666
3	,0125
2	,008333
1	,004166

Farth.	*Decimals*
3	,003125
2	,002083
1	,001042

TABLE II.
Troy weight.
1*lb.* the integer
This table will ferve for inches, months, or dozens.

Ounces	*Decimals*
11	,916666
10	,833333
9	,75
8	,666666
7	,583333
6	,5
5	,416666
4	,333333
3	,25
2	,166666
1	.083333

Penny weight	*Decimals*
10	,041666
9	,0375
8	,033333
7	,029166
6	,025
5	,020833
4	,016666
3	,0125
2	,008333
1	,004166

Grains	*Decimals*
20	,003472
10	,001736
9	,001562
8	,001389
7	,0 01215
6	,001042
5	,000868
4	,000694
3	,000521
2	,000347
1	,000173
½	,000086

TABLE III.
Avoirdupoife
112*lb.* the integ.

Qrs.	*Decimals*
1	,25
2	,5
3	,75

Pounds	*Decimals*
20	,178571
14	,125
13	,116071
12	,107143
11	,098214
10	,089286
9	,080357
8	,071428
7	,0625
6	,053571
5	,044643
4	,035714
3	,026786
2	,017857
1	,008928

Ounces	*Decimals*
10	,00558
9	,005022
8	,004464
7	,003906
6	,003348
5	,00279
4	,002232
3	,001674
2	,001116
1	,000558

Drams	*Decimals*
10	,000348
9	,000313
8	,000279
7	,000244
6	,000209
5	,000174
4	,000139
3	,000104
2	,000069
1	,000034
½	,000027

TABLE IV.
Avoird. weight
1*lb.* the integer

Ounces	*Decimals*
8	,5
7	,4375
6	,375
5	,3125
4	,25
3	,1875
2	,125
1	,0625

Drams	*Decimals*
8	,03125
7	,027343
6	,023437
5	,019531
4	,015625
3	,011718
2	,007812
1	,003906

TABLE V.
Liquid meafure
1 Tun the int.

Gall.	*Decimals*
100	,396825
90	,357141
80	,317462
70	,27
60	,238095
50	,198412
40	,15873
30	,119047
20	,079365
10	,039682
9	,035714
8	,031746
7	,027
6	,023809
5	,019841
4	,015873
3	,011904
2	,007936
1	,003968

DECIMAL TABLES of Coin, Weight, and Measure.

Pints	Decimals
4	,001984
3	,001488
2	,000992
1	,000496

A Hogshead the int.

Gall.	Decimals
30	,47619
20	,31746
10	,15873
9	,142857
8	,126984
7	,111111
6	,095238
5	,079365
4	,063492
3	,047619
2	,031746
1	,015873

Pints	Decimals
3	,005952
2	,003968
1	,001984

TABLE VI.

Measure

Liquid 1 *Gall.* *Dry* 1 *Quar.*

Integer

Pints	Decim.	Bush
4	,5	4
3	,375	3
2	,25	2
1	,125	1

Q.pt.	Decim.	Peck
3	,09375	3
2	,0625	2
1	,03125	1

Decimals	Q. Pk.
,023437	3
,015615	2
,007812	1

Decimals	Pints
,005859	3
,003906	2
,001953	1

TABLE VII.

Long measure

1 mile the int.

Yards	Decimals
1000	,568182
900	,511364
800	,454545
700	,397727
600	,340909
500	,284091
400	,227272
300	,170545
200	,113636
100	,056818
90	,051136
80	,045454
70	,039773
60	,054091
50	,028409
40	,022727
30	,017045
20	,011364
10	,005682
9	,005114
8	,004545
7	,003977
6	,003409
5	,002841
4	,002273
3	,001704
2	,001139
1	,000568

Feet	Decimals
2	,0003787
1	,0001894

Inches	Decimals
6	,0000947
3	,0000474
2	,0000315
1	,0000158

TABLE VIII.

TIME

1 Year the integer

Days	Decimals
300	,821918
200	,547945
100	,273963
90	,246575
80	,219178
70	,191781
60	,164383
50	,136986
40	,109589
30	,082192
20	,054794
10	,027397
9	,024657
8	,021918
7	,019178
6	,016438
5	,013699
4	,010959
3	,008219
2	,005479
1	,002739

1 Day the integer

Hours	Decimals
20	,833333
10	,416666
9	,375
8	,333333
7	,291666
6	,25
5	,208333
4	,166666
3	,125
2	,083333
1	,041666

Minute	Decimals
50	,034722
40	,027777
30	,020833
20	,013888
10	,006944
9	,00625
8	,005555
7	,004861
6	,004166
5	,003472
4	,002777
3	,002083
2	,001388
1	,000694

TABLE IX.

Cloth measure

1 Yard the int.

Qrs.	Decimals
3	,75
2	,5
1	,25

Nails	Decimals
3	,1875
2	,125
1	,0625

TABLE X.

Lead weight

1 Fother the int.

Hund.	Decimals
10	,51282
9	,461538
8	,410256
7	,358974
6	,307692
5	,25641
4	,205128
3	,153846
2	,102564
1	,051282

Qrs.	Decimals
2	,025641
1	,01282

Pounds	Decimals
14	,0064102
13	,0059523
12	,0054945
11	,0050366
10	,0045789
9	,0041208
8	,003663
7	,0032051
6	,0027472
5	,0022893
4	,0018315
3	,0013736
2	,0009157
1	,0004578

TABLE XI.

Of motion

A sign of the zodiac the int.

D°	Decimals
1	,033333
2	,066666
3	,1
4	,133333
5	,166666
6	,2
7	,233333
8	,266666
9	,3
10	,333333
11	,366666
12	,4
13	,433333
14	,466666
15	,5
16	,533333
17	,566666
18	,6
19	,633333
20	,666666
21	,7
22	,733333
23	,766666
24	,8
25	,833333
26	,866666
27	,9
28	,933333
29	,966666

Note. The uſe of the preceding tables is ſo obvious and natural, even by a bare inſpection, that I preſume it is needleſs to ſay any thing about that; the following examples being ſufficient to teſtify the great uſe and excellency of ſuch tables, and will at the ſame time give the learner a clear knowledge of the uſe of them.

EXAMPLE 1. What is the decimal part of a pound for 15*s.* 9*d*?

In table I. you find againſt	15 Shillings - -	,75
	9 Pence - - -	,0375
	The anſwer is -	,7875

E. 2. What decimal part of a pound is 18*s.* 6¾*d*?

In table I. you find againſt	18 Shillings - - -	,9
	6 Pence - -	,025
	3 Farthings - -	,003125
	The anſwer is - - -	,928125

E. 3. What decimal part of a pound troy is 7 ounces?
In table II. you find againſt 7 ounces - ,583333 Anſwer

E. 4. What decimal part of an hundred weight is 12 pounds 4 oz?

In table III. you find againſt	12 Pounds - - -	,107143
	4 Ounces - - -	,002232
	The anſwer - -	,109375

E. 5. What decimal part of a mile is 300 yards 2 feet?

In table VII. you find againſt	300 Yards - -	,170454
	2 Feet - - - -	,0003787
	The anſwer - -	,1708327

By the preceding tables all the ſpecies of money, weight, meaſure, &c. contained therein, by the above method are immediately turned into decimals, and are then worked with the ſame pleaſure and facility as whole numbers.

CASE 3. To find the value of any decimal fraction, in money, weight, meaſure, &c.

RULE. Multiply the given decimal by the parts of the next inferior denomination, and cut off towards the right-hand of the product ſo many figures as there are places in the given decimal, and thoſe on the left will be integers; then multiply the remaining decimals by the next inferior denomination, and cut off for decimals as before; thus proceed till you have brought it into the loweſt parts of the integer. A few examples will make this plain to the young practitioner.

EXAMPLE 1. What is the value of ,725 of a pound ſterling?

	,725
	20
Shillings	14,500
	12
Pence -	6,0

Anſwer 14*s.* 6*d.*

Note.

Note. As often as cyphers fall on the right-hand of your work, always drop them, for they are of no value.

E. 2. What is the value of ,72083 of a crown?

	5	
Shillings	3,60416	
	12	
Pence -	7,24999	Answer 3*s.* $7\frac{1}{4}$*d.*
	4	
Farthings	,99999	

Note. If the multiplicand be a compound repetend, and the multiplier only a single digit, to the product of the first figure on the right-hand, add as many units as there are tens in the product of the left-hand place of the repetend.

Thus in the above example, ,72083 being a repetend as above described, I multiply by 5, the shillings in a crown, saying 5 times 3 is 15, there being only one ten in that product, I set down 6, which is one more, and then proceed as in common multiplication with the remainder of the multiplicand. Again ,60416 I multiply by 12, the pence in a shilling, saying 12 times 6 is 72, there being seven tens in that product, I add 7 to the 2 remaining, which makes 9, which I set down, and proceed as before with the remainder of the multiplicand, continuing thus till the work is finished.

E. 3. What is the value of ,36 of a shilling?

```
 ,36
  12
 ----
 4,32
   4
 ----
 1,28   Answer 4¼d.
```

E. 4. What is the value of ,9 of a guinea?

```
 ,9
 21
 ----
 18,9
 12
 ----
 10,8
  4
 ----
 3,2   Answer 18s. 10¾d.
```

E. 5. What is the value of ,775 of an ounce troy?

```
       ,775
         20
       ----
dwts.  15,5
         24
       ----
grs.   12,0   Ans. 15 dwts. 12 grs.
```

E. 6. What is the value of ,3375 of a ton?

```
      ,3375
         20
      ------
Cwt.  6,7500
          4
      ------
qrs.  3,00   Ans. 6 Cwt. 3 qrs.
```

E. 7. What is the value of ,175 of a hundred weight?

```
     ,175
        4
     ----
     ,700
       28
     ----
lb.  19,6   Answer 19 lb. 9 oz.
```

E. 8. What is the value of ,8375 of an acre?

```
          ,8375
              4
         ------
Roods    3,3500
             40
         ------
Perches  14,0   Ans. 3 r. 14 per.
```

E. 9.

E. 9. What is the value of ,933593 of a barrel of ale, London measure?

,933593
32
1867186
2800779
Gall. 29,874976
4
Quarts 3,499904
2
Pints ,999808

Answer 29 *galls.* 3 *qts.* 1 *pt.* nearly

E. 10. What is the value of ,342 of a day?

,342
24
1368
684
Hours 8,208
60
Minutes 12,48
60
Seconds 28,8
60
Thirds 48,0

	Hours.	*min.*	*sec.*	*thirds.*
Answer	8	12	28	48

E. 11. What is the value of ,241 of a chaldron of coals?

,241
36
1446
723
Bushels 8,676
4
Pecks 2,704

Answer 8 *bushels*, 2 *pecks*

E. 12. What is the value of ,53373 of a year?

,53373
13
160119
53373
Months 6,93849
4
Weeks 3,75396
7
Days 5,27772
24
111088
55544
Hours 6,66528
60
Minutes 39,9168
60
Seconds 55,008

	Mo.	*w.*	*d.*	*h.*	*m.*	*sec.*
Answer	6	3	5	6	39	55+

These examples I think sufficient to shew the method of reducing decimals into the known parts of any species of quantity.

LI. Extraction of the SQUARE ROOT.

EXTRACTION of the square root, is finding such a number that being multiplied by itself shall give the respective power, out of which the root is to be extracted; as if 36 be proposed to be extracted, you will find its root to be 6, for $6 \times 6 = 36$, the given number.

T A B L E.

Roots	1	2	3	4	5	6	7	8	9
Squares	1	4	9	16	25	36	49	64	81

To

To extract the ſquare root of any number, obſerve the following

RULE. 1. Begin at the units place, and point the given number into periods of two figures each.

2. Find the greateſt ſquare that is contained in the firſt period, towards the left-hand; ſet the root in the quotient, and ſubtract the ſquare from the figures of that period.

3. To the remainder bring down the two figures under the next point for a dividend.

4. Double the quotient or root, and place it for a diviſor; ſeek how often the diviſor is contained in the dividend (reſerving the units place) and put the anſwer in the quotient, and alſo on the right-hand of the diviſor; then multiply the diviſor by the laſt figure put in the quotient (as in common diviſion) the product ſubtract from the dividend, and to the remainder bring down the next period, and proceed thus till all the figures or periods are brought down.

Note. If at laſt there be no remainder, the quotient will be the true root; but if any thing remain, annex two cyphers, and work as has been taught above, and for every two cyphers thus annexed, there will be one decimal place in the root.

Inſtead of doubling the quotient every time for a diviſor, you may always add the laſt quotient figure to the laſt diviſor, for a new diviſor, and proceed as before.

EXAMPLE 1. Let it be required to extract the ſquare root of 393129?

```
           .  .  .
           393129(627 Root
           36
     122)331
     +2  244
    1247) 8729
          8729
```

EXPLANATION. The number being ſeparated or pointed into periods of two figures each, then the neareſt ſquare to 39 the firſt period, is 36, whoſe root 6, I place in the quotient, and ſubtract the ſquare 36 from 39, the remainder is 3.

Then I bring down 31, the next point, and annex it to 3, and the new dividend is 331, then I double the quotient 6 for a diviſor, which is 12, and ſeek how oft 12 in 33? the anſwer is 2, which I place in the quotient, and alſo after 12; then the diviſor becomes 122, which multiplied by 2, the product is 244, which ſubtracted from 331, the remainder is 87.

Laſtly, I bring down 29, the next point, and the dividend is 8729; then I double the quotient 62, which is 124, for a new diviſor, and ſeek how oft 124 in 872? the anſwer is 7 times. Then I multiply 1247 by 7, and ſubtract the product 8729 from the laſt dividend, and there remains nothing; therefore 393129 is found to be a ſquare number, and 627 its root.

To prove the work; if you ſquare the root, and to that product add the remainder (if any) that ſum ſhall be equal to the number firſt given, thus: $627 \times 627 = 393129$, the given div. in the laſt example.

E. 2.

E. 2. What is the square root of 321489?

```
321489)567 Root
25
106) 714
+6   636
1127) 7889
      7889
      ....
```

E. 3. What is the square root of 814602573?

```
814602573,0000(28541,24
4                (Root
48)414
   384
365) 3060
     2825
5704) 23525
      22816
57081) 70973
       57081
570822) 1389200
        1141644
5708244) 24755600
         22832976
Remains 1922624
```

Note. If the root of a mixed number is proposed to be extracted, make the number of decimal places even, by annexing cyphers to the right-hand of the given square, that a point may fall on the units place of the whole number.

E. 4. What is the square root of 436,5?

```
436,50000000(20,8925 Rt.
4
408) 3650
     3264
4169) 38600
      37521
41782) 107900
        83564
417845) 2433600
        2089225
Remains 344375
```

E. 5. What is the square root of ,000729?

```
,000729(,027 Root
4
47)329
   329
   ...
```

E. 6. What is the square Root of 2?

```
2,000000000000(1,414213 Root
1
24)100
   96
281) 400
     281
2824) 11900
      11296
28282) 60400
        56564
282841) 383600
        282841
2828423) 10075900
          8485269
Remains 1590631
```

Note. When the root is to be extracted to a great number of places, the work may be much abbreviated by proceeding by the common method, till you have one figure more than half the number there is to be in the root, and then dividing the remainder according to the contraction in division of decimals. See the above example worked by this method.

E. 7.

```
 . . . . . . .
 2,000000000000(1,414213
 1               Root, the
 ----            same as
24) 100          before
     96
    ----
281) 400
     281
     -----
2824) 11900
      11296
      -----
2828)   604
        565
        ---
         39
         28
         --
         11
          8
          -
Remainder 3
```

E. 8.

```
 . . . . . . .
 2,000000000000(1,414213
 1                  Root, as
 ----               before
24)100
    96
   ----
281) 400
     281
     -----
2824)  11900
       11296
      ------
2828)   6040
        5656
        ----
        3840
        2828
        ----
        9120
        8484
        ----
Remainder 636
```

Note. If common division be used, you must bring down as many figures, as there were periods to come down when you began with division. See the last example.

Numbers like those in examp. 8, are called surds, whose square root cannot be exactly found; but by annexing cyphers as above, you may come extremely near the truth, and the further you proceed, the more exact will the root be; but for common purposes four or five places of decimals are sufficient.

To extract the square root of VULGAR FRACTIONS;

RULE. Reduce the fraction or fractional parts to their lowest terms, and if it be a mixed number, to an improper fraction; then extract the square root of the numerator for a new numerator, and the square root of the denominator for a new denominator. But if the fraction be not a complete power, then reduce it to a decimal, and proceed as taught before.

EXAMPLE 1. What is the square root of $\frac{288}{646}$?
First $\frac{288}{646}$ in its lowest terms is $= \frac{4}{9}$; then $\sqrt{\frac{4}{9}} = \frac{2}{3}$ the root required.

E. 2. What is the square root of $\frac{2704}{4225}$?
First $\frac{2704}{4225} = \frac{16}{25}$ in its lowest terms; then $\sqrt{\frac{16}{25}} = \frac{4}{5}$ the root required.

E. 3. What is the square root of $\frac{9216}{12544}$?
First $\frac{9216}{12544} = \frac{36}{49}$ in its lowest terms; then $\sqrt{\frac{36}{49}} = \frac{6}{7}$ the root required.

LII. The USE of the SQUARE ROOT.

CASE 1.

TO find a mean proportion between any two given numbers,

RULE. Multiply the two given numbers together, and extract the square root of the product, which root will be the mean proportional sought.

EXAMPLE 1. What is the mean proportional between 7 and 9? Firſt $9 \times 7 = 63$;

Then 63(7,93 Anſwer
49
149)1400
1341
1583) 5900
4749
1151

E. 2. What is the mean proportional between 36 and 64?

Firſt $36 \times 64 = 2304$;

Then 2304(48 Anſwer
16
88) 704
704
0

Therefore, as 36 : 48 :: 48 : 64 proof

CASE 2. To find the ſide of a ſquare, equal in area to any given ſuperficies,

RULE. Extract the ſquare root of the given ſuperficies, which root will be the ſide of the ſquare ſought.

E. 3. If the area of a circle be 33124, I demand the ſide of a ſquare, whoſe ſuperficial content ſhall be equal thereto?

33124(182 Anſwer
1
28)231
224
362) 724
724
...

E. 4. A gentleman has a piece of ground in the form of a parallelogram, whoſe longeſt ſide is 134 chains, and ſhorteſt 80 chains, which he intends to change for a ſquare piece of ground of the ſame area, which is to be incloſed out of a large field; you are required to find the length of the ſide? Firſt $134 \times 80 = 10720$; then

10720,00(103,5 Anſwer
1
203) 0720
609
2065)11100
10325
775

CASE 3. To find the diameter of a circle, equal in area to an ellipſis, whoſe tranſverſe and conjugate axes are given,

RULE. Multiply the two axes of the ellipſis together; and the ſquare root of the product is the diameter of a circle equal to the ellipſis.

E. 5.

E. 5. Suppose the transverse axes of an ellipsis be 36, and the conjugate 23,5, what is the diameter of a circle equal thereto?

First $23,5 \times 36 = 8460$; then

$\dot{8}4\dot{6}0,\dot{0}0\dot{0}0(91,97$ Answer
81
181) 360
181
1829) 17900
16461
18387) 143900
128709
15191

CASE 4. Having the area of a circle, to find the diameter.

RULE. As 355 : 452 :: or, as : 1,273239 :: the area to the square of the diameter; or, multiply the square root of the area by 1,12837, and the product will be the answer.

E. 6. Required the diameter of a circle, that will comprehend within its circumference the quantity of an acre of land?

First, an acre of land contains 4840 square yards, then 355 : 452 :: 4840 : 6162,4788 square of the diameter.

$\dot{6}1\dot{6}2,\dot{4}7\dot{8}8(78,5$ yards, the (diameter
49
148) 1262
1184
1565) 7847
7825
2288

Note. ,7854, and 3,1416, are areas of circles, whose diameters are 1 and 2, and ,079577 is the area of a circle, whose circumference is 1; likewise 452, and 1,273239, are squares of the diameters of circles, whose areas are 355; and 1, and 1,12831, is the diameter of a circle, whose area is equal to a square whose side is 1.

E. 7. *In the midst of a meadow well stored with grass,*
I took just three acres to tether my horse;
How long must the cord be, that feeding all round,
He mayn't graze less or more than three acres of ground?

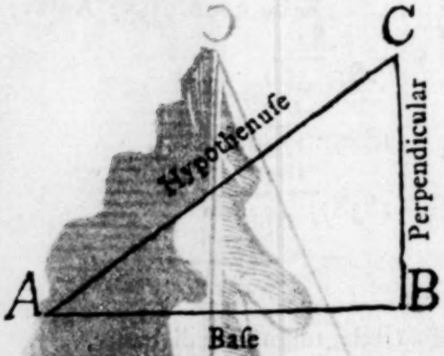
First $4840 \times 3 = 14520$ yards, the content of the three acres; then, as 355 : 452 :: 14520 : 18487,4 yards square of the diameter

$\dot{1}8\dot{4}8\dot{7},\dot{4}0\dot{0}0(135,96$ Diameter
1
23) 84
69
265) 1587
1325
2709) 26240
24381
27186) 185900
163116
22784

Therefore 2) 135,96 the diameter

67,98 yards, length (of the cord required

CASE 5. Any two sides of a right-angled triangle, A, B, C, being given, to find the remaining side.



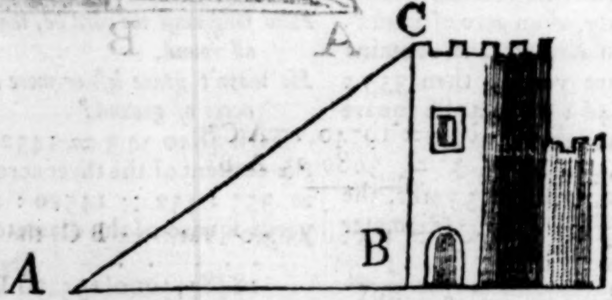
1. The base and perpendicular being given, to find the hypothenuse,

RULE. Square each side, add the squares together, and the square root of this sum gives the hypothenuse required.

2. If the hypothenuse and one side be given, to find the other side,

RULE. From the square of the hypothenuse, subtract the square of the given side, the square root of the remainder gives the side required.

E. 8. The top of a castle from the ground is 45 yards high, and surrounded with a ditch 60 yards broad; what length must a ladder be, to reach from the outside of the ditch to the top of the castle?



In the above figure, A B = the breadth of the ditch = 60 yards; B C = 45 yards, the height of the castle; and A C the length of the ladder required. First $60 \times 60 = 3600$
And $45 \times 45 = 2025$

$$
\begin{array}{r l}
5625(75 & \text{Yards} = \text{A C, the} \\
49 & \text{(length of the ladder} \\
145)\ 725 & \\
725 & \\
\ldots &
\end{array}
$$

E. 9. At Matlock, near the Peak, in Derbyshire, where are many surprising curiosities in nature, is a rock by the side of the river Derwent, rising perpendicular to a wonderful height, which being inaccessible, I endeavoured to measure, and found by a mathematical method, that the distance between the place of observation and the foot of the rock, to be $55\frac{1}{2}$ yards, and from the top of the rock to the said place, to be $140\frac{1}{2}$ yards, (nearly); required the height of this stupendous rock?

In

In the annexed figure, A is the place of observation; A B the distance to the foot of the rock $= 55\frac{1}{2}$ yards; A C the distance from the top of the rock to the said place $= 140\frac{1}{2}$ yards, and B C the perpendicular height of the rock, which is required.

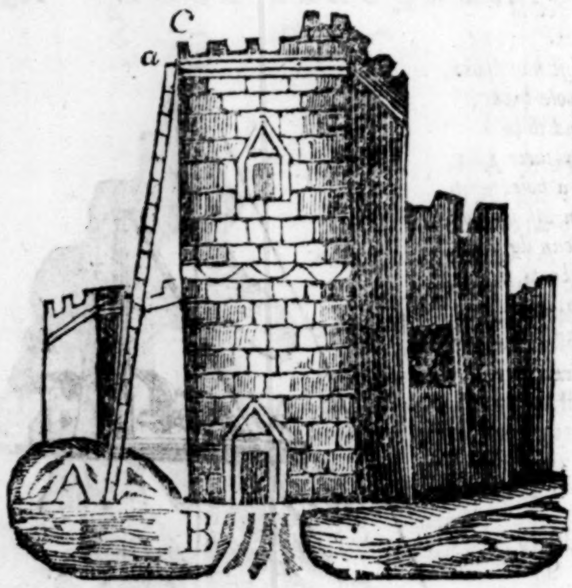
First $140,5 \times 140,5 = 19740,25$ A C^2
And $55,5 \times 55,5 = 3080,25$ A B^2

B C^2 = 16660(129,07 yards = B C, the height (required

```
        16660(129,07
        1
   22)  66
        44
  249)  2260
        2241
24907)  190000
        174349
        15651
```

E. 10. *A castle wall there was, whose height was found,*
To be one hundred feet from th' top to th' ground;
Against the wall a ladder stood upright,
Of the same length the castle was in height.
A waggish youth did the ladder slide,
(The bottom of it) ten feet from the side;
Now I would know how far the top did fall,
By pulling out the ladder from the wall?

In



In the annexed fig. B C = 100 feet, the height of the castle; A B = 10 feet, the distance of the ladder from the wall; A a the ladder = BC, and aC the distance the ladder fell from the top of the castle, which is required.

First 100 × 100 = 10000 = A a^2 the ladder
And 10 × 10 = 100 = A B^2 the ladder's distance from the wall

Difference 9900 = B a^2

9900,0000000000(99,49874 = B a
81

189) 1800
1701

1984) 9900
7936

19889) 196400
179001

198988) 1739900
1591804

1989967) 14809600
13929769

19899744) 87983100
79598976

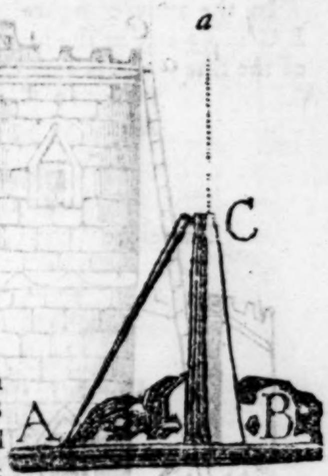
8384124

Then 100
− 99,49874

,50126 feet, = a C = 6,01512 inches, the distance the ladder fell from the top.

E. 11. *As I was walking out one day,*
Which happened on the first of May,
As luck would have it, I did spy
A may-pole raised up on high,
The which at first me much surpriz'd,
Not being before-hand advertis'd
Of such a strange, uncommon sight;
I said, I would not stir that night,

Nor



Nor rest content until I'd found
Its height exact from off the ground;
But when these words I just had spoke,
A blast of wind the may-pole broke,
Whose broken piece I found to be
Exact in length yards sixty-three,
Which by its fall broke up a hole,
Twice fifteen yards from off the pole;
But this being all that I can do,
The may-pole now being broke in two,
Unequal parts, to aid a friend,
Ye youths, pray then an answer send?

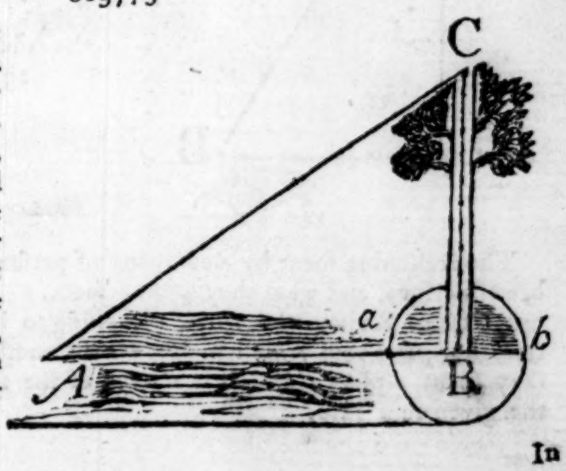
In the annexed figure, A C = the length of the piece broken off, = 63 yards; A B = the distance the top of the piece fell from the bottom = 30 yards, and B C = the length of the pole that was left standing.

First $63 \times 63 = 3969 = AC^2$ or Ca^2.
And $30 \times 30 = 900 = AB^2$

Difference 3069,00000000(55,3985 yards = B C
25
105) 569
525
1103) 4400
3309
11069) 109100
99621
110788) 947900
886304
1107965) 6159600
5539825
619775

Therefore 63
+55,3985
Answer 118,3985 Yards

E. 12. The height of an elm, growing in the middle of a circular island 30 feet in diameter, plumbs 53 feet, and a line stretched from the top of the tree straight to the hither edge of the water, 112 feet; what then is the breadth of the moat, supposing the land on the other side the water to be level?



In

In the annexed figure, $a\,b = 30$ feet $=$ the diameter of the iſland; B C $= 53$ feet $=$ the height of the elm; A C $= 112$ feet, the length of the line; and A $a =$ the breadth of the moat, which is required.

First $112 \times 112 = 12544 = A\,C^2$
And $53 \times 53 = 2809 = B\,C^2$

Difference $9735 = A\,B^2$

Therefore 9735,00 (98,66 $=$ A B
81 — 15,00 Radius of the iſland

188) 1635 83,66 $=$ A a the breadth of the moat (required, anſwer
1504

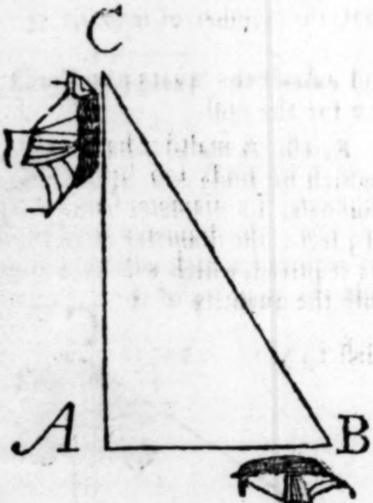
1966) 13100
11796

19726) 130400
118356

12044

E. 13. Two ſhips ſet ſail from the ſame port, one of them goes due eaſt, 50 leagues; the other due north, 84; how far are they aſunder?

In the following figure, A is the port where the two ſhips ſailed from; one north to C $= 84$ leagues, the other eaſt to B $= 50$ leagues; conſequently B C is the diſtance they are from one another, which is required.



First $50 \times 50 = 2500 = A\,B^2$
And $84 \times 84 = 7056 = A\,C^2$

Sum 9556,(97,75 leagues
81 $=$ B C, the diſtance

187) 1456
1309

1947) 14700
13629

19545) 107100
97725

9375

The reckoning ſpent by a company of perſons, to find out the number of perſons, and what they ſpent a-piece,

RULE. Reduce the whole reckoning to its loweſt name, and extract the ſquare root of it, which gives the number of perſons, and what they ſpent a-piece; which is always of the ſame name you reduced the given ſum into.

E. 14.

E. 14. A certain company being at a public-houſe, their reckoning came to 6*s.* $0\frac{1}{4}$*d.* the number of perſons in company were equal to the farthings each ſpent; query, the number in company, and what each ſpent?

First 6*s.* $0\frac{1}{4}$*d.* = 289 *qrs.*

```
Then 289(17 men, anſwer
     1
   -----
  27)189
     189
     ...
```

Again, if 17 *m.* : 289 *qrs.* :: 1 *m.*

```
17)289(17 qrs. = 4¼d. a-piece,
   17          (anſwer
   ---
   119
   119
   ...
```

E. 15. A company of men drinking till the reckoning came to 30*s.* 1*d.* I demand how many there were in company, and what they paid a-piece? Anſwer 19 men, paid 19*d.* a-piece.

E. 16. Suppoſe 75625 ſoldiers were ordered into a ſquare battalia, how many men muſt there be in rank and in file?

```
     75625(275 Men in rank and file, anſwer
     4
  47)356
     329
 545) 2725
      2725
```

To place any number of men, ſo that the number of men in rank may be double to them in the file,

RULE. Take half the number, and extract the ſquare root for the file, which file you muſt multiply by 2 for the rank.

E. 17. Suppoſe 35912 men to be martialled in battle array, and the number of men in rank to be double to them in file; query, the number in rank and file?

```
First 2)35912

     17956(134 men in file  }
     1     × 2              } Anſwer
  23) 79   268 men in rank  }
      69
 264) 1056
      1056
      ....
```

E. 18. A maltſter hath a kiln, which he finds too little for his buſineſs, its diameter being only 15 feet; the diameter of another is required, which will hold double the quantity of the old one?

```
First 15 × 15 = 225 the ſquare of
                  2  (the diam.
                ---
                450 (21,2 the
                4  (diam. req.
             41) 50
                 41
            422) 900
                 844
                  56
```

E. 19. A maltster hath a kiln, which he finds too large for his business, its diameter being 21,2 feet; the diameter of another, which will hold half the quantity, is required?

First 21,2 × 21,2 = 450 nearly; then 2)450

225(15 feet, the answer
1
25)125
125
. . .

By having the bung and head diameters of a cask given, to find the diagonal line,

RULE. Add the square of half the sum of the head and bung diameters, to the square of half the length; the square root of that sum is the diagonal of the cask.

E. 20. Let 25 be the bung, 22 the head diameter, and 30 inches the length of the cask; what is the diagonal line?

Head 22
Bung 25
Sum 47
Half 23,5
23,5
1175
705
470
Square 552,25
Add 225

Length 30
Half 15
15
Square 225

777,25(27,87 the diagonal (line
4
47)377
329
548) 4825
4384
5567) 44100
38969
5131

E. 21. The semi-diameter of the earth being 3984,58 miles, and the perpendicular height of a mountain 3 miles; how far may it be seen at sea, the eye of the spectator being supposed to be on the surface of the water?

3984,58 = Semi-diameter of the earth
+3 The height of the mountain
3987,58 × 3987,58 = 15900794,2564
3987,58 × 3984,58 = 15876877,7764
23916,4800

23916,4800(154,64 miles, (the answer
1
25)139
125
304) 1416
1216
3086) 20048
18516
30924) 153200
123696
29504

LIII. Extraction of the CUBE ROOT.

TO extract the cube root, is to find out a number, which being multiplied into itself, and then again into the product, produceth the given number.

As the cube root of 512 is 8, consequently $8 \times 8 \times 8 = 512$, the given number; and so of others, as in the following

T A B L E.

Roots	1	2	3	4	5	6	7	8	9
Cube	1	8	27	64	125	216	343	512	729

RULE. 1. Make a point over every third figure given, beginning at the units place; seek the greatest cube to the first point on the left-hand (by the table) whose root place in the quotient; then subtract its cube from the period, and to the remainder (if any) bring down the three next figures, or your next period, and call it your dividend.

2. Find a divisor, by calling your quotient figure, with a cypher joined to it, *r*; then three times the square of *r* will be your divisor, seek how often it is contained in the dividend, and put the answer in the quotient, as in division, only with this difference, call the said quotient figure last put up *e*, and multiply your divisor by it, and place the produce underneath the dividend; then multiply the square of *e* by three times *r*, and place it also under the dividend. Lastly, cube the figure you called *e*, and place it under the dividend; then add the three products together, which gives the subtrahend, which subtract from your last dividend, and to the remainder bring down the next period, and proceed as before.

EXAMPLE 1. What is the cube root of 32768?

$$
\begin{array}{rl}
 & 32768(32 \text{ The root} \\
 & \underline{27} \\
3r^2 = 2700) & \underline{5768} \text{ Dividend} \\
 & \left.\begin{array}{r} 5400 = 3rre \\ 360 = 3ree \\ 8 = 3^e \end{array}\right\} \begin{array}{l}\text{here } r = 30 \\ \text{and } e,\ 2\end{array} \\
 & \overline{\underline{5768}} \text{ Subtrahend, equal to the last dividend} \\
 & \cdots
\end{array}
$$

EXPLANATION. The nearest cube to 32, the first period, is 27, which is set under, and subtracted therefrom, and 3, the root of the said cube, is placed in the quotient, and to the remainder 5, the

period

period 768 is annexed, which makes 5768 for a dividend; then a cypher is joined to the quotient figure 3, making 30, which is called *r*, and being squared, and that square multiplied by 3, produces 2700 for a divisor, which being contained twice in the dividend, 2 is placed in the quotient, and called *e*, by which the divisor is multiplied, and the product 5400 set under the dividend. Then 3 times $r = 90$, is multiplied by 4, the square of *e*, and the product 360 is placed under 5400; and lastly, 8 the cube of *e*, is placed under, and added to the other two numbers under the dividend; and the sum 5768 being the same as the dividend, and no more periods to be brought down, the work is finished, and 32768 is found to be a cube number, and 32 its cube root.

E. 2. What is the cube root of 21024576?

$$
\begin{array}{rll}
 & \dot{2}1\dot{0}24\dot{5}76(276 & \text{The root} \\
 & 8 & \\
3rr = 1200) & 13024 & \text{Dividend} \\
 & 8400 = 3rre & \\
 & 2940 = 3ree & \left.\right\} \text{here } r = 20 \text{ and } e = 7 \\
 & 343 = eee & \\
 & 11683 & \text{Subtrahend} \\
3rr = 218700) & 1341576 & \\
 & 1312200 = 3rre & \\
 & 29160 = 3ree & \left.\right\} \text{here } r = 270 \text{ and } e = 6 \\
 & 216 = eee & \\
 & 1341576 & \text{Remainder} \\
 & \dots\dots\dots &
\end{array}
$$

E. 3. What is the cube root of 924?

$$
\begin{array}{rll}
 & \dot{9}24(9,7 & \text{The root} \\
 & 729 & \\
3rr = 24300) & 195000 & \text{Dividend} \\
 & 170100 = 3rre & \\
 & 13230 = 3ree & \left.\right\} \text{here } r = 90 \text{ and } e = 7 \\
 & 343 = eee & \\
 & 183673 & \text{Subtrahend} \\
\text{Remains} & 11327 &
\end{array}
$$

E. 4.

E. 4. What is the cube root of 92398647 ?

92398647(452,08 + The root
64
4800) 28398 Dividend

24000
3000
125
} here $r = 40$ and $e = 5$

27125 Subtrahend

607500) 1273647 Dividend

1215000
5400
8
} here $r = 450$ and $e = 2$

1220408 Subtrahend

6129120000) 53239000000 Dividend

49032960000
8678400
512
} here $r = 45200$ and $e = 8$

49041638912 Subtrahend

4197361088 Remains

Now $452{,}08 \times 452{,}08 \times 452{,}08 + 4197361088 = 92398647$ proof.

Another concise method of extracting the CUBE ROOT.

RULE. 1. Point every third figure of the given number, beginning at the units place; then find the nearest cube to the first point, subtract, and bring the three next figures in the next period to the remainder for a resolvend.

2. Square the quotient, and multiply it by 3, for a divisor; find how often it is contained in the resolvend, rejecting units and tens, and put the answer in the quotient.

3. Square this new figure, and put it on the right-hand of the divisor; but if the new figure should be 1, 2, or 3, then put 01, 04, or 09, to the right-hand.

4. Multiply the last figure in the quotient by 30, and multiply it by the former figures; add this product to the divisor, and multiply the sum by the last figure in the quotient; subtract that product from the resolvend, bring down the next three figures, and proceed as before.

E. 5. What is the cube root of 32768 ?

32768(32 the root
27
2884) 5768
5768
....

EXPLANATION. The square of $3 \times 3 = 27$, the divisor; and the square of 2 is 4, which (per rule) is 04, this put on the right-hand

hand of the divisor 27, makes 2704; then $2 \times 30 \times 3 = 180$, which added to 2704, makes 2884, for a new divisor, which multiplied by 2, the last figure in the quotient, the product is 5768, to be set under the dividend and subtracted therefrom, and nothing remains; therefore 32768 is found to be a cube number, and 32 its cube root; the same as example 1 in this section.

E. 6. What is the cube root of 618470208?

```
     .  .  .
  618470208(852 Root
  512
20425)106470
      102125
2172604) 4345208
         4345208
         .......
```

E. 7. What is the cube root of 27407028375?

```
      .  .  .  .
  27407028375(3015 Root
  27
270901) 407028
        270901
27225475)136127375
         136127375
         .........
```

M. de la Hire has given us a very odd property common to all powers, which *M. Carre* had observed with regard to the number 6, which is this: that all the natural cubic numbers, 8, 27, 64, 125, whose root is less than 6, being divided by 6, the remainder of the division is the root itself; and if we go further, 216, the cube of 6, being divided by 6, leaves no remainder, but the divisor 6 is the root itself. Again, 343, the cube of 7, being divided by 6, leaves 1, which added to the divisor 6, makes 7 the root, &c.

The above gentleman, on considering this property of 6, has found that all numbers, raised to any power whatever, have divisors, which have the same effect with regard thereto, that 6 hath with regard to cubic numbers.

For finding of these divisors, observe the following

RULES. 1. If the exponent of the power of a number be even, i. e. if the number be raised to the second, fourth, sixth power, &c. it must be divided by 2; the remainder of the division, in case there be any, added to 2, or to a multiple of 2, gives the root of this number, corresponding to its power, i. e. the second, sixth, &c. root.

2. If the exponent of the power be an uneven number, i. e. if the number be raised to the third, fifth, seventh power, &c, the double of that exponent will be the divisor, which has the property mentioned.

Thus it is found in 6, double of 3, the exponent of the power of all the cubes; thus also 10 is the divisor of all numbers raised to the fifth power, &c.

To extract the cube root of a VULGAR FRACTION,

RULE. Extract the cube root of the numerator for a new numerator, and the cube root of the denominator for a new denominator; and this new fraction will be the cube root of the given fraction.

The fractions must be reduced to their lowest terms; if it be a mixed number, to an improper fraction; and if a surd to a decimal.

EXAMPLE 1. What is the cube root of $\frac{27}{343}$?

First $\sqrt[3]{27} = 3$; and the $\sqrt[3]{343} = 7$; then $\frac{3}{7}$ is the root required.

E. 2.

E. 2. What is the cube root of $\frac{352}{1188}$?

First $\frac{352}{1188} = \frac{8}{27}$; then $\sqrt[3]{\frac{8}{27}} = \frac{2}{3}$ the root.

E. 3. What is the cube root of $13\frac{4}{7}$?

First $13\frac{4}{7} = \frac{95}{7}$, or $\frac{95000}{7000}$; then $\sqrt[3]{\frac{95000}{7000}} = \frac{45}{19} = 2\frac{7}{19}$ the root.

The extraction of roots of higher powers, are of little or no use in practical arithmetic; I shall therefore leave this rule with the following observations.

1. The biquadrate of any number is found by extracting the square root of the given number first, and then the square root of that root.

2. The root of the square cubed, or sixth power of any number, is found by extracting the square root of the given number, then extract the cube root of that square root, which will give the sixth power required.

3. The root of the biquadrate squared, or eighth power, is found by extracting the square root of the given number, which will reduce it to a biquadrate, which proceed with as before.

4. The root of the cube cubed, or ninth power of any number, is found by extracting the cube root of the given number, and the result will be a cubic resolvend; or extract the cube root also, which will be the root of the ninth power.

LIV. The USE of the CUBE ROOT.

CASE 1.

TO find the side of a cube that will be equal in solidity to any given solid, as a globe, cylinder, cone, &c.

RULE. Extract the cube root of the solid content, of the given body, which root will be the side of the cube required?

EXAMPLE 1. There is a stone of a cubic form, which contains 432 solid feet; what is the superficial content of one of its sides?

```
        432(7,55 + Side of the cube
        343
 15775) 89000
        78875
1698775)10125000       Then 7,55 × 7,55 =
        8493875        57,0925 the content req.

   Remains 1631125
```

E. 2. The content of a globe is 1728 solid inches, what is the side of a cube equal thereto?

```
     1728(12 Inches, the side of the cube
     1
364) 728
     728
     ...
```

CASE

CASE 2. Having the dimensions of any solid body, to find the dimensions of another similar solid, that shall be any number of times greater or less than the solid given.

RULE. Multiply the cube of each side by the difference between the solid given and that required, if greater, or divide by the difference if less than the solid given; then extract the cube root of each product or quotient, which will give the dimensions of the solid required.

E. 3. Suppose the length of a ship's keel to be 125 feet, the breadth of the midship beam 25 feet, and the depth of the hold 15 feet; I demand the dimensions of another ship of the same form, that shall carry three times the burthen?

First - 125
125
625
250
125
15625
125
78125
31250
15625
1953125
3
. . .
5859375(180,28 keel
1
604)4859
4832
9730804) 27375000
19461608
974593744) 7913392000
7796749952
Remains 116642048

Secondly 25
25
125
50
625
25
3125
1250
15625
3
. .
46875(36,05 midship (beam
27
3276)19875
19656
38754025) 219000000
193770125
Remains 25229875

Thirdly 15
15
75
15
225
15
1125
225
3375
3
. .
10125(21,6 Depth in (the hold

E. 4. Suppose I lend my neighbour a stack of hay 12 feet in length, breadth, and depth, and he returns me 2 stacks, each of whose sides is 6 feet, how much will remain due?

$12 \times 12 \times 12 = 1728$ Solid feet borrowed
$6 \times 6 \times 6 \times 2 = 432$ Solid feet repaid

Answer 1296 Solid feet unpaid

For $1728 \div 432 = 4$; consequently $\frac{3}{4}$ is still unpaid.

E. 5.

E. 5. What dimensions must I give to a joiner, to make a cubical box, that will hold 2000 oranges, of $2\frac{1}{2}$ inches diameter each, supposing the oranges globular, keeping that form, and laid in rows exactly at the top of each other?

First $2,5 \times 2,5 \times 2,5 \times 2000 = 31250$ the solidity of the box

31250(31,498 Inches, the side of the box

```
             31250(31,498
             27
    2791)    4250
             2791
  292036)    1459000
             1168144
29663661)    290856000
             266972949
2975616124)  23883051000
             23804928992
```

CASE 3. To find two mean proportionals between two given numbers.

RULE. Divide the greater extreme by the less, and the cube root of the quotient multiplied by the less extreme, gives the lesser mean; multiply the said cube root by the lesser mean, and the product will be the greater mean proportional.

E. 6. What are the two mean proportionals between 6 and 384?

First 6)384(64 whose cube root is 4

$\times$ 6 The lesser extreme

24 Lesser mean
4

96 Greater mean

For, as 6 : 24 :: 96 : 384

```
   96
   24
  ----
  384
 192
  ----
6) 2304
  ----
   384 Proof
```

CASE 4. Having the dimensions and capacity of a solid, to find the dimensions of a similar solid, of a different capacity.

RULE. Like solids are in triplicate proportion to their homologous sides; therefore it will be, as the cube of a dimension : is to its given weight :: so is the cube of any like dimension : to the weight sought.

E. 7. Suppose a cannon ball of 4 inches diameter weighs 18 *lb.* I demand the diameter of another that weighs 141 *lb*?

First $4 \times 4 \times 4 = 64$, cube of the diameter.

	lb.		*in.*		*lb.*		*in.*	
Then as	18	:	64	::	141	:	501,3	Cube of the diameter

 501,333

$\dot{5}01,\dot{3}33(7,9$ + Inches, the diameter required.
343
16671)158333
150039
8294

E. 8. There is a ball or globe of marble, whose diameter is 6 inches, and its weight 11 pounds, what will be the diameter of another globe of the same marble, that weighs 500 pounds?

First $6 \times 6 \times 6 = 216$, cube of the diameter

Then as 11 *lb.* : 216 *in.* :: 500 *lb.* : 9818,181 *in.* cube of the diameter

$\dot{9}81\dot{8},18\dot{1}(21,4$ inches, the diameter sought
8
1261)1818
1261
134836) 557181
539344
17837

LV. The SINGLE RULE of THREE, IN DECIMALS.

RULE.

REDUCE the fractional parts into decimals of the highest name mentioned; then state the question, and proceed as taught in sect. XII. and XIII.

EXAMPLE 1. If $2\frac{1}{2}$ pounds of tea cost 1*l.* 5*s.* what will $14\frac{3}{4}$ come to at the same rate?

First $2\frac{1}{2} = 2,5$; and 1*l.* 5*s.* $= 1,25$; also $14\frac{3}{4} = 14,75$.

Then, as 2,5 : 1,25 :: 14,75
1,25
7375
2950
1475
2,5 { 5) 18,4375
5) 36875
Answer 7,375 = 7*l.* 7*s.* 6*d.*

E. 2. Supposing the earth to be 81000000 miles distant from the sun; I would know at what distance from him another body must be placed, so as to receive light and heat, double to that of the earth? First $81000000 \times 81000000 = 6561000000000000$; then reciprocally,

As 1 : 656100000000000 :: 2 : 328050000000000

Therefore 328050000000000(57275649 miles, anſwer
```
        25
  107)780
      749
  1142) 3150
        2284
  11447) 86600
         80129
  114545) 647100
          572725
  1145506) 7437500
           6873036
  11455124) 56446400
            45820496
  114551289) 1062590400
             1030961601
               31628799
```

Note. The effects, or degrees of light, heat and attraction, are reciprocally proportional to the ſquares of their diſtances from the centre, whence they are propagated.

E. 3. If the diameter of the earth is 7970 miles, of the moon 2170 miles, ſuppoſing them both to be exact ſpheres (as they are not); what compariſon is there between them in point of magnitude?

First $7970 \times 7970 \times 7970 = 506261573000$
And $2170 \times 2170 \times 2170 = 10218313000$
As 10218313000 : 506261573000 :: 1

10218313|000)506261573|000(49,5445 Times larger than the moon, anſwer.

Note. The quantity of matter contained in all ſpheres, is directly in proportion to the cubes of their diameters.

E. 4. Suppoſe a ſtone let go into an abyſs, ſhould be ſtopped at the end of the eleventh ſecond after its delivery, what ſpace would it have gone through?

Firſt $11 \times 11 = 121$, and the ſquare of 1 is 1;

Then, as 1 : 16,083 :: 121
```
          121
        16083
       32166
      16083
Anſwer 1946,043 Feet
```

Note. The velocity acquired by heavy bodies falling near the earth's ſurface, is 16,083 feet in the firſt ſecond; and as 16,083 is to the ſquare of one ſecond, or 1, ſo is the given diſtance to the ſquare of the ſeconds required. Or by multiplying 16,083 feet, the deſcent of an heavy body near the earth's ſurface, in one ſecond of time, by as many of the odd numbers, beginning from unity, as there are ſeconds in any given time, viz. by 1 for the firſt, 3 for the ſecond, 5 for the

 third,

third, 7 for the fourth, &c. the sum total will give the space it has passed, any where on this side the centre of the earth ?

E. 5. What is the difference between the depth of two wells, into each of which should a stone be dropped at the same instant, one will meet with the bottom at 6 seconds, the other at 10?

First $10 \times 10 = 100$; and $6 \times 6 = 36$, square of their descents

Then as 1 : 16,083 :: $\left\{ \begin{array}{l} 100 : 1608,3 \\ 36 : 579 \end{array} \right\}$ their depths

Answer 1029,3 difference

E. 6. In what time would a musquet ball, dropped from the top of St. Martin's steeple, in Birmingham, said to be 300 feet high, be at the bottom ?

First, as 16,083 : 1 :: 300

16,083)300,0000000(18,6532

18,6532(4,318 + seconds, answer
16

83) 265
249

861) 1632
861

8628) 77100
69024
8076

E. 7. A ball descending by the force of gravity from the top of a tower, was observed to fall half the way in the last second of time; required the tower's height, and the whole time of descent ?

First, the square root of $1 = 1$, and the square root of $2 = 1,4142$, from which take 1, and there remains ,4142 ; then, as ,4142 : 1,4142 :: 1 : 3,414 sec. the descent

Now $3,414 \times 3,414 = 11,6554$

And, as 1 : 16,083 :: 11,6554

16,083

349662
932432
6993240
116554

Answer, feet 187,4537982

LVI. The DOUBLE RULE of THREE IN DECIMALS.

EXAMPLE 1.

If 100 men can dig a trench 5 feet long in 24 hours, what length of such a trench can 9800 men dig, in 10 hours ?

	men.		feet.		men.
Stated thus ; as	1000	:	500	::	9800
	24 h.	:	—	::	10
	24000				98000
					500

24|000)4900|000(2041,6 =

2041 feet 7 inches, the answer,

E. 2.

E. 2. When the bushel of wheat was sold at 10*s.* the four-penny loaf weighed $4\frac{1}{2}$ *lb.* what should the six-penny loaf weigh, when the bushel of wheat sells at 15*s*?

	s.		*lb.*		*s.*
First reciprocally, as	10	:	4,5	::	15
	4	:	—	::	6

15
4
———
60

4,5
10
———
45,0
6
———
6|0)27|0
———
4,3 = $4\frac{1}{2}$ *lb.* Answer

E. 3. A young hare starts 5 rods, before a greyhound, and is not perceived by him till she has been up 34 seconds; she scuds away at the rate of 12 miles an hour, and the dog, on view, makes after her at the rate of 20; how long will the course hold, and what ground will he run, beginning with the outsetting of the dog?

First 34″ = ,009444 hours, and 5 rods = ,015625 miles; then

As 1 *h.* : 12 *m.* :: ,009444 *h.* : ,113328 *m.*

,113328
,015625
———
,128953 Miles = 680,9 feet, the hare had started

Now 20 — 12 = 8 Dog gained in running 20

Therefore, as 8 *m.* : 20 *m.* :: ,128953 *m.* : 2,57906 furlongs = $1702\frac{1}{4}$ feet, run by the greyhound.

Again, as 8 *m.* : 1 *h.* :: ,128953 *m.* : ,016119 *h.* = 58″,0284 time run by the greyhound.

Note. It hath been found by experiment, that a pendulum 39,2 inches long, in our latitude, vibrates 60 times in one minute; and that the length of pendulums are to one another reciprocally as the square of the number of their vibrations made in the same space of time.

E. 4. What difference is there between the length of a pendulum that vibrates half a second, or 120 times in a minute, and another that swings double seconds, or 30 times in a minute? First 60 × 60 = 3600; then reciprocally,

As 3600 : 39,2 :: 900 = 30^2
3600
———
2352
1176
———
9|00)1411,200
———
156,8 Length of the pendulum that vibrates double seconds, or 30 times in a minute.

Again,
As 3600 : 39,2 :: 14400 = 120^2
3600
———
2352
1176
———
144|00)1411,200 (9,8 Length of the pend. that vib. half sec.
1296
———
1152
1152
———

156,8
9,8
———
Ans. 147 in. = 12 *f.* 3 *i.*

E. 5.

E. 5. Obferved, that while a ftone was defcending to meafure the depth of a well, a ftring and plummet, that from the point of fufpenfion, or the place where it was held, to the centre of ofcillation, or that part of the bob, which being divided by the circular line, ftruck from the centre aforefaid, would divide it into two equal parts of equal weight, meafured juft 18 inches, had made 8 vibrations ; pray what was the depth, allowing the fame as in page 78 for the return of found to the ear ?

First 60
60
———
in. in.
39,2 : 3600 :: 18
39,2
———
72
324
108
———
18 { 3) 141120,0
———
6) 47040
———
7840 Seconds

7840,00000000(88,5437 vibr. (in 1 minute
64
———
168)1440
1344
———
1765) 9600
8825
———
17704) 77500
70816
———
177083) 668400
531249
———
1770867) 13715100
12396069
———
319031

Then 6|0) 88,5437
———
1,475728 Vibrations in a fecond

Alfo 1,475728)8,000000(5,421 feconds in 8 vibrations

Now 5,421
5,421
———
5421
10842
21684
27105
———
Then, as 1 : 16,083 :: 29,387241
16,083
———
88161723
235097928
1763234460
29387241
———
472,634997003 Feet found was returned through

Again, as 1150 : 1 :: 472,634997003
1
———
115|0)472,634997003(,410986953 Seconds, time found was returning.

∴ 5,421 — ,411 = 5,01 feconds, time of the bodies defcent ; and 5,01 × 5,01 = 25,1 ; alfo 25,1 × 16,083 = 403,674 feet, the depth of the well, Anfwer.

E. 6.

E. 6. *In Derbyshire, a wonder of the Peak,*
Is Eldon-hole, as poets often speak,
Whose depth exactly, none could e'er descry,
Tho' atheist Hobbs his utmost skill did try,
Who wrote de Mirabilibus Pecci.
And burlesque Cotton does strange things rehearse
In rustic words, and Hudibrastic verse,
How he this monstrous orifice did plumb,
But could not at the bottom of it come
With 16 *hundred yards of rope let loose;*
And tells a story of a woman's goose:
Fabulous the one, so may the other be,
Erroneous too, without philosophy;
But I the depth have found exactly true
By gravity; a method something new:
As heavy bodies do accelerate,
In spaces known first to our Newton *great;*
Four pond'rous stones into the well let fall
In measur'd time, agreed in numbers all:
A pendulum sixty-one inches long,
By which the time I measured was not wrong,
Vibrated freely whilst that each stone fell,
Eight times; by which the depth I'd have you tell,
Allowing rightly for th' approach of sound
That your own works may not themselves confound?

First $60 \times 60 = 3600$; then reciprocally,

In.		*sec.*		*in.*		*sec.*
As 39,2	:	3600	::	61	:	2313,4426

$\therefore \sqrt[2]{2313,4426} = 48,09$

Then, as $48,09 : 1 :: 8 : ,1663$; and $,1663 \times 60 = 9,99$, or $10''$ nearly, the time the pendulum made 8 vibrations.

Now $10 \times 10 = 100$; then, as $^21 : 16,083 :: 100 : 1608,3$ feet, sound was returning. Again $1150 : 1 :: 1608,3 : 1,398$ seconds, the time sound was ascending; $\therefore 10 - 1,398 = 8,602$ seconds, time of the bodies descent; and $8,602 \times 8,602 = 74$, nearly.

Also $74 \times 16,083 = 1200,142$ feet $= 400,473$ yards, the depth of Eldon-hole.

LVII. FELLOWSHIP.

RULE.

DIVIDE the whole gain or loss by the whole stock, and multiply the quotient by each person's particular stock, and the several products, will be the respective gain or loss of each.

EXAMPLE 1. Three merchants, A, B, and C, freight a ship with 96 ton of wine, thus; A put on board 24, B 32, and C 40 ton; but the extremity of the weather obliged them to cast 12 ton overboard; how much must each merchant bear of this loss?

First $24 + 32 + 40 = 96$, the whole stock;
And $12t. \div 96 = ,125$, the quotient.

$$\text{Then } \left.\begin{array}{l} 24 \text{ A's} \\ 32 \text{ B's} \\ 40 \text{ C's} \end{array}\right\} \text{Stock} \times ,125 = \left\{\begin{array}{l} 3 \text{ A's loss} \\ 4 \text{ B's} \\ 5 \text{ C's} \end{array}\right.$$

Proof 12 tons

E. 2.

E. 2. Four men trade together, A puts in 200*l.* B 150*l.* C 85*l.* and D 70*l.* they gain 60*l.* what is the ſhare of each ?

First 200 + 150 + 85 + 70 = 505 the whole ſtock ;
And 60*l.* ÷ 505 = ,11881 the quotient; then

			£.	s.	d.
200 A's	Stock × ,11881 =	23,762 =	23	15	2,9 A's gain
150 B's		17,8215 =	17	16	5,2 B's
85 C's		10,09885 =	10	1	11,8 C's
70 D's		8,3167 =	8	6	4,1 D's
		Proof - £.	60	0	0

E. 3. *In honour of* Criſpin, *the cordwainers, they*
Prepared a feaſt, to be jovial and gay;
Six tanners, eight curriers, at firſt took their place ;
Sixteen cordwainers next, all with regular grace ;
Then the coblers next, who were twenty and one,
At table ſat down with their hoſt merry John :
When dinner was over full bumpers did paſs,
Some drank a full noggin, and ſome a wide glaſs ;
Carouſing and ſinging they paſs the long day,
No ſons of great Bacchus more jovial than they.
At laſt for the reck'ning the tanners did call,
Whilſt ſome of the coblers did nothing but bawl
For old hock, or ſtingo ;—the landlord came in
With his ſcores round a trencher—to work did begin,
And found that Ten Pounds was the ſhot to defray,
Then tell me, Tyro, what each had to pay,
When the tanners and th' other, agreed very true,
In proportion to pay, as five, four, three, and two?

It is plain by the queſtion, that as often as each tanner paid 5*s.* the others paid 4*s.* 3*s.* and 2*s.* a-piece ; which ſum multiply by the number of each trade or occupation, thus :

	s.	£.
5 × 6 =	30 =	1,5
4 × 8 =	32 =	1,6
3 × 16 =	48 =	2,4
2 × 21 =	42 =	2,1
	152 =	7,6

And 10*l.* ÷ 7,6 = 1,31578 the quotient.

Then 1,5	× 1,31578 =	1,97367 the 6 tanners' ſhare
1,6		2,105248 the 8 curriers
2,4		3,157872 the 16 cordwainers
2,1		2,763138 the 21 coblers

	s.	d.	
Alſo 6)1,97367(,328945 =	6	$6\frac{3}{4}$	+ each tanner's ſhare
8)2,105248(,263156 =	5	3	+ each currier's
16)3,157872(,197367 =	3	$11\frac{1}{4}$	+ each cordwainer's
21)2,763138(,131578 =	2	$7\frac{1}{2}$	+ each cobler's

LVIII.

LVIII.. DOUBLE FELLOWSHIP.

WHEN the shares of partners are continued in company unequal times, they occasion the name fellowship with time, or double fellowship; which is performed by the following

RULE. Divide the whole gain or loss, by the first term or sum of the products; the quotient is a common multiplier, by which multiplying the several products, you will have the several shares required.

EXAMPLE 1. Three merchants, A, B and C, enter into partnership thus; A puts into the stock 65*l.* for 8 months; B puts in 78*l.* for 12 months; and C puts in 84*l.* for 6 months; with these they traffic, and gain 166*l.* 12*s.* it is required to find each man's share of the gain, proportionable to his stock and time of employing it?

£.		mo.		pro.		
65	×	8	=	520	A's	Stock multiplied into his time
78	×	12	=	936	B's	
84	×	6	=	504	C's	
			Sum	1960		

1960)166,6(,085 the common multiplier

Then	520 × ,085	=	44,2	=	44	4	0	A's	Gain
And	936 × ,085	=	79,56	=	79	11	$2\frac{1}{2}$	B's	
Also	504 × ,085	=	42,84	=	42	16	$9\frac{1}{2}$	C's	
				Proof £.	166	12	0		

E. 2. Four merchants trade after this manner; A puts in 100*l.* for 8 months; B puts in 80*l.* for 5 months, and then puts in 40*l.* more for 3 months longer; C puts in 176*l.* for 4 months, and then takes out 50*l.* for four months more; D puts in 230*l.* for 6 months, and then takes out the whole: they gained 212*l.* 10*s.* what is the gain of each merchant?

£.		mo.			products.	
100	×	8	=		800	A's Stock and time
80	×	5	=	400	760	B's
120	×	3	=	+ 360		
176	×	4	=	704	1208	C's
126	×	4	=	+ 504		
230	×	6	=		1380	D's
				Sum - -	4148	

4148)212,50(,05123 the common multiplier

				£.	
Then	800 × ,05123	= 40,984	=	40 19 8	A's Share
And	760 × ,05123	= 38,9348	=	38 18 $8\frac{3}{4}$	B's
Alfo	1208 × ,05123	= 61,4760	=	61 17 $8\frac{3}{4}$	C's
Likewife	1380 × ,05123	= 70,6974	=	70 13 $11\frac{1}{4}$	D's
		Proof -		£. 212 10 0	

E. 3. Three merchants, A, B and C, trade together; A puts in 120*l.* for 8 months; B 250*l.* for 4 months; and C 100*l.* for 5 months; they gained 184*l.* 10*s.* what is each man's fhare of the gain?

£.
First 120 × 8 = 960 A's Stock and time
250 × 4 = 1000 B's
100 × 5 = 500 C's

Sum 2460)184,5(,075 the quotient

$$\text{Then}\ \left.\begin{array}{r}960\\1000\\500\end{array}\right\}\times ,075=\left\{\begin{array}{ll}72 & \text{A's Gain}\\75 & \text{B's}\\37,5 & \text{C's}\end{array}\right.$$

Proof £. 184,5 = 184*l.* 10*s.*

E. 5. Four merchants, A, B, C and D, enter into partnerfhip thus; A put in 64*l.* 10*s.* for $4\frac{1}{2}$ months; B put in 78*l.* 15*s.* for 6 months; C put in 112*l.* 14*s.* for $8\frac{3}{4}$ months; and D 125*l.* 5*s.* for $5\frac{1}{4}$ months; they gain 108*l.* 18*s.* $4\frac{1}{2}$*d.* what is due to each in proportion to their ftocks and time they were employed?

	£.		mo.			
First	64,5	×	4,5	=	290,25	A's Stock and time
	78,75	×	6	=	472,5	B's
	112,7	×	8,75	=	986,125	C's
	125,25	×	5,25	=	657,5625	D's
			Sum	-	2406,4375	

And 2406,4375)108,91875(,045261 the quotient

$$\text{Then}\ \left.\begin{array}{r}290,25\\472,5\\986,125\\657,5625\end{array}\right\}\times ,045261=\left\{\begin{array}{ll}13,1370 & \text{A's Gain}\\21,3859 & \text{B's}\\44,633 & \text{C's}\\29,762 & \text{D's}\end{array}\right.$$

Proof - 108,9179 = 108*l.* 18*s.* $4\frac{1}{2}$*d.*

Note. Queftions of this kind feldom occur in bufinefs, and therefore to enlarge on this head would be entirely ufelefs.

LIX. SIMPLE INTEREST.

INTEREST is the premium allowed for the loan of money, &c. See ſection XVIII. page 110.

There are ſeveral ways of computing ſimple intereſt; as by the ſingle and double rules of three, &c. But all computations relating to ſimple intereſt, are grounded upon arithmetical progreſſion; I ſhall from thence make uſe of ſuch general theorems, as will ſuit with all caſes. In order to that, here are five letters to be obſerved, viz.

Let
- P = any principal or ſum put to intereſt.
- I = the intereſt.
- T = the time of the principal's continuance at intereſt.
- A = the amount, or principal and its intereſt.
- R = the ratio, or rate per cent. per annum.

The ratio is the ſimple intereſt of 1*l.* for one year, at any given rate; and is thus found:

	l.		*l.*		*l.*		
Viz.	100	:	5	:: 1 :	,05	the ratio at 5 per cent. per annum.
Or,	100	:	4	:: 1 :	,04	the ratio at 4 per cent. per annum.

And in this manner the ratios in the following table are found.

TABLE.

Rate.		*Ratio.*
3		,03
$3\frac{1}{2}$		,035
$3\frac{3}{4}$		,0375
4	=	,04
$4\frac{1}{4}$		,0425
$4\frac{1}{2}$		,045
5		,05

When the principal, time, and rate per cent. are given, to find the intereſt,

RULE. Multiply the principal, rate, and time continually into one another; the product is the intereſt ſought.

Or, if p = the principal, t = the time, r = the rate, and I = the intereſt.

THEOREM 1. $ptr = I$.

EXAMPLE 1. What is the intereſt of 241*l.* for 3 years, at 4 per cent. per annum?

By theorem 241 = p.
3 = t.
———
723 = pt.
,04 = r.
———
£. 28,92 = ptr.
20
———
s. 18,40
12
———
d. 4,80
4
———
qrs. 3,2 Anſ. 28*l.* 18*s.* $4\frac{3}{4}$*d.*+

E. 2. What is the intereſt of 842*l.* 10*s.* for four years, at 5 per cent per annum?

By the rule 842,5 Principal
,05 Ratio
———
42,125
4 No. of years
———
£. 168,500
20
———
s. 10,000

Anſwer 168*l.* 10*s.*

E. 3. What is the intereſt of 20,000*l.* for ſeven years, at $4\frac{1}{2}$ per cent. per annum?

$$\begin{array}{rl} 20000 & = p. \\ ,045 & = r. \\ \hline 100000 & \\ 80000 & \\ \hline 900,000 & = pr. \\ 7 & = t. \\ \hline £.\ 6300 & = ptr. \end{array}$$

Anſwer

E. 4. What is the intereſt of 482*l.* 17*s.* 6*d.* for $6\frac{1}{2}$ years, at $4\frac{1}{2}$ per cent. per annum?

$$\begin{array}{rl} 482,875 & = p. \\ ,045 & = r. \\ \hline 2414375 & \\ 1931500 & \\ \hline 21,729375 & = pr. \\ 6,5 & = t. \\ \hline 108646875 & \\ 130376250 & \\ \hline \end{array}$$

Anſwer 141,2409375 = 141*l.* 4*s.* $9\frac{3}{4}$*d.*

LX. When the Intereſt required is for days only.

RULE.

MULTIPLY the intereſt of 1*l.* for one day, at the given rate, by the principal and number of days for the anſwer.

The intereſt of 1*l.* for one day is thus found; viz.

d.		l.				d.

As 365 : ,05 :: 1 : ,0001369863, &c. the intereſt of 1*l.* for one day, at 5 per cent. And in this manner the following table is made.

TABLE.

per cent.		Decimals.
5	=	,0001369863
$4\frac{1}{2}$	=	,0001232876
4	=	,0001095890
$3\frac{1}{2}$	=	,0000958904
3	=	,0000821917

EXAMPLE 1. What is the intereſt of 547*l.* 15*s.* for 320 days, at 5 per cent. per annum?

$$\begin{array}{rl} ,0001369863 & \text{Ratio} \\ 547,75 & \text{Principal} \\ \hline 6849315 & \\ 9589041 & \\ 9589041 & \\ 5479452 & \\ 6849315 & \\ \hline ,075034245825 & \\ 320 & \text{Number of days} \\ \hline 15006849165 00 & \\ 225102737475 & \\ \hline \end{array}$$

Anſwer 24,010958664000 = 24*l.* 0*s.* $2\frac{1}{4}$*d.*

Again

Again thus: 547,75 Principal
320 Number of days

1095500
164325

* 73|00)1752|80,00(24,0109 = 24*l.* 0*s.* 2½*d.* the answer as before.

E. 2. What is the interest of 150*l.* from the 18th day of January to the 11th of November, at 5 per cent. per annum?

$$
\begin{array}{r l}
,0001369863 & = r. \\
150 & = p. \\
\hline
68493150 & \\
1369863 & \\
\hline
,0205479450 & \\
297 & = t. \\
\hline
1438356150 & \\
1849315050 & \\
410958900 & \\
\hline
\text{Answer } 6,1027396650 & = 6l.\ 2s.\ 0\tfrac{1}{2}d.
\end{array}
$$

Again thus: 150 Principal
297 Number of days

1050
1350
300

73|00)445|50(6,102 = 6*l.* 2*s.* 0½*d.* as before.

E. 3. What is the interest of 40*l.* for 50 days, at 3 per cent?

$$
\begin{array}{r l}
,00008219178 & = r. \\
40 & = p. \\
\hline
,003287671120 & = pr. \\
50 & = t. \\
\hline
\text{Answer } ,1643835,6000 & = ptr = 3s.\ 3\tfrac{1}{4}d.
\end{array}
$$

When the principal, time, and rate per cent. are given, to find the amount,

RULE. Find the interest by theorem 1, which added to the principal, will give the amount.

THEOREM 2. $ptr + p = A$.

EXAMPLE 1. What will 312*l.* 10*s.* amount to in 3 years, at 4 per cent. per annum?

* The reason of this contraction may be seen in section XVIII. page 120.

312,5

$$\begin{array}{rl} 312,5 & = p. \\ ,04 & = r. \\ \hline 12,500 & = pr. \\ 3 & = t. \\ \hline 37,500 & = ptr. \\ 312,5 & = p. \\ \hline \text{Answer - £} \cdot 350,000 & = ptr + p. \end{array}$$

E. 2. What will 672*l.* amount to in $8\frac{1}{2}$ years, at $4\frac{1}{2}$ per cent. per annum?

$$\begin{array}{rl} 672 & \text{Principal} \\ ,045 & \\ \hline 3360 & \\ 2688 & \\ \hline 30,240 & \\ 8,5 & \text{Time} \\ \hline 151200 & \\ 241920 & \\ \hline 257,0400 & = \text{Interest} \\ 672 & \text{Principal} \\ \hline \end{array}$$

Anf. $929,0400 = 929l.\ 0s.\ 9\frac{1}{2}d.$

E. 3. What will 500*l.* amount to in 6 years 120 days, at $4\frac{3}{4}$ per cent. per annum?

$$\begin{array}{rl} 6,328767 & = t. \\ ,0475 & = r. \\ \hline 31643835 & \\ 44301369 & \\ 25315068 & \\ \hline ,3006164325 & = tr. \\ 500 & = p. \\ \hline 150,30821625 & = ptr. \\ 500 & = p. \\ \hline \end{array}$$

Anf. $650,30821625 = ptr + p = (650l.\ 6s.\ 1\frac{3}{4}d.$

When the rate, time, and interest, are given, to find the principal,

RULE. Divide the interest by the product of the rate and time, the quotient is the principal.

THEOREM 3. $\frac{I}{tr} = p.$

EXAMPLE 1. What principal being put to interest for 2 years, will gain 60*l.* at 5 per cent. per annum?

First $2 \times ,05 = ,10$ the product of the ratio and time

Then 10)60,00(600*l.* the answer

E. 2. What principal being put to interest for 3 years, will gain 69*l.* 13*s.* 6*d.* at 5 per cent. per annum?

First $3 \times ,05 = ,15$ product of ratio and time

$$,15 \begin{cases} 5)69,675 \\ 3)\ 13935 \end{cases}$$

Answer $464,5 = 464l.\ 10s.$

E. 3.

E. 3. What principal being put to interest for $4\frac{1}{2}$ years, will gain 58*l.* 14*s.* 6*d.* at 4 per cent. per annum?

First 4,5 × ,04 = ,18 product of ratio and time

```
    ⎧ 2)58,725
 18 ⎨ ————————
    ⎩ 9)293,625
       ————————
        326,25 = 326l. 5s. the answer
```

When the amount, rate, and time are given, to find the principal,

RULE. Multiply the rate by the time; add unity or 1, to the product for a divisor, by which sum divide the amount, the quotient will be the principal.

THEOREM 4. $\frac{a}{tr+1}=p$.

EXAMPLE 1. What principal will amount to 4700*l.* in 5 years, at $3\frac{1}{2}$ per cent. per annum?

5 × ,035 + 1 = 1,175; then per theorem, 4700*l.* ÷ 1,175 = 4000*l.* the answer.

```
The work at length:  ,035
                        5
                     ————
                      175
                  + 1,000
                  ———————
                  1,175)4700,000(4000 Answer as before
                        4700
                        ————
                           0
```

E. 2. What principal being put to interest will amount to 354*l.* 4*s.* $0\frac{1}{2}$*d.* in 7 years, at $2\frac{1}{2}$ per cent. per annum?

```
   ,035
      7
   ————
   ,245
 +1,000
 ——————
 1,245)354,202083(284,499 =
       2490       284l. 9s. 11¾d.
       ————       Answer
       10520
        9960
        ————
         5602
         4980
         ————
          6220
          4980
         —————
          12408
          11205
         ——————
           12033
           11205
           —————
             828
```

E. 3. What principal being put to interest will amount to 40*l.* in 3 years, at 5 per cent. per annum?

First ,05 × 3 + 1 = 1,15; then

```
1,15)40,00000(34,7826 = 34l.
     345           (15s. 7¾d. Ans.
     ———
      550
      460
      ———
       900
       805
       ———
        950
        920
        ———
         300
         230
         ———
          700
          690
          ———
           10
```

When

When the principal, intereſt and rate are given, to find the time,

RULE. Divide the intereſt by the product of the principal and rate, the quotient is the time.

THEOREM 5. $\frac{I}{p\,r} = t.$

EXAMPLE 1. In what time will 200*l.* gain 60*l.* at 5 per cent, per annum?

$$200 = p.$$
$$,05 = r.$$

$$p\,r = \quad 10,00)60 = I.$$

Anſwer 6 Years $= t.$

E. 2. In what time will 260*l.* gain 64*l.* 7*s.* at $4\frac{1}{2}$ per cent. per annum? Firſt 260*l.* $\times$,045 $=$ 11,7 the product of the principal and rate; and 64*l.* 7*s.* $=$ 64,35 the intereſt.

Then 11,7)64,35(5,5 $= 5\frac{1}{2}$ years, the anſwer.

```
       585
       ---
       585
       585
       ---
         0
```

E. 3. In what time will 500*l.* gain 130*l.* 9*s.* at $3\frac{1}{2}$ per cent. per annum? Firſt 500*l.* $\times$,035 $=$ 17,5 product of the principal and rate; and 130*l.* 9*s.* $=$ 130,45 the intereſt;

Then 17,5)130,45)7,4 $=$ 7 years 146 days, anſwer

When the amount, principal and rate are given, to find the time,

RULE. Divide the amount leſs the principal, by the product of the principal and rate, the quotient is the time.

THEOREM 6. $\frac{a - p}{p\,r} = t.$

EXAMPLE 1. In what time will 284*l.* amount to 354*l.* at 5 per cent. per annum?

In this example $a = 354$, $P = 284$, and $r = ,05$.

Then per theorem $\frac{354 - 284}{284 \times ,05} = \frac{70}{14,2} = 4,9295 = 4$ years, 339 days, the anſwer. The work at length:

```
 284    354
 ,05    284
 ----   ----
14,20  ) 70,00000(4,9295 = 4 years, 339 days, anſwer
         568
         ----
         1320
         1278
         ----
           420
           284
          ----
          1360
          1278
          ----
            820
            710
            ---
            110
```

E. 2.

E. 2. In what time will 336*l.* 5*s.* amount to 423*l.* 10*s.* at $4\frac{1}{2}$ per cent per annum?

In this example, $a = 423,5$, $p = 336, 25$, and $r = ,045$

Then per theorem, $\frac{423,5 - 336,25}{336,25 \times ,045} = \frac{87,25}{15,13125} = 5766 = 5$ years, 279 days, the answer.

The work at length,

```
              423,5
              336,25
              ------
  15,13125) 87,25000 (5,766 = 5 years, 279 days, answer.
            7565625
            --------
            11593750
            10591875
            --------
             10018750                  336,25
              9078750                    ,045
              -------                  ------
               9400000                 168125
               9078750                134500
               -------                --------
                321250                15,13125
```

When the principal, interest, and time are given, to find the rate per cent,

RULE. Divide the interest by the product of the principal and time, the quotient is the rate.

THEOREM 7. $\frac{I}{pt} = r.$

EXAMPLE 1. At what rate per cent will 260l. gain 64*l.* 7*s.* in $5\frac{1}{2}$ years?

In this example, $I = 64,35$, $p = 260$, and $t = 5,5$.

Then per theorem, $\frac{64,35}{260 \times 5,5} = \frac{64,35}{1430} = ,045$, or $4\frac{1}{2}$ per cent. the answer.

The work at length,

```
     260
     5,5
    ----
    1300
    1300
    -----
  1430,0)64,35(,045 Rate = 4½ the answer.
         572
         ---
          715
          715
          ---
          ...
```

E. 2. At what rate per cent. will 216*l.* 10*s.* gain 43*l.* 6*s.* in 4 years?

```
  216,5
      4
-------
  866, )43,30(,05 = 5 per cent. answer.
        4330
        ....
```

When the principal, amount, and time are given, to find the rate,

RULE. Take the difference between the amount and principal, and divide it by the product of the principal and time, the quotient is the rate.

THEOREM 8. $\frac{a-p}{pt}=r.$

EXAMPLE 1. At what rate per cent. will 142*l.* 5*s.* amount to 177*l.* 2*s.* $0\frac{1}{4}d.$ in $3\frac{1}{2}$ years?

In this example, $a = 177{,}1010416$, $p = 142{,}25$, and $t = 7$.

Then per theorem, $\frac{177{,}1010416 - 142{,}25}{142{,}25 \times 7} = \frac{34.8510416}{995{,}75} =$ $,03498 = 3l.\ 9s.\ 11\frac{1}{2}d.$ per cent. the answer.

The work at length,

```
 142,25    177,1010416
      7    142,25
 ------    -----------
 995,75 )  34,8510416(,03|498 = 3l. 9s. 11½d.
           298725           20     per cent.
           ------        -----     the answer
            497854       9,960     as above.
            399300          12
            ------       -----
             985541      11,52
             896175          4
             ------      -----
              893666      2,08
              796600
              ------
               97066
```

E. 2. At what rate per cent. will 260*l.* amount to 324*l.* 7*s.* in $5\frac{1}{2}$ years?

In this example $a = 324{,}35$, $p = 260$, and $t = 5{,}5$.

Then per theorem, $\frac{324{,}35 - 260}{260 \times 5{,}5} = \frac{64{,}35}{1430} =$,045, or $4\frac{1}{2}$ per cent. the answer. The work at length,

```
  260
  5,5
 ----
 1300
 130
 ----
 1430,0)64,350(,045, or 4½ per cent. the answer.
        5720
        ----
         7150
         7150
         ....
```

E. 3. At what rate per cent. will 672*l*. 5*s*. amount to 847*l*. 17*s*. 6*d*. in $5\frac{1}{2}$ years?

In this example, $a = 847,875$, $p = 672,25$, and $t = 5,5$.

Then per theorem $\frac{847,875 - 672,25}{672,25 \times 5,5} = \frac{175,62500}{3697,375} = ,0475$, or $4\frac{3}{4}$ per cent. the answer.

LXI. Of Annuities or Pensions in Arrears, &c.

AN annuity is a yearly income arising from money, &c. and is either paid for a term of years, or upon a life.

Annuities or pensions are said to be in arrears, when they are payable or due either yearly, half-yearly, or quarterly, and are unpaid for any number of payments, and each payment at the time it is due, simple interest is allowed, at a certain rate per cent.

When the annuity, rate and time are given, to find the amount; that is, when U, R, T, are given, to find A.

Here U represents the annuity, pension, or yearly rent, A, T and R, as before,

RULE. Multiply the time by itself, and that product by the annuity; from this subtract the product of the annuity multiplied by the time, and divide the remainder by 2; multiply this quotient by the ratio of the rate per cent. and to this product add the time multiplied by the annuity, the sum will be the amount.

THEOREM 9. $\frac{ttu - tu}{2} \times r : + tu = A$, the amount.

EXAMPLE 1. If an annuity of 50*l*. be forborne or unpaid 5 years, what will it amount to in that time, at 5 per cent?

In this example, $u = 50$, $t = 5$, and $r = ,05$.

Then per theorem, $\frac{50 \times 5 \times 5 - 50 \times 5}{2} \times ,05 : + 50 \times 5 =$

$\frac{1250 - 250}{2} \times ,05 : + 250 = 500 \times ,05 + 250 = 275l.$ Answ.

The work at length,

$$\begin{array}{rl}
50 & = u. \\
5 & = t. \\
\hline
250 & = tu. \\
5 & \\
\hline
1250 & = ttu. \\
-250 & \\
\hline
2)1000 & \\
\hline
500 & \\
,05 & = r. \\
\hline
25,00 & \\
+250 & \\
\hline
\end{array}$$

Answer £. 275 as before.

Note. : + in the 9th theorem, denotes that all its succeeding terms must be added to all its preceding terms.

E. 2. If a houſe be let upon a leaſe for 7 years, at 45*l.* per annum, I deſire to know the amount for that time, at 4 per cent. per annum?

In this example, $u = 45$, $t = 7$, and $r = ,04$.

Then per theorem, $\frac{45 \times 7 \times 7 - 45 \times 7}{2} \times ,04 : + 45 \times 7 =$ 352,8 = 352*l.* 16*s.* the anſwer.

At length thus,

$$\begin{array}{r} 45 \\ \times 7 \\ \hline 315 \\ \times \;\; 7 \\ \hline 2205 \\ -315 \\ \hline \div 2)\;1890 \\ \hline 945 \\ \times ,04 \\ \hline 37,80 \\ +315 \\ \hline \end{array}$$

Note. When the annuity, &c. is to be paid half-yearly, or quarterly, then for half-yearly payments take half the ratio, half the annuity, &c. and twice the number of years; and for quarterly payments, take a fourth part of the ratio, a fourth part of the annuity, and four times the number of years, and then proceed as before directed.

Anſwer 352,8 = 352*l.* 16*s.* as above.

E. 3. If 125*l.* yearly rent, penſion, &c. be forborne, or unpaid 3 years, what will it amount to in that time, at 3 per cent. for each payment as it becomes due?

In this example, $u = 125$, $t = 3$, and $r = ,03$.

Then per theorem $\frac{125 \times 3 \times 3 - 125 \times 3}{2} \times ,03 : + 125 \times 3$ = 386,25 = 386*l.* 5*s.* the anſwer.

E. 4. If a ſalary of 125*l.* payable every half-year, remain unpaid for 3 years, what will it amount to in that time, at 3 per cent. per annum?

In this example $u = 62,5$, $t = 6$, and $r = ,015$, per note.

Then per theorem, $\frac{62,5 \times 6 \times 6 - 62,5 \times 6}{2} \times ,015 : + 62,5 \times 6$ = 389,0625 = 389*l.* 1*s.* 3*d.* the anſwer.

E. 5. If a ſalary of 125*l.* payable every quarter, was left unpaid for three years, what would it amount to in that time, at 3 per cent. per annum?

In this example, $u = 31,25$, $t = 12$, $r = ,0075$, per note.

Then per theorem $\frac{31,25 \times 12 \times 12 - 31,25 \times 12}{2} \times ,0075 : + 31,25 \times 12$ = 390,46875 = 390*l.* 9*s.* 4½*d.* the anſwer.

At

At length thus,

$$\begin{array}{r} 31,25 \\ \times\ 12 \\ \hline 375,00 \\ \times\ 12 \\ \hline 4500 \\ -375 \\ \hline 2)\,4125 \\ \hline 2062,5 \\ \times\ ,0075 \\ \hline 103125 \\ 144375 \\ \hline 15,16875 \\ +375 \\ \hline \end{array}$$

Note. It may be observed by comparing the answers of the three last examples, that the half-yearly payments are more advantageous than the yearly, and quarterly more than the half-yearly.

Answer 390,16875 = 390*l.* 9*s.* 4½*d.* as before.

When the amount, rate and time are given, to find the annuity; or when A, R and T are given, to find U,

RULE. Multiply the square of the time by the ratio, to which product add twice the time; and from that sum subtract the time multiplied by the ratio, for a divisor: multiply the amount of the annuity by 2 for a dividend: the quotient arising from this division will be the annuity required.

THEOREM 10. $\dfrac{2a}{ttr + 2t - tr} = U$, the annuity.

EXAMPLE 1. If the amount of an annuity for 5 years, at 5 per cent. be 275*l.* what is the annuity?

In this example, $a = 275$, $t = 5$, and $r = ,05$.

Then per theorem, $\dfrac{275 \times 2}{5 \times 5 \times ,05 + 5 \times 2 - 5 \times ,05} = \dfrac{550}{11} =$ 50*l.* the answer.

The work at length,

$$\begin{array}{r} 5 \\ \times\ 5 \\ \hline 25 \\ \times\ ,05 \\ \hline 1,25 \\ +\ 10 \\ \hline 11,25 \\ -\ 25 \\ \hline \end{array} \qquad \begin{array}{r} 5 \\ 2 \\ \hline 10 \end{array} \qquad \begin{array}{r} 5 \\ ,05 \\ \hline ,25 \end{array} \qquad \begin{array}{r} 275 \\ 2 \\ \hline 550 \end{array}$$

11,00)550(50*l.* the answer as before.
55
―
0

Note. When the payments are half-yearly, take 4 times the amount of the annuity, or 4 *a*; if quarterly, 8 times the amount, or 8 *a*, and proceed with the rate and time as before directed; see page 244.

E. 2. If a ſalary payable yearly amounts to 352*l.* 16*s.* in 7 years, at 4 per cent. what is the ſalary?

In this example, $a = 352{,}8$, $t = 7$, and $r = {,}04$.

Then per theorem, $\dfrac{352{,}8 \times 2}{7 \times 7 \times {,}04 + 7 \times 2 - 7 \times {,}04} = \dfrac{705{,}6}{15{,}68} =$ 45*l.* the anſwer.

The work at length,

$$\begin{array}{rrrr}
7 & 7 & 7 & 352{,}8 \\
\times 7 & {,}04 & 2 & 2 \\
\hline
49 & {,}28 & 14 & 705{,}6 \\
\times\, {,}04 & & & \\
\hline
1{,}96 & & & \\
+\,14 & & & \\
\hline
15{,}96 & & & \\
-\,{,}28 & & & \\
\hline
\end{array}$$

15,68)705,60(45*l.* Anſwer, as before.

$$\begin{array}{r}
6272 \\
\hline
7840 \\
7840 \\
\hline
\cdots\cdot
\end{array}$$

E. 3. The amount of a ſalary payable half-yearly, for 5 years, at 5 per cent. is 278*l.* 2*s.* 6*d.* what is the ſalary?

Theorem for half-yearly payments $\dfrac{4a}{ttr + 2t - tr} = U$, the ann.

In the above example, $a = 278{,}125$, $t = 10$, and $r = {,}025$.

Then per theorem, $\dfrac{278{,}125 \times 4}{10 \times 10 \times {,}025 + 10 \times 2 - 10 \times {,}025} = \dfrac{1112{,}500}{22{,}250} = 50l.$ the anſwer.

The work at length,

$$\begin{array}{rrrr}
10 & 10 & 10 & 278{,}125 \\
10 & 2 & {,}025 & 4 \\
\hline
100 & 20 & {,}250 & 1112500 \\
{,}025 & & & \\
\hline
500 & & & \\
200 & & & \\
\hline
2{,}500 & & & \\
+\,20 & & & \\
\hline
22{,}500 & & & \\
-\,250 & & & \\
\hline
\end{array}$$

22,250)1112,500(50*l.* anſwer, as before.

$$\begin{array}{r}
111250 \\
\hline
\cdots\cdot\cdot 0
\end{array}$$

E. 4.

E. 4. If the amount of an annuity, payable quarterly, be 1629*l.* 7*s.* 6*d.* for 6 years, at 3 per cent. what is the annuity?

Theorem for quarterly payments $\frac{8a}{ttr + 2t - tr} = U$ the annuity.

In this example, $a = 1629{,}375$, $t = 24$, and $r = {,}0075$.

Then per theorem, $\frac{1629{,}375 \times 8}{24 \times 24 \times {,}0075 + 24 \times 2 - 24 \times {,}0075}$

$= \frac{1303{,}5}{52{,}14} = 150$*l.* the anſwer.

The work at length,

$$\begin{array}{r} 24 \\ 24 \\ \hline 96 \\ 48 \\ \hline 576 \\ {,}0075 \\ \hline 2880 \\ 4032 \\ \hline 4{,}3200 \\ +48 \\ \hline 52{,}3200 \\ -\ {,}1800 \\ \hline \end{array} \quad \begin{array}{r} 24 \\ 2 \\ \hline 48 \end{array} \quad \begin{array}{r} 24 \\ {,}0075 \\ \hline 120 \\ 168 \\ \hline {,}1800 \end{array} \quad \begin{array}{r} 1629{,}375 \\ 8 \\ \hline 13035{,}000 \end{array}$$

52,1400)13035,00(250*l.* anſwer, as before.

When the annuity, amount, and time are given, to find the rate of intereſt; or when U, A and T are given, to find R,

RULE. From twice the amount, ſubtract twice the annuity multiplied by the time, and divide the remainder by the ſquare of the time multiplied by the annuity, made leſs by the time multiplied by the annuity, and the quotient will be the ratio of the rate per cent.

THEOREM II. $\frac{2a - 2ut}{utt - ut} = R$, the rate of intereſt.

Note. If the payments be half-yearly, the amount and annuity muſt each be multiplied by 4 (that is $4a - 4ut$ muſt be taken for a dividend); if quarterly, by 8 (viz. $8a - 8ut$); in every other reſpect proceed as before-mentioned. See page 244.

EXAMPLE 1. If an annuity of 50*l.* per annum amount to 275*l.* in 5 years, what is the rate per cent?

In this example, $a = 275$, $u = 50$, and $t = 5$.

Then per theorem, $\frac{275 \times 2 - 50 \times 2 \times 5}{50 \times 5 \times 5 - 50 \times 5} = \frac{50{,}00}{1000} = {,}05$, or 5 per cent. the anſwer.

The

The work at length,				
	275	50	50	50
	2	2	5	5
	550	100	250	250
	— 500	5	5	
1000)	50,00	500	1250	
			— 250	
Anſwer	,05, or 5 per cent.		1000	

E. 2. If a ſalary of 250*l.* per annum, amounts to 1612*l.* 10*s.* in 6 years, what is the rate per cent?

In this example, $a = 1612{,}5$, $u = 250$, and $t = 6$.

Then per theorem, $\frac{1612{,}5 \times 2 - 250 \times 2 \times 6}{250 \times 6 \times 6 - 250 \times 6} = \frac{225{,}00}{7500} =$,03, or 3 per cent. the anſwer.

The work at length,

		250
		6
	250	1500
	2	6
1612,5	500	9000
2	6	— 1500
3225,0	3000	7500 Diviſor
— 3000		

7500) 225,00(,03, or 3 per cent. anſwer.
22500
.....

E. 3. If a ſalary of 50*l.* per annum, payable half-yearly, amounts to 278*l.* 2*s.* 6*d.* in 5 years, what is the rate per cent?

Theorem for half-yearly payments, $\frac{4a - 4ut}{utt - ut} = r$.

In this example, $a = 278{,}125$, $u = 25$, and $t = 10$.

Then per theorem, $\frac{278{,}125 \times 4 - 25 \times 4 \times 10}{25 \times 10 \times 10 - 25 \times 10} = \frac{112{,}50}{2250} =$,05, or 5 per cent. the anſwer.

The work at length,

			25
			10
278,125	25	25	250
4	4	10	10
1112,500	100	250	2500
—1000	10		— 250
	1000		2250

2250) 112,500(,05, or 5 per cent. the anſwer.
11250
.....

E. 4.

E. 4. Suppose a pension of 250*l.* per annum, payable quarterly, amounts to 1629*l.* 7*s.* 6*d.* in 6 years, what is the rate per cent?

Theorem for quarterly payments, $\frac{8a - 8ut}{utt - ut} = r$.

In this example, $a = 1629{,}375$, $u = 62{,}5$, and $t = 24$.

Then per theorem, $\frac{1629{,}375 \times 8 - 62{,}5 \times 8 \times 24}{62{,}5 \times 24 \times 24 - 62{,}5 \times 24} = \frac{1035{,}00}{34500}$ $= {,}03$, or 3 per cent. the answer.

When the annuity, amount, and rate are given, to find the time; or, when U, A and R, are given, to find T,

RULE. Divide 2 by the ratio, and subtract 1 from the quotient; then square the remainder, and divide the square by 4; to this quotient add twice the amount divided by the annuity, multiplied by the ratio, and extract the square root of the sum. Lastly, from this root subtract half the number found by dividing 2 by the ratio, and subtracting 1 from the quotient: this result will be the time.

THEOREM 12. First $\frac{2}{r} - 1 = x$. Then $\sqrt{\frac{2a}{ur} + \frac{xx}{4}} : -\frac{x}{2} = T$, the time.

EXAMPLE 1. In what time will an annuity of 50*l.* per annum amount to 275*l.* at 5 per cent?

In this example, $a = 275$, $u = 50$, and $r = {,}05$.

Then per theorem 1st, $\frac{2}{{,}05} - 1 = 39 = x$.

Then $\sqrt{\frac{275 \times 2}{50 \times {,}05} + \frac{39 \times 39}{4}} : -\frac{39}{2} = \sqrt{\frac{550}{2{,}50} + \frac{1521}{4}} : - 19{,}5 = \sqrt{600{,}50} - 19{,}5 = 24{,}5 - 19{,}5 = 5$ years, the answer.

The work at length,

```
,05)2,00          39              50 = u.
    ----          39             ,05 = r.
     40          ----            ----
    - 1          351             2,50 = ur.
    ---         117
     39 = x.    ----
               4)1521
               ------
 275 = a.      380,25 = xx ÷ 4   2,5|0)55|0,00(220
   2          +220                     50
 ---          ------                   --
 550 = 2a.    600,25(24,5              50
               4      19,5             50
             44)200   5,0 Years, answ.  --
               176                      0
              -----
             485) 2425
                  2425
```

If payments are half-yearly, the time will be equal to the number of half years; if quarterly, the time will be equal to the number of quarterly payments; with the ratio and annuity proceed as before.

E. 2. If an annuity of 250*l.* per annum, payable half-yearly, amounts to 1623*l.* 15*s.* at 3 per cent. what time was the payments forborne?

In this example, $a = 1623{,}75$, $u = 125$, and $r = {,}015$.

Then per theorem first, $\frac{2}{,015} - 1 = 132{,}8 = x$.

Then $\sqrt{\frac{1623{,}75 \times 2}{125 \times ,015} + \frac{132{,}8 \times 132{,}8}{4}} : - \frac{132{,}8}{2} =$

$\sqrt{\frac{3247{,}5}{1{,}875} + \frac{17507{,}68}{4}} : - 66{,}1 = \sqrt{6108{,}92} - 66{,}1 = 78.$

$1 - 66{,}1 = 12$ Half years, or 6 years, the time required.

LXII. Of the Present Worth of Annuities, &c.

WHEN the annuity, rate and time are given, to find the present worth; or when U, T and R are given, to find P. Here P represents the present worth; U, T and R, as before.

RULE. Square the time, and multiply the product by the ratio; to this add twice the time; then from the sum subtract the time multiplied by the ratio, and let the remainder stand for a dividend. Next, multiply twice the ratio by the time, and add 2 to the product for a divisor; lastly, divide the one by the other, and multiply the quotient by the annuity, pension, &c. and the product will be the present worth.

THEOREM 13. $\frac{ttr - tr + 2t}{2tr + 2} : \times u = P.$

The same is to be observed here, for half-yearly and quarterly payments, as before-mentioned.

EXAMPLE 1. What is the present worth of 200*l.* per annum, to continue 6 years, at 5 per cent?

$$\begin{array}{r}
6 = t. \\
6 \\
\hline
36 \\
,05 \\
\hline
1{,}80 \\
-,30 \\
\hline
1{,}50 \\
+12 \\
\hline
\end{array}
\qquad
\begin{array}{r}
6 \\
2 \\
\hline
12 = 2t.
\end{array}
\qquad
\begin{array}{r}
6 = t. \\
,05 = r. \\
\hline
,30 = tr. \\
2 \\
\hline
,60 \\
+2, \\
\hline
\text{Divisor } 2{,}60 = tr \times 2 + 2
\end{array}$$

$$\begin{array}{r}
2{,}60)13{,}50(5{,}1923 \\
200 \\
\hline
1038{,}4600 = 1038l.\ 9s.\ 2\tfrac{1}{4}d. \text{ Answer.}
\end{array}$$

E. 2.

E. 2. What is the preſent worth of a houſe, whoſe yearly rent is 75*l.* per annum, to continue 9 years, at 6 per cent?

9	9	9	9
9	,06	2	,06
81	,54	18	,54
,06			2
4,86			1,08
— ,54			+ 2
4,32			3,08 Diviſor
+ 18,			

3,08) 22,32 (7,24675 × 75 = 543,50625 = 543*l.* 10*s.* $1\frac{1}{2}$*d.* Anſ.

E. 3. What is the preſent worth of 40*l.* per annum, payable half-yearly, at 5 per cent. and to continue 6 years?

12	12	12	12
12	,025	2	,025
144	,300	24	,300
,025			2
720			,600
288			+ 2
3,600			2,6 Diviſor
— ,300			
3,300			
+ 24			

2,6) 27,30 (10,5 × 20 = 210*l.* the anſwer.

E. 4. What is the preſent worth of 50*l.* per annum, payable quarterly, at 5 per cent. to continue 6 years?

24	24	24	24
24	,0125	2	,0125
96	120	48	120
48	48		48
576	24		24
,0125	,3000		,3000
2880			× 2
1152			,6
576			+ 2
7,2000			2,6 Diviſor
— ,3000			
6,9000			
+ 48			

2,6) 54,9000 (21,114 × 12,5 = 263,9250 = 263*l.* 18*s.* 6*d.* Anſ.

 When

When the present worth, time, and ratio are given, to find the annuity, rent, &c. Or when P, T and R, are given, to find U,

RULE. Multiply the ratio by the time, and add 1 to the product for a dividend; then multiply the square of the time by the ratio; subtract the product of the time and ratio, and to the remainder add twice the time for a divisor; lastly, multiply the quotient of these two numbers by twice the present worth, the product will be the annuity, &c.

THEOREM 14. $\dfrac{tr + 1}{ttr - tr + 2t} : \times\ 2p = U$, the annuity, &c.

EXAMPLE 1. There is an annuity of 6 years to come, I desire to know the yearly value, when the present worth at 3 per cent. is 1366*l.* 10*s.* 6*d.*

In this example, $t = 6$, $r = ,03$, and $p = 1366,525$, to find U.

Then per theorem, $\dfrac{\overline{6 \times ,03} + 1}{6 \times 6 \times ,03 - 6 \times ,03 + 6 \times 2} : \times$

$1366,525 \times 2 = \dfrac{1,18}{12,9} \times 2733,05 = ,0914728 \times 1733,05 =$ 250*l.* the answer.

Note. If the payments be half-yearly, take $\frac{1}{2}$ of the ratio, and multiply by four times the present worth; if quarterly, $\frac{1}{4}$ of the ratio, and multiply by eight times the present worth, and proceed with the time and ratio as before.

E. 2. There is an annuity payable half-yearly, for 6 years to come; what is the yearly income, when the present worth, at 3 per cent. is 1376*l.* 5*s*?

In this example, $t = 12$, $r = ,015$, and $p = 1376,25$.

Then per theorem, $\dfrac{\overline{12 \times ,015} + 1}{12 \times 12 \times ,015 - 12 \times ,015 + 12 \times 2} : \times$

$1376,25 \times 4 =$ 250*l.* 0*s.* 8½*d.* the answer.

E. 3. The present worth of an annuity, payable quarterly, for 6 years to come, at 3 per cent. is 1380*l.* 17*s.* 6*d.* I desire to know the annuity?

In this example, $t = 24$, $r = ,0075$, and $p = 1380,875$.

Then per theorem, $\dfrac{\overline{24 \times ,0075} + 1}{24 \times 24 \times ,0075 - 24 \times ,0075 + 24 \times 2}$

$\times\ 1380,875 \times 8 = \dfrac{1,18}{52,14} \times 11047 =$ 250*l.* the answer.

When the annuity, present worth and time are given, to find the ratio; or when U, P and T are given, to find R, the ratio,

RULE.

RULE. Multiply the annuity by twice the time, from which product subtract twice the present worth, and reserve the remainder for a dividend; next multiply twice the present worth by the time, and reserve this product; again, multiply the annuity by the square of the time, from which subtract the annuity multiplied by the time, and let this remainder taken from the product last reserved, be your divisor. Lastly, the quotient arising from the division of these two numbers will be the ratio required.

THEOREM 15. $\frac{\overline{2ut - p} \times 2}{2pt - ut - utt} = R.$

EXAMPLE. At what rate per cent. will an annuity of 30*l.* 10*s.* per annum, to continue 10 years, produce the present worth of 250*l.* 6*s.* 8*d*?

$30,5 = u.$
$\times\ 10 = t.$
$305 = tu.$
$\times\ 10 = t.$
$3050 = ttu.$
$-\ 305 = tu.$
$2745 = ttu - tu.$
$-\ 5006,6 = 2pt.$

Divisor 2261,6

And $500,6 = 2p.$
Also $610,0 = 2tu.$

Dividend $109,3 = 2p - 2tu.$

Then 2261,6)109,3 } See division of
109 } repetends.

2035,5) 8840(,04343 = *R.* = 4*l.*
8142 (7*s.* 11*d.* Answer.
698
610
88
81
7
6
1

When the payments are half-yearly, or quarterly, proceed with the annuity and time as before directed, and the quotient will be the answer (i. e.) if for half-yearly, the quotient will be half the ratio, and if for quarterly, a fourth part of the ratio.

When the annuity, present worth, and ratio are given, to find the time; or when U, P and R are given, to find T.

THEOREM 16. $\sqrt{\frac{2p}{ru} + \frac{xx}{4}} \pm \frac{1}{2}x = T.$

EXAMPLE. In what time will 7*l.* per annum pay a debt of 120*l.* 8*s.* at 6*l.* per cent?

In this example, $u = 7$, $r = ,06$, and $p = 120,4$, to find T.

First

First - - - - - - 240,8 $= 2p.$

And - - - - - - 34,4 $= \frac{2\,p.}{u.}$

From which take - 33,3 $= \frac{2}{R.}$

To the remainder - 1,06 $= \frac{2}{r.} - \frac{2p}{u}$

Add unity - - 1 $= +\,1$

The sum is - - - 2,06 $= \frac{2}{r.} - \frac{2p}{u} + 1 = x$ by substitution.

$$\left\{\begin{array}{r} 1{,}03 = \tfrac{1}{2}x. \\ 103 = \tfrac{1}{2}x. \\ \hline 9)309 \\ 344 \\ 10333 \\ \hline 1{,}0677 = \tfrac{1}{4}xx. \end{array}\right.$$

Then - - - - - - 1,03 $= \frac{1}{2}x.$

And - - - - - - 1,067 $= \frac{1}{4}xx.$

Again - - - - - ,42 $= ru.$

And - - - - - - 573,3 $= \frac{2\,p.}{ru.}$

Then - - - - - - 574,4 $= \frac{2p}{ru} + \frac{xx.}{4}$

Square root of that - 23,96 $= \sqrt{\frac{2p}{ru} + \frac{xx.}{4}}$

To which add - - 1,03 $= \frac{1}{2}x.$

The sum is - - - 24,99 $= 25$ nearly the time, answer.

When the payments are half-yearly, or quarterly, proceed with the annuity and ratio as before, the quotient will be the number of payments.

A TABLE, ſhewing the intereſt of any ſum of money, from a million to a pound, for any number of days, at any rate of intereſt.

Sum.	£.	s.	d.	q.	Sum .	£.	s.	d.	q.	Sum	£.	s.	d.	q.
1000000	2739	14	6	9,99	10000	27	7	11	1,37	100	0	5	5	3,15
900000	2465	15	0	3,29	9000	24	13	1	3,23	90	0	4	11	5,71
800000	2191	15	7	1,59	8000	21	18	4	1,10	80	0	4	4	2,41
700000	1917	16	1	3,89	7000	19	3	6	2,96	70	0	3	10	0,11
600000	1643	16	8	2,19	6000	18	8	9	0,82	60	0	3	3	1,81
500000	1369	17	3	0,49	5000	13	[illegible]	11	2,58	50	0	2	8	3,51
400000	1095	17	9	2,95	4000	10	19	2	0,55	40	0	2	2	1,21
300000	821	18	4	1,09	3000	8	4	4	2,41	30	0	1	7	2,90
200000	547	18	10	3,40	2000	5	9	7	0,27	20	0	1	1	0,60
100000	273	19	5	1,70	1000	2	14	9	2,14	10	0	0	6	2,30
90000	246	11	6	0,32	900	2	9	3	2,12	9	0	0	5	3,67
80000	219	3	6	0,96	800	2	3	10	0,11	8	0	0	5	1,40
70000	191	15	7	1,59	700	1	18	4	1,10	7	0	0	4	2,41
60000	164	7	8	0,22	600	1	12	10	2,80	6	0	0	3	3,76
50000	136	19	8	2,85	500	1	7	5	3,70	5	0	0	3	1,15
40000	109	11	9	1,48	400	1	1	11	0,50	4	0	0	2	2,52
30000	82	3	10	0,11	300	0	16	5	1,40	3	0	0	1	3,80
20000	54	15	10	2,74	200	0	10	11	2,30	2	0	0	1	1,26
10000	27	7	11	1,37	100	0	5	5	3,15	1	0	0	0	2,63

Note.

Note: The decimals in the foregoing table are 100th parts of a farthing.

The design of tables of interest (both simple and compound) is ease and expedition in practical calculations; and the rules expressed in words for answering questions of interest are tedious and intricate, and the reason not easily understood? the operations themselves are, for the most part, very laborious; for which reason I have thought proper to insert this and the following tables of simple interest, by the help of which all questions relating thereto may be quickly resolved, that fall within the compass of the tables.

The USE of the preceding TABLE.

RULE. Multiply the sum by the number of days, and the product thereof by the rate of interest per cent. then cut off the two last figures to the right-hand, and enter the table with what remains to the left, against which numbers collected, you have the interest for the given sum.

EXAMPLE. What is the interest of 100*l.* at 5 per cent. for 365 days?

Number of days 365
× 100*l.* Sum
36500
× 5 Rate per cent.
1825|00

		£.	s.	d.	q. parts.
Then, in the table, against	1000 is	2	14	9	2,14
	800	2	3	10	0,11
	20	0	1	1	0,60
	5	0	0	3	1,15
	1825	*l.* 5	0	0	0,00 Answer.

And in the same way may the interest of any other given number of pounds be found for any given number of days.

Questions where principal, annuity, amount, &c. are concerned, are likewise to be solved by the tables; for there are similar numbers in the tables analogous to those given; and therefore having three terms given, a proportion or analogy must be made by the rule of three, between the numbers given in the question, and those in the proper table for the same rate and time, in order to find the fourth term, which is either the thing itself which is sought, or it will shew it by the table. And as 1 is commonly a term in the proportion, the question will generally be solved by multiplication or division.

If any thing is wanting to make the proportion, or to carry on the process, it must be found from what is given in the question.

TABLE

TABLE 1. Of the amount of 1*l.* for years, at simple intereſt.

Years.	3 per Cent.	3½ per Cent.	4 per Cent.	4½ per Cent.	5 per Cent.
1	1,03	1,035	1,04	1,045	1,05
2	1,06	1,070	1,08	1,090	1,10
3	1,09	1,105	1,12	1,135	1,15
4	1,12	1,140	1,16	1,180	1,20
5	1,15	1,175	1,20	1,225	1,25
6	1,18	1,210	1,24	1,270	1,30
7	1,21	1,245	1,28	1,315	1,35
8	1,24	1,280	1,32	1,360	1,40
9	1,27	1,315	1,36	1,405	1,45
10	1,30	1,350	1,40	1,450	1,50
11	1,33	1,385	1,44	1,495	1,55
12	1,36	1,420	1,48	1,540	1,60
13	1,39	1,455	1,52	1,585	1,65
14	1,42	1,490	1,56	1,630	1,70
15	1,45	1,525	1,60	1,675	1,75
16	1,48	1,560	1,64	1,720	1,80
17	1,51	1,595	1,68	1,765	1,85
18	1,54	1,630	1,72	1,810	1,90
19	1,57	1,665	1,76	1,855	1,95
20	1,60	1,700	1,80	1,900	2,00
21	1,63	1,735	1,84	1,945	2,05
22	1,66	1,770	1,88	1,990	2,10
23	1,69	1,805	1,92	2,035	2,15
24	1,72	1,840	1,96	2,080	2,20
25	1,75	1,875	2,00	2,125	2,25
26	1,78	1,910	2,04	2,170	2,30
27	1,81	1,945	2,08	2,215	2,35
28	1,84	1,980	2,12	2,260	2,40
29	1,87	2,015	2,16	2,305	2,45
30	1,90	2,050	2,20	2,350	2,50
31	1,93	2,085	2,24	2,395	2,55
32	1,96	2,120	2,28	2,440	2,60
33	1,99	2,155	2,32	2,485	2,65
34	2,02	2,190	2,36	2,530	2,70
35	2,05	2,225	2,40	2,575	2,75
36	2,08	2,260	2,44	2,620	2,80
37	2,11	2,295	2,48	2,665	2,85
38	2,14	2,330	2,52	2,710	2,90
39	2,17	2,365	2,56	2,755	2,95
40	2,20	2,400	2,60	2,800	3,00
41	2,23	2,435	2,64	2,845	3,05
42	2,26	2,470	2,68	2,890	3,10
43	2,29	2,505	2,72	2,935	3,15
44	2,32	2,540	2,76	2,980	3,20
45	2,35	2,575	2,80	3,025	3,25
46	2,38	2,610	2,84	3,070	3,30
47	2,41	2,645	2,88	3,115	3,35
48	2,44	2,680	2,92	3,160	3,40
49	2,47	2,715	2,96	3,205	3,45
50	2,50	2,750	3,00	3,250	3,50
51	2,53	2,785	3,04	3,295	3,55
52	2,56	2,820	3,08	3,340	3,60
53	2,59	2,855	3,12	3,385	3,65
54	2,62	2,890	3,16	3,430	3,70
55	2,65	2,925	3,20	3,475	3,75
56	2,68	2,960	3,24	3,520	3,80
57	2,71	2,995	3,28	3,565	3,85

The USE of the foregoing TABLE.

EXAMPLE 1. If 250*l.* be put out to intereſt, what will it amount to in 21 years, at 4 per cent?

Firſt by table 1, the amount of 1*l.* for 21 years, at 4 per cent. is 1,84; then ſay,

	Prin.		*Amount.*		*Prin.*
As	1	:	1,84	::	250
			250		
			9200		
			368		

Anſwer £. 460,00 the amount required.

E. 2. What principal, put out for 21 years, will amount to 460*l.* at 4 per cent?

Firſt, by table 1, the amount of 1*l.* is 1,84, for the given time and rate; then ſay,

	Amount.		*Prin.*		*Amount.*
As	1,84	:	1	::	460

1,84)460(250*l.* the principal required.
368
920
920
0

E. 3. At what rate of ſimple intereſt will 250*l.* amount to 460*l.* in 21 years?

		Prin.		*Amount.*		*Prin.*
By table 1ſt ſay,	If	250	:	460	::	1
				1		

250)460(1,84 the amount of 1*l.* which being ſought for againſt 21 years, you find in the column under 4 per cent. the rate of int. required.
250
2100
2000
1000
1000
0

TABLE 2. The amount of 1*l.* annuity for years, at simple intereſt.

Years.	3 per Cent.	$3\frac{1}{2}$ per Cent.	4 per Cent.	$4\frac{1}{2}$ per Cent.	5 per Cent.
1	1,00	1,000	1,00	1,000	1,00
2	2,03	2,035	2,04	2,045	2,05
3	3,09	3,105	3,12	3,135	3,15
4	4,18	4,210	4,24	4,270	4,30
5	5,30	5,350	5,40	5,450	5,50
6	6,45	6,525	6,60	6,675	6,75
7	7,63	7,735	7,84	7,945	8,05
8	8,84	8,980	9,12	9,260	9,40
9	10,08	10,260	10,44	10,620	10,80
10	11,35	11,575	11,80	12,025	12,25
11	12,65	12,925	13,20	13,475	13,75
12	13,98	14,310	14,64	14,970	15,30
13	15,34	15,730	16,12	16,510	16,90
14	16,73	17,185	17,64	18,095	18,55
15	18,15	18,675	19,20	19,725	20,25
16	19,60	20,200	20,80	21,400	22,00
17	21,08	21,760	22,44	23,120	23,80
18	22,59	23,355	24,12	24,885	25,65
19	24,13	24,985	25,84	26,695	27,55
20	25,70	26,650	27,60	28,550	29,50
21	27,30	28,350	29,40	30,450	31,50
22	28,93	30,085	31,24	32,395	33,55
23	30,59	31,855	33,12	34,385	35,65
24	32,28	33,660	35,04	36,420	37,80
25	34,00	35,500	37,00	38,500	40,00
26	35,75	37,375	39,00	40,625	42,25
27	37,53	39,285	41,04	42,795	44,55
28	39,34	41,230	43,12	45,010	46,90
29	41,18	43,210	45,24	47,270	49,30
30	43,05	45,225	47,40	49,575	51,75
31	44,95	47,275	49,60	51,925	54,25
32	46,88	49.360	51,84	54,320	56,80
33	48,84	51,480	54,12	56,760	59,40
34	50,83	53,635	56,44	59,245	62,05
35	52,85	55,825	58,80	61,775	64,75
36	54,90	58,050	61,20	64,350	67,50
37	56,98	60,310	63,64	66,970	70,30
38	59,09	62,605	66,12	69,635	73,15
39	61,23	64,935	68,64	72,345	76,05
40	63,40	67,300	71,20	75,100	79,00
41	65,60	69,700	73,80	77,900	82,00
42	67,83	72,135	76,44	80,745	85,05
43	70,09	74,605	79,12	83,635	88,15
44	72,38	77,110	81,84	86,570	91,30
45	74,70	79.650	84,60	89,550	94,50
46	77,05	82,225	87,40	92,575	97,75
47	79,43	84,835	90,24	95,645	101,05
48	81,84	87,480	93,12	98,760	104,40
49	84,28	90,160	96,04	101,920	107,80
50	86,75	92,875	99,00	105,125	111,25
51	89,25	95,625	102,00	108,375	114,75
52	91,78	98,410	105,04	111,670	118,30
53	94.34	101,230	108,12	115,010	121,90
54	96,93	104,085	111,24	118,395	125,55
55	99,55	106,975	114,40	121,825	129,25
56	102,20	109,900	117,60	125,300	133,00
57	104,88	112,860	120,84	128,820	136,80

The USE of the foregoing TABLE.

EXAMPLE 1. If 320*l.* yearly rent be forborne for 12 years, what will be in arrear at that time, at $4\frac{1}{2}$ per cent?

By table 2. the amount of 1*l.* annuity for 12 years, is 14,97;

	Ann.		*Amount.*		*Ann.*
Then say,	If 1	:	14,97	::	320
			320		
			29940		
			4491		
		Answer	4790,40 = 4790*l.* 8*s.* the arrear.		

E. 2. In what time will 320*l.* yearly rent amount to 4790,4*l.* at $4\frac{1}{2}$ per cent?

	Rent.		*Amount.*		*Rent.*
As	320	:	4790,4	::	1
			1		

320)4790,4(14,97 the amount of 1*l.* annuity, which being found in the column under $4\frac{1}{2}$ per cent. in table 2d, stands over-against 12 years, the time sought.

E. 3. If 640*l.* yearly rent be forborne 24 years, what will be in arrear at that time, at 5 per cent?

The amount of 1*l.* annuity for 24 years, in the table, is 37,8;

	Ann.		*Amount.*		*Ann.*
Then say,	If 1	:	37,8	::	640
			640		
			1512		
			2268		
		Answer	24192,0 the arrear sought.		

E. 4. In what time 640*l.* yearly rent, amount to 24192*l.* at 5 per cent?

	Rent.		*Amount.*		*Rent.*
As	640	:	24192	::	1
			1		

640)24192(37,8 the amount of 1*l.* annuity; which being found in the column under 5 per cent. in the table, stands over-against 24 years, the time sought.

TABLE 3. The DISCOUNT of 1*l.* for days, at the rate of 3, 3½, 4, 4½, and 5 per cent. per annum.

Days.	3 per Cent.	3½ per Cent.	4 per Cent.	4½ per Cent.	5 per Cent.
1	,0000822	,0000959	,0001096	,0001233	,0001370
2	,0001644	,0001917	,0002191	,0002465	,0002739
3	,0002465	,0002876	,0003287	,0003679	,0004108
4	,0003287	,0003834	,0004382	,0004929	,0005477
5	,0004108	,0004792	,0005477	,0006161	,0006845
6	,0004929	,0005750	,0006571	,0007392	,0008212
7	,0005750	,0006708	,0007665	,0008623	,0009580
8	,0006571	,0007666	,0008759	,0009853	,0010947
9	,0007392	,0008623	,0009853	,0011084	,0012314
10	,0008212	,0009580	,0010947	,0012314	,0013680
20	,0016411	,0019141	,0021870	,0024597	,0027322
30	,0024597	,0028685	,0032769	,0036850	,0040928
40	,0032769	,0038210	,0043644	,0049073	,0054496
50	,0040928	,0047716	,0054496	,0061266	,0068027
60	,0049073	,0057205	,0065234	,0073429	,0081522
70	,0057205	,0066676	,0076128	,0085563	,0094980
80	,0065324	,0076128	,0086909	,0097667	,0108401
90	,0073429	,0085563	,0097667	,0109741	,0121786
100	,0081522	,0094980	,0108401	,0121786	,0135135
110	,0089601	,0104379	,0119112	,0133802	,0148448
120	,0097667	,0113760	,0129800	,0145788	,0161725
130	,0105720	,0123123	,0140465	,0157746	,0174966
140	,0113760	,0132468	,0151006	,0169674	,0188172
150	,0121786	,0141796	,0161725	,0181574	,0201342
160	,0129780	,0151106	,0172321	,0193444	,0214477
170	,0137801	,0160399	,0182894	,0205286	,0227577
180	,0145788	,0169674	,0193444	,0217100	,0240642
190	,0153763	,0178932	,0203972	,0228885	,0253672
200	,0161725	,0188172	,0214477	,0240642	,0266667
210	,0169674	,0197395	,0224960	,0252370	,0279627
220	,0177610	,0206601	,0235420	,0264070	,0292553
230	,0185534	,0215789	,0245858	,0275743	,0305445
240	,0193444	,0224959	,0256273	,0287387	,0318302
250	,0201342	,0234114	,0266667	,0299003	,0331126
260	,0209227	,0243251	,0277038	,0310592	,0343915
270	,0217100	,0252370	,0287387	,0322153	,0356671
280	,0224960	,0261473	,0297714	,0333686	,0369393
290	,0232807	,0270558	,0308019	,0345192	,0382082
300	,0240642	,0279627	,0318302	,0356671	,0394737
310	,0248464	,0288679	,0328564	,0368122	,0407352
320	,0256273	,0297714	,0338804	,0379547	,0419948
330	,0264070	,0306732	,0349022	,0390444	,0432503
340	,0271855	,0315734	,0359218	,0402314	,0445026
350	,0279627	,0324718	,0369393	,0413657	,0457516
360	,0287387	,0333686	,0379547	,0424974	,0469974
361	,0288162	,0334582	,0380561	,0426104	,0471218
362	,0288937	,0335478	,0381575	,0427234	,0472462
363	,0289712	,0336374	,0382588	,0428364	,0473705
364	,0290487	,0337269	,0383602	,0429493	,0474948
365	,0291262	,0338164	,0384615	,0430622	,0476191

The

The USE of TABLE 3, of DISCOUNT.

EXAMPLE 1. What is the discount of 83*l.* 10*s.* for 200 days, at 4 per cent? In the table, under 4 per cent. and against 200 days, is ,0214477

	,0214477			
	× 83,5 Principal sum			
	1072385			
	643431			
	1715816			
		£.	s.	d.
Answer	1,79088295 =	1	15	$9\frac{3}{4}$

E. 2. What is the discount of 100*l.* for 1 year, at 5 per cent?

In the table under 5 per cent. and against 365 days, is - -	,047619			
Which multiplied by the sum -	100	£.	s.	d.
Answer	4,7619 =	4	15	$2\frac{1}{4}$
Now the interest of 100*l.* for 1 year, at 5 per cent. is		5	0	0
The difference of discount and interest is - - -		0	4	$9\frac{3}{4}$

Whence (as I observed in section 20) it is evident, he who allows interest for discount, wrongs himself considerably.

E. 3. What is the discount of 8462*l.* at 3 per cent. for a year?

The discount of 1*l.* for 365 days, at 3 per cent. in the table, is - - -	,0291262			
Which multiplied by the principal sum	8462			
	582524			
	1747572			
	1165048			
	2330096	£.	s.	d.
Answer	246,4659044 =	246	9	$3\frac{3}{4}$

Note. This table of discount is perfectly true for all the days expressed therein, and is sufficiently exact for any use. None but a table of discount for every day, can be perfect; because every day's discount differs, being still less, as the number of days increase.

☞ If the number of days be a mixed one, resolve them into pure numbers.

LXIII. COMPOUND INTEREST.

WHAT compound interest is, I have already shewn in section XIX. page 121.

RULE. Multiply the principal by the amount of 1*l.* at the given rate per cent. as often as there are numbers of years; the last product is the amount, from which if you subtract the principal, the remainder will be the interest.

EXAMPLE

EXAMPLE 1. What is the compound intereſt of 221*l.* forborne 3 years, at 5 per cent. per annum?

	£.
Principal - - -	221
	1,05
	1105
	221
Firſt year's amount	232,05
	1,05
	116025
	23205
Second year's amt.	243,6525
	1,05
	12182625
	2436525
Third year's amt.	255,835125
Principal -	221
Intereſt - -	34,835125 =

(35*l.* 16*s.* $8\frac{1}{4}$*d.* Anſwer.

E. 2. What is the compound intereſt of 320*l.* forborne 4 years, at 5 per cent. per annum?

Principal - - -	320
	1,05
	1600
	320
Firſt year's amount -	336,00
	1,05
	1680
	336
Second year's amount	352,80
	1,05
	17640
	3528
Third year's amt.	370,440
	1,05
	185220
	37044
4th year's amt.	388,9620
Principal	320
Intereſt -	68,9620 = 68*l.*

(19*s.* $2\frac{3}{4}$*d.* Anſwer.

The amount of 1*l.* for one year, at any given rate, may be found by the following proportion, thus:

As $\begin{cases} 100 : 105 :: 1 : 1{,}05 \text{ the ratio at 5 per cent.} \\ 100 : 106 :: 1 : 1{,}06 \text{ the ratio at 6 per cent. \&c.} \end{cases}$

E. 3. What is the amount of 500*l.* for 6 years, at 5 per cent. per annum?

Principal - -	500
	1,05
	2500
	500
The firſt year's amount	525,00
	1,05
	2625
	525
Second year's amount	551,25
	1,05
	275625
	55125
Third year's amount	578,8125
	1,05
	28940625
	5788125
4th year's amount	607,753125
	1,05
	3038765625
	607753125
5th year's amt.	638,14078125
	1,05
	319070390625
	63814078125
6th y. amt.	670,0478203125 =

(670*l.* 0*s.* $11\frac{1}{4}$*d.* anſwer.

These

These examples being so easy to be understood, I shall omit giving any more, and proceed to shew the construction and use of a set of tables, which are absolutely necessary for those that do not understand logarithms or algebra; and as nothing shall be wanting in this system to make it complete, I have inserted the six following tables for the purposes of compound interest, and continued each table for 50 years; whereby any question of compound interest for the rates contained therein may be expeditiously answered.

The CONSTRUCTION of the following TABLES.

Table 1st is constructed by a continual multiplication of the amount of 1*l.* for a day, being the root of its amount for a year, extracted to the 365th power.

The amount of 1*l.* for a day, at 5 per cent. being 1,0001336, then 1,0001336 × 1,0001336 = 1,0002673, the amount for 2 days; and 1,0001336 × 1,0001336 × 1,0001336 = 1,0004011, the amount of 1*l.* at 5 per cent. for 3 days, compound interest, &c.

Table 2d is constructed by involving the amount of 1*l.* for a year, to the power of the number of years; thus, the amount of 1*l.* for 2 years, at 5 per cent. will be 1,05 × 1,05 = 1,1025; 1,05 × 1,05 × 1,05 = 1,157625 the amount of 1*l.* for 3 years, at 5 per cent. &c.

Table 3d is constructed thus, 1 ÷ 1,05 = ,952381, first year's present worth at 5 per cent. and ,952381 ÷ 1,05 = ,9070295, the second year's present worth; and ,90703 ÷ 1,05 = ,8638376, the third year's present worth; and after the same method are all the other years in the table found, to 50 years, inclusive.

The 4th table is constructed thus, take the first year's amount, which is 1*l.* and multiply it by 1,05 + 1 = 2,05 = the second year's amount, which also multiplied by 1,05 + 1 = 3,1525 = third year's amount, &c.

The 5th table is constructed thus, divide 1 by 1,05 = ,95238, the present worth for the first year, which ÷ 1,05 = ,90703, added to the first year's present worth = 1,85941, the second year's present worth; again ,90703 ÷ 1,05 and quotient, added to 1,85941 = 2,72324 = the third year's present worth, &c.

The 6th table is constructed thus, find the present worth of 1*l.* per annum in the 5th table, at the assigned rate and time, and divide unity or 1 thereby, the quotient will be the annuity that 1*l.* will purchase, at the same rate, for the same time.

TABLE

TABLE I. The amount of one pound for days.

Days.	3 per Cent.	3½ per Cent.	4 per Cent.	4½ per Cent.	5 per Cent.
1	1,0000809	1,0000942	1,0001074	1,0001206	1,0001336
2	1,0001619	1,0001885	1,0002149	1,0002412	1,0002673
3	1,0002429	1,0002827	1,0003224	1,0003618	1,0004011
4	1,0003240	1,0003770	1,0004299	1,0004824	1,0005348
5	1,0004050	1,0004713	1,0005374	1,0006031	1,0006685
6	1,0004860	1,0005656	1,0006449	1,0007238	1,0008023
7	1,0005670	1,0006600	1,0007524	1,0008445	1,0009361
8	1,0006480	1,0007542	1,0008600	1,0009652	1,0010699
9	1,0007291	1,0008486	1,0009675	1,0010859	1,0012037
10	1,0008101	1,0009429	1,0010751	1,0012066	1,0013376
20	1,0016209	1,0018867	1,0021513	1,0024148	1,0026770
30	1,0024324	1,0028315	1,0032288	1,0036243	1,0040182
40	1,0032445	1,0037771	1,0043074	1,0048354	1,0053611
50	1,0040573	1,0047236	1,0053871	1,0060479	1,0067059
60	1,0048708	1,0056710	1,0064680	1,0072618	1,0080525
70	1,0056849	1,0066193	1,0075501	1,0084773	1,0094009
80	1,0064996	1,0075685	1,0086333	1,0096942	1,0107511
90	1,0073151	1,0085186	1,0097177	1,0109125	1,0121031
100	1,0081311	1,0094696	1,0108033	1,0121324	1,0134569
110	1,0089479	1,0104214	1,0118900	1,0133537	1,0148125
120	1,0097653	1,0113742	1,0129779	1,0145765	1,0161699
130	1,0105834	1,0123279	1,0140670	1,0158027	1,0175291
140	1,0114021	1,0132825	1,0151572	1,0170265	1,0188902
150	1,0122215	1,0142379	1,0162487	1,0182537	1,0202531
160	1,0130415	1,0151943	1,0173412	1,0194824	1,0216178
170	1,0138623	1,0161516	1,0184350	1,0207126	1,0229843
180	1,0146837	1,0171098	1,0195299	1,0219442	1,0243527
190	1,0155057	1,0180689	1,0206261	1,0231774	1,0257228
200	1,0163284	1,0190288	1,0217233	1,0244120	1,0270949
210	1,0171518	1,0199897	1,0228218	1,0256481	1,0284687
220	1,0179759	1,0209515	1,0239215	1,0268858	1,0298444
230	1,0188006	1,0219142	1,0250233	1,0281249	1,0312219
240	1,0196260	1,0228778	1,0261243	1,0293655	1,0326013
250	1,0204520	1,0238424	1,0272275	1,0306076	1,0339825
260	1,0212788	1,0248078	1,0283319	1,0318512	1,0353656
270	1,0221062	1,0257741	1,0294375	1,0330963	1,0367505
280	1,0229342	1,0267414	1,0305443	1,0343429	1,0381373
290	1,0237630	1,0277096	1,0316522	1,0355910	1,0395259
300	1,0245924	1,0286786	1,0327614	1,0368406	1,0409164
310	1,0254225	1,0296486	1,0338717	1,0383917	1,0423087
320	1,0262532	1,0306195	1,0349832	1,0393444	1,0437029
330	1,0270847	1,0315914	1,0360960	1,0405985	1,0450990
340	1,0279168	1,0325641	1,0372099	1,0418542	1,0464969
350	1,0287495	1,0335378	1,0383250	1,0431114	1,0478967
360	1,0295830	1,0345123	1,0394413	1,0443700	1,0492984
361	1,0296664	1,0346098	1,0395530	1,0444960	1,0494387
362	1,0297497	1,0347073	1,0396648	1,0446220	1,0495790
363	1,0298331	1,0348049	1,0397765	1,0447479	1,0497193
364	1,0299165	1,0349024	1,0398882	1,0448739	1,0498596
365	1,0300000	1,0350000	1,0400000	1,0450000	1,0500000

TABLE

TABLE 2. The amount of one pound for years.

Years.	3 per Cent.	3½ per Cent.	4 per Cent.	4½ per Cent.	5 per Cent.
1	1,0300000	1,0350000	1,0400000	1,0450000	1,0500000
2	1,0609000	1,0712250	1.0816000	1,0920250	1,1025000
3	1.0927270	1,1087178	1,1248640	1,1411661	1,1576250
4	1,1255088	1,1475230	1,1698586	1,1925186	1,2155063
5	1,1592740	1,1876863	1,2166529	1,2461819	1,2762816
6	1,1940523	1,2292553	1,2653190	1,3022601	1,3400956
7	1,2298738	1,2722792	1.3159318	1.3608613	1,4071064
8	1,2667700	1,3168098	1,3685691	1.4221006	1,4774554
9	1,3047731	1,3628973	1,4233118	1,4860951	1,5513282
10	1,3439163	1,4105987	1,4802443	1,5529694	1,6288946
11	1,3842338	1,4599697	1,5394541	1,6228530	1,7103393
12	1,4257608	1,5110686	1,6010322	1,6958814	1,7958563
13	1,4685337	1,5639560	1,6650735	1,7721961	1,8156491
14	1,5125897	1,6186945	1,7316764	1,8519449	1,9799316
15	1,5579674	1,6753488	1,8009435	1,9352824	2,0789282
16	1,6047064	1,7339860	1,8729812	2,0223701	2.1828746
17	1,6528476	1,7946755	1,9479005	2,1133768	2,2920183
18	1,7024330	1,8574892	2.0258165	2,2084787	2,4066192
19	1,7535060	1,9225013	2,1068492	2,3078603	2,5269502
20	1,8061112	1,9897888	2,1911231	2.4117140	2,6532977
21	1,8602945	2,0594314	2,2787681	2.5202111	2,7859626
22	1,9161034	2,1315115	2,3699188	2,6336520	2.9252607
23	1,9735865	2,2061144	2,4647155	2,7521663	3,0715238
24	2,0327941	2,2833284	2,5633042	2.8760138	3,2251000
25	2,0937779	2,3632449	2,6658363	3.0054344	3,3863549
26	2,1565912	2,4459585	2,7724697	3,1406790	3,5556727
27	2,2212890	2,5315871	2,8833685	3,2820095	3,7334563
28	2,2879276	2,6201719	2,9987033	3,4296999	3,9201291
29	2,3565655	2,7118779	3,1186514	3,5840364	4,1161356
30	2,4272624	2,8067937	3,2433975	3,7453181	4,3219424
31	2,5000803	2,9050314	3,3731334	3,9138574	4,5380395
32	2,5750827	3,0067075	3.5080587	4,0899810	4,7649415
33	2,6523352	3,1119423	3.6483811	4,2740301	5,0031885
34	2,7319053	3,2208603	3.7943163	4,4663615	5,2533480
35	2,8138624	3,3335904	3,9460889	4,6673478	5,5160154
36	2,8982783	3,4502661	4,1039325	4,8773784	5,7918161
37	2,9852266	3,5710254	4,2680898	5,0968604	6,0814069
38	3,0747834	3,6960113	4.4388134	5.3262192	6,3854773
39	3,1670269	3,8253717	4,6163659	5,5658990	6,7047511
40	3,2620377	3,9592597	4,8010206	5,8163645	7,0399887
41	3,3598989	4,0978338	4,9930614	6,0781009	7,3919881
42	3,4606958	4,2412579	5,1927839	6,3516154	7,7615875
43	3,5645167	4,3897020	5,4004952	6,6374381	8,1496669
44	3,6714522	4,5433416	5,6165150	6,9361229	8,5571503
45	3,7815958	4,7023585	5,8411756	7,2482484	8,9850078
46	3,8950437	4,8669411	6,0748227	7,5744196	9,4342582
47	4,0118950	5,0372840	6,3178156	7,9152684	9,9059711
48	4,1322518	5,2135889	6,5705282	8,2714555	10,4012696
49	4,2562194	5,3960645	6,8333493	8,6436710	10,9213331
50	4,3839060	5,5849268	7,1066833	9,0326362	11,4674000

TABLE 3. The present worth of one pound for years.

Years.	3 per Cent.	3½ per Cent.	4 per Cent.	4½ per Cent.	5 per Cent.
1	,9708738	,9661836	,9615385	,9569378	,9523810
2	,9425959	,9335107	,9245562	,9157299	,9070295
3	,9151417	,9019427	,8889964	,8762966	,8638376
4	,8884870	,8714422	,8548042	,8385613	,8227025
5	,8626088	,8419732	,8219271	,8024511	,7835262
6	,8374843	,8135006	,7903145	,7678957	,7462154
7	,8130915	,7859910	,7599178	,7348285	,7106813
8	,7894092	,7594116	,7306902	,7031851	,6768394
9	,7664167	,7337710	,7025867	,6729044	,6446089
10	,7440939	,7089188	,6755642	,6439277	,6139133
11	,7224213	,6849457	,6495809	,6161987	,5846793
12	,7013799	,6617833	,6245971	,5896639	,5568374
13	,6809513	,6394041	,6005741	,5642716	,5303214
14	,6611178	,6177818	,5774751	,5399729	,5050679
15	,6418619	,5968906	,5552645	,5167204	,4810171
16	,6231669	,5767059	,5339082	,4944693	,4581115
17	,6050164	,5572038	,5133733	,4731764	,4362967
18	,5873946	,5383611	,4936281	,4528004	,4155207
19	,5702860	,5201557	,4746424	,4333018	,3957340
20	,5536758	,5025659	,4563870	,4146429	,3768895
21	,5375493	,4855709	,4388336	,3967874	,3589424
22	,5218925	,4691506	,4219554	,3797009	,3418499
23	,5066917	,4532856	,4057263	,3633501	,3255713
24	,4919337	,4379571	,3901215	,3477035	,3100679
25	,4776056	,4231470	,3751168	,3327306	,2953028
26	,4636947	,4088378	,3606892	,3184025	,2812407
27	,4501891	,3950123	,3468166	,3046914	,2678483
28	,4370768	,3816543	,3334775	,2915707	,2550936
29	,4243464	,3687482	,3206514	,2790150	,2429463
30	,4119868	,3562784	,3083187	,2670000	,2313775
31	,3999871	,3442304	,2964603	,2555024	,2203595
32	,3883370	,3325897	,2850579	,2444999	,2098662
33	,3770263	,3213427	,2740942	,2339712	,1998762
34	,3660449	,3104761	,2635521	,2238959	,1903548
35	,3553834	,2999769	,2534155	,2142544	,1812903
36	,3450324	,2898327	,2436687	,2050282	,1726574
37	,3349829	,2800316	,2342969	,1961992	,1644356
38	,3252262	,2705619	,2252854	,1877504	,1566054
39	,3157536	,2614125	,2166206	,1796655	,1491479
40	,3065568	,2525725	,2082890	,1719287	,1420457
41	,2976280	,2440314	,2002779	,1645251	,1352816
42	,2889592	,2357791	,1925749	,1574403	,1288396
43	,2805429	,2278059	,1851682	,1506605	,1227044
44	,2723718	,2201023	,1780464	,1441728	,1168613
45	,2644386	,2126594	,1711984	,1379644	,1112965
46	,2567365	,2054679	,1646139	,1320233	,1059967
47	,2492588	,1985197	,1582826	.1263381	,1009492
48	,2419988	,1918065	,1521948	,1208977	,0961421
49	,2349503	,1853202	,1463411	,1156916	,0915639
50	,2281071	,1790534	,1407126	,1107097	,0872037

TABLE

TABLE 4. The amount of one pound per annum, or annuity for years.

Years.	3 per Cent.	$3\frac{1}{2}$ per Cent.	4 per Cent.	$4\frac{1}{2}$ per Cent.	5 per Cent.
1	1,0000000	1,0000000	1,0000000	1,0000000	1,0000000
2	2,0300000	2,0350000	2,0400000	2,0450000	2,0500000
3	3,0909000	3,1062250	3,1216000	3,1370250	3,1525000
4	4,1836270	4,2149429	4,2464640	4,2781919	4,3101250
5	5,3091358	5,3624659	5,4163226	5,4707097	5,5256312
6	6,4684099	6,5501522	6,6329755	6,7168917	6,8019128
7	7,6624622	7,7794075	7,8982945	8,0191518	8,1420084
8	8,8923360	9,0516866	9,2142263	9,3800136	9,5491089
9	10,1591061	10,3684958	10,5827953	10,8021142	11,0265643
10	11,4638793	11,7313931	12,0061071	12,2882094	12,5778925
11	12,8077957	13,1419919	13,4863514	13,8411788	14,2067871
12	14,1920296	14,6019616	15,0258055	15,4640318	15,9171265
13	15,6177904	16,1130303	16,6268377	17,1599133	17,7129828
14	17,0863242	17,6769864	18,2919112	18,9321094	19,5986320
15	18,5989139	19,2956809	20,0235876	20,7840543	21,5785636
16	20,1568813	20,9710297	21,8245311	22,7193367	23,6574918
17	21,7615877	22,7050158	23,6975124	24,7417069	25,8403664
18	23,4144354	24,4996913	25,6454129	26,8550837	28,1323847
19	25,1168684	26,3571805	27,6712294	29,0635625	30,5390039
20	26,8703745	28,2796818	29,7780786	31,3714228	33,0659541
21	28,6764857	30,2694707	31,9692017	33,7831368	35,7192518
22	30,5367803	32,3289022	34,2479698	36,3033779	38,5052144
23	32,4528837	34,4604137	36,6178886	38,9370299	41,4304751
24	34,4264702	36,6665282	39,0826041	41,6891963	44,5019989
25	36,4592643	38,9498567	41,6459083	44,5652101	47,7270988
26	38,5530422	41,3131017	44,3117446	47,5706446	51,1134538
27	40,7096335	43,7590602	47,0842144	50,7113236	54,6691265
28	42,9309225	46,2906273	49,9675830	53,9933332	58,4025828
29	45,2188502	48,9107993	52,9662863	57,4230332	62,3227119
30	47,5754167	51,6226773	56,0849377	61,0070698	66,4388475
31	50,0026782	54,4294719	59,3283352	64,7523878	70,7607899
32	52,5027585	57,3345025	62,7014687	68,6662452	75,2988294
33	55,0778413	60,3412101	66,2095274	72,7562263	80,0637708
34	57,7301765	63,4531524	69,8579045	77,0302565	85,0669594
35	60,4620818	66,6740127	73,6522248	81,4966180	90,3203073
36	63,2759443	70,0076032	77,5983138	86,1639658	95,8363227
37	66,1742226	73,4578693	81,7022464	91,0413443	101,6281388
38	69,1594493	77,0288947	85,9703362	96,1382048	107,7095458
39	72,2342327	80,7249060	90,4091497	101,4644249	114,0950231
40	75,4012597	84,5502778	95,0255157	107,0303231	120,7997742
41	78,6632975	88,5095375	99,8265363	112,8466876	127,8397829
42	82,0231964	92,6073713	104,8195978	118,9247885	135,2317511
43	85,4838923	96,8486293	110,0123817	125,2764040	142,9933786
44	89,0484191	101,2383313	115,4128169	131,9138422	151,1430056
45	92,7198614	105,7816729	121,0293920	138,8499651	159,7001559
46	96,5014172	110,4840315	126,8705677	146,0982135	168,6851637
47	100,3965000	115,3509726	132,9453904	153,6726331	178,1194218
48	104,4083960	120,3882566	139,2632060	161,5879016	188,0253929
49	108,5406479	125,6018456	145,8337342	169,8593572	198,4266626
50	112,7968673	130,9979102	152,6670836	178,5030282	209,3479957

TABLE 5. The present worth of 1*l.* per ann. or annuity for years.

Years.	3 per Cent.	$3\frac{1}{2}$ per Cent.	4 per Cent.	$4\frac{1}{2}$ per Cent.	5 per Cent.
1	0,9708738	0,9661836	0,9615385	0,9569378	0,9523809
2	1,9134697	1,8996943	1,8860947	1,8726076	1,8594103
3	2,8286114	2,8016379	2,7750910	2,7489644	2,7232480
4	3,7170984	3,6730792	3,6298952	3,5875257	3,5459505
5	4,5797072	4,5150524	4,4518223	4,3899707	4,3294767
6	5,4171914	5,3285530	5,2421369	5,1578725	5,0756921
7	6,2302829	6,1145439	6,0020547	5,8927009	5,7863734
8	7,0196922	6,8739555	6,7327448	6,5958861	6,4632128
9	7,7861089	7,6076865	7,4353314	7,2687905	7,1078217
10	8,5302028	8,3166053	8,1108955	7,9127182	7,7217349
11	9,5256241	9,0015510	8,7604703	8,5289169	8,3064142
12	9,9540040	9,6633343	9,3850733	9,1185808	8,8632516
13	10,6349563	10,3027385	9,9856473	9,6828524	9,3925730
14	11,2960731	10,9205203	10,5631223	10,2228253	9,8986409
15	11,9379351	11,5174109	11,1183868	10,7395457	10,3796500
16	12,5611020	12,0941168	11,6522949	11,2340151	10,8377695
17	13,1661185	12,6513206	12,1656680	11,7071914	11,2740662
18	13,7535131	13,1896812	12,6592961	12,1599918	11,6895869
19	14,3237991	13,7098374	13,1339385	12,5932936	12,0853208
20	14,8774748	14,2124033	13,5903253	13,0079365	12,4622103
21	15,4150241	14,6979742	14,0291589	13,4047239	12,8211527
22	15,9369166	15,1671248	14,4511142	13,7844248	13,1630026
23	16,4436084	15,6204105	14,8568405	14,4954784	13,4885739
24	16,9355421	16,0583676	15,2469619	14,1477749	13,7986418
25	17,4131477	16,4815146	15,6220787	14,8282089	14,0939445
26	17,8768420	16,8903523	15,9827678	15,1466115	14,3751853
27	18,3270315	17,2853645	16,3295844	15,4513028	14,6430336
28	18,7641082	17,6670188	16,6630618	15,7428735	14,8981272
29	19,1884546	18,0357670	16,9837132	16,0218885	15,1410735
30	19,6004113	18,3920454	17,2920318	16,2888885	15,3724510
31	20,0004285	18,7362758	17,5884921	16,5443909	15,5928104
32	20,3887655	19,0688656	17,8735500	16,7888909	15,8026766
33	20,7657918	19,3902082	18,1476441	17,0228621	16,0025491
34	21,1318367	19,7006842	18,4111916	17,2467580	16,1929039
35	21,4872200	20,0006612	18,6646116	17,4610124	16,3741942
36	21,8322525	20,2904938	18,9082803	17,6660406	16,5468516
37	22,1672354	20,5705254	19,1425771	17,8622398	16,7112872
38	22,4924616	20,8410874	19,3678625	18,0499902	16,8678926
39	22,8082151	21,1024999	19,5844831	18,2296557	17,0170406
40	23,1147719	21,3550723	19,7927721	18,4015844	17,1590862
41	23,4123999	21,5991037	19,9930500	18,5661095	17,2943678
42	23,7013592	21,8348828	20,1856250	18,7235498	17,4232074
43	23,9819021	22,0626887	20,3707931	18,8742103	17,5459118
44	24,2542739	22,2827910	20,5488395	19,0183831	17,6627732
45	24,5187125	22,4954503	20,7200378	19,1563474	17,7740697
46	24,7754490	22,7009181	20,8846517	19,2883707	17,8800663
47	25,0247078	22,8994378	21,0429342	19,4147088	17,9610155
48	25,2667066	23,0912443	21,1951289	19,5356066	18,0771576
49	25,5016569	23,2765645	21,3414700	19,6512981	18,1681715
50	25,7297640	23,4556179	21,4821826	19,7620078	18,2559253

TABLE

TABLE 6. The annuity which 1*l.* will purchase for any number of years.

Years.	3 per Cent.	$3\frac{1}{2}$ per Cent.	4 per Cent.	$4\frac{1}{2}$ per Cent.	5 per Cent.
1	1,0300000	1,0350000	1,0400000	1,0450000	1,0500000
2	,5226108	,5264005	,5301961	,5339976	,5378049
3	,3535304	,3569342	,3603485	,3637734	,3672086
4	,2690271	,2722511	,2754901	,2787437	,2820118
5	,2183546	,2214814	,2246271	,2277910	,2309748
6	,1845975	,1876682	,1907619	,1938784	,1970157
7	,1605064	,1635445	,1666096	,1097015	,1728198
8	,1424564	,1454767	,1485279	,1516097	,1547218
9	,1284339	,1314460	,1344930	,1375745	,1406901
10	,1172305	,1202414	,1232909	,1263788	,1295046
11	,1080775	,1110920	,1141490	,1172482	,1203890
12	,1004621	,1034840	,1065522	,1096662	,1128254
13	,0940295	,0970616	,1001437	,1032754	,1064558
14	,0885263	,0915707	,0946690	,0978203	,1010240
15	,0837666	,0868251	,0899411	,0931138	,0963423
16	,0796109	,0826848	,0858200	,0890154	,0922699
17	,0759525	,0790431	,0821985	,0854176	,0886991
18	,0727087	,0758168	,0789933	,0822369	,0855462
19	,0698139	,0729403	,0761386	,0794073	,0827450
20	,0672157	,0703610	,0735818	,0768761	,0802426
21	,0648718	,0680366	,0712801	,0746006	,0779961
22	,0627474	,0659321	,0691988	,0725457	,0759705
23	,0608139	,0640188	,0673091	,0706825	,0741368
24	,0590474	,0622728	,0655868	,0689870	,0724709
25	,0574279	,0606740	,0640121	,0674390	,0709545
26	,0559383	,0592054	,0625674	,0660214	,0695643
27	,0545642	,0578524	,0612385	,0647195	,0682919
28	,0532932	,0566027	,0600130	,0635208	,0671225
29	,0521147	,0554454	,0588799	,0624146	,0660455
30	,0510193	,0543713	,0578301	,0613915	,0650514
31	,0499989	,0533724	,0568554	,0604435	,0641321
32	,0490466	,0524415	,0559486	,0595632	,0632804
33	,0481561	,0515724	,0551036	,0587445	,0624900
34	,0473220	,0507597	,0543148	,0579819	,0617554
35	,0465393	,0499984	,0535773	,0572705	,0610717
36	,0458038	,0492842	,0528869	,0566058	,0604345
37	,0451116	,0486133	,0522396	,0559840	,0598398
38	,0444593	,0479821	,0516319	,0554017	,0592842
39	,0438439	,0473878	,0510608	,0548557	,0587646
40	,0432624	,0468273	,0505235	,0543431	,0582782
41	,0427124	,0462982	,0500174	,0538616	,0578223
42	,0421917	,0457983	,0495402	,0534087	,0573947
43	,0416981	,0453254	,0490899	,0529824	,0569933
44	,0412299	,0448777	,0486645	,0525807	,0566163
45	,0407852	,0444534	,0482625	,0522020	,0562617
46	,0403625	,0440511	,0478821	,0518447	,0559282
47	,0399605	,0436692	,0475219	,0515073	,0556142
48	,0395778	,0433065	,0471807	,0511886	,0553184
49	,0392131	,0429617	,0468571	,0508872	,0550397
50	,0388655	,0426337	,0465502	,0506021	,0547767

The

The USE of the foregoing TABLES.

The use of all these tables depends on the following general

RULE. Multiply the tabular number which stands against the given number of days, or years, and under the given rate of interest, by the principal sum, and the product will be the answer to the question.

EXAMPLE 1. What will 246*l.* amount to, in 30 days, at 5 per cent. per annum? In the first table against 5 per cent. and against 30 days, stands 1,0040182 the tabular number
246

60241092
40160728
20080364

Answer - 246,9884772 = 246*l.* 19*s.* 9*d.*

E. 2. What will 246*l.* amount to, in 30 years, at 5 per cent. per annum? In table 2, against 30 years, at 5 per cent. is 4,3219424
246

259316544
172877696
86438848

Answer - 1063,1978304 = (1063*l.* 3*s.* 11¼*d.*

E. 3. What is the present worth of an annuity of 246*l.* to continue 30 years, at 5 per cent. per annum? In table 3, against 30 years, at 5 per cent. is ,2313775
246

13882650
9255100
4627550

Answer 56,9188650 = 56*l.* 18*s.* 4½*d.* the present worth.

E. 4. What is the amount of an annuity of 246*l.* per annum, forborne or unpaid 30 years, at 5 per cent. per annum? In table 4, against 30 years, at 5 per cent. is 66,4388475
246

3986330850
2657553900
1328776950

Answer - 16343,9564850 = 16343*l.* 19*s.* 1½*d.* (the amount required.

E. 5.

E. 5. What is the present worth of an annuity of 246*l.* to continue 30 years, at 5 per cent. per annum? In table 5, against 30 years at 5 per cent. is 15,372451

```
 15,372451
       246
 ---------
  92234706
 61489804
30744902
----------
3781,622946
```

Answer - 3781,622946 = 3781*l.* 12*s.* 5½*d.* the present worth.

E. 6. What is the annuity which 246*l.* will purchase, to continue 30 years, reckoning 5 per cent? In table 6, against 30 years, at 5 per cent. is ,0650514

```
  ,0650514
       246
  --------
   3903084
  2602056
 1301028
----------
16,0026444
```

Anf. 16,0026444 = 16*l.* 0*s.* 0¾*d.* the purchased annuity per annum.

If the amount of any sum be sought, for a number of days, which are not in the first table, and years which are not in the second; divide the given number of days or years into two such numbers as are in the table; then multiply the amount pertaining to each into each other, the product will be the amount for the time required.

E. 7. What will 523*l.* amount to, in 194 days, at 5 per cent. per annum?

In table 1, against 190 days, under 5 per cent. is -	1,0257228
And against 4 days, at the same rate, is - -	1,0005348
The product is the amount of 1*l.* for 194 days, viz.	1,0262714
Which multiply by the principal sum, viz. -	523
The product is the answer	536,7399840 = (536*l.* 14*s.* 9½*d.*

E. 8. What is the amount of 150*l.* in 91 years, at 5 per cent?

In table 2, against 50 years, under 5 per cent. is	11,4674000
And against 41 years, at 5 per cent. is - -	7,3919881
The product is the amount of 1*l.* for 91 years, viz.	84,7668833
Which multiply by the principal sum, viz. -	150
	12715,032495 = (12715*l.* 0*s.* 7½*d.*

Note. The other tables of compound interest, cannot be extended in this manner.

QUESTIONS

QUESTIONS for exercise, to shew the extensive use of the tables.

Q. 1. A person having 12 years to run in a lease of an estate of 60*l.* per annum, for 40 years, would know what present money he must pay, in order to complete the lease by adding 28 years thereto, computing at 5 per cent. compound interest? By table 5, the present value of 1*l.* per annum, at 5 per cent. for 40 years, is 17,1590862

By the same table, the value of 1*l.* per annum, at that rate, for 12 years to run, is - - } 8,8632516

Difference -	8,2958346
Multiply by -	60
Answer	£. 497,7500760

Q. 2. Which is the most advantageous, a term of 15 years in an estate of 100*l.* per annum; or the reversion of such an estate for ever, after the expiration of the said 15 years, computing at the rate of 5 per cent. per annum, compound interest?

A Freehold estate of 100*l.* per annum, at 5 per cent. is worth - - . } £. 2000

In table 5, the present value of the same estate, at the same rate, for 15 years, is } 1037,965

The difference is - 962,035 val. of rever.

Hence it appears that the first term of 15 years is better than the reversion for ever afterwards, by 75,930 = 75*l.* 18*s.* 7*d.* the answer.

Q. 3. What annuity, to continue 14 years, may be purchased with 1000*l.* due at the end of 5 years; the annuity to commence presently, at 5 per cent? By table 3, the present worth of 1000*l.* due 5 years hence, at 5 per cent. may be found equal to 783,5262; and by table 6, it may be found that the annuity which 783,5262 will purchase, for 14 years, at the rate of 5 per cent. is 79,1518 = 79*l.* 3*s.* 0¼*d.* per annum, the answer.

Q. 4. For a lease of certain profits for 7 years, A offers to pay 150*l.* gratuity, and 300*l.* per annum; B offers 400*l.* gratuity, and 250*l.* per annum; C bids 650*l.* gratuity, purchase without any yearly rent; query, which is the best offer, and what is the difference, computing at 5 per cent? By table 5, the present worth of 300*l.* per annum, for 7 years, at 4 per cent. is — 1800,6164

To which add —	150
The value of A's offer —	1950,6164

The present worth of 250*l.* per annum, for 7 years	1500,5136
Add — —	400
The value of B's offer —	1900,5136

The

The present worth of 200*l*. per annum, for 7 years	1200,4109
Add —— ——	650,
The value of C's offer —	1850,4109
D's offer — —	1800

Hence it appears that A's is the best offer; and that rejecting the decimals, he bids 50*l*. more than B, 100*l*. more than C, and 150*l*. more than D.

Q. 5. What annuity is sufficient to pay off a debt of 50 millions, in 30 years, at 4*l*. per cent. compound interest?

In table 6, against 30 years, under 4 per cent. is -	,0578301
Which multiply by the debt —— ——	50000000
The annuity sought —— ——	£. 2891505
So that supposing the national debt to be 50 millions, the interest at 4 per cent. would be ——	2000000

Then it would require a sinking fund of 2891505*l*. per annum to clear the whole debt in 30 years.

Q. 6. A son previous to his marriage, is minded to have 50*l*. a year freehold settled on his family; and to have immediate possession of it, offers his father in lieu an annuity for his life, valued at 12 years purchase, discounting at 4 per cent. thereon; whereas he is content the estate should be valued at a discount of 3 per cent. and consequently will be worth $33\frac{1}{3}$ years purchase; pray what had the father for his life?

First $33,3 \times 50 = 1666,6 = 1666$*l*. 13*s*. 4*d*. nearly the value of the annuity. Then per table 6, 1*l*. for 12 years, at 4 per cent. will purchase ,1065522 per annum.

$\because$ $1666,6 \times ,1065522 = 177,587 = 177$*l*. 11*s*. $8\frac{3}{4}$*d*. the answer.

LXIV. Concerning DIVISORS.

IT being often necessary in arithmetical calculations, to find such multipliers, or numbers, which may be divided by any number of given divisors, without any remainder, or remainders; by which means many pleasant questions, not reducible to any other rule in common arithmetic, may be solved.

To find the least number that can be divided by any number of divisors, with a remainder,

RULE. Multiply all the prime numbers, and the root of such as are square or cube numbers, continually; the product will be the number required.

Note, A prime number is such as hath no measure but itself and unity, and consequently cannot be produced by the multiplication of two or more integers; as, 1, 2, 3, 5, 7, 11, &c. are prime numbers.

Composite numbers are such as are divisible by some numbers besides unity; as 8 is divisible by 4 and 2, &c.

 A number

A number that will divide feveral numbers exactly, is called a common meafure, as 3 is a common meafure to 12 and 15.

EXAMPLE 1. Required the three leaft numbers, which divided by 20, fhall leave 19 for a remainder; but if divided by 19, fhall leave 18; if divided by 18, fhall leave 17; and fo on, always leaving one lefs than the divifor, to unity?

Firft 1, 2, 3, 5, 7, 11, 13, 17, and 19, are prime numbers:

Alfo $\sqrt[2]{4} = 2$, $\sqrt[3]{8} = 2$, $\sqrt[2]{9} = 3$, and $\sqrt[4]{16} = 2$, and all the reft are compofite numbers. Therefore,

$1 \times 2 \times 3 \times 2 \times 5 \times 7 \times 2 \times 3 \times 11 \times 13 \times 2 \times 17 \times 19 =$ 232792560, the leaft number that can be divided by the given divifors without a remainder.

Then the number $232792560 - 1 = 232792559$, the firft number.

And $\overline{232792560 \times 2} - 1 = 465585119$, the fecond number.

Alfo $\overline{232792560 \times 3} - 1 = 698377679$, the third number.

And after this manner may the other numbers be found.

E. 2. What is the leaft number that can be divided by the nine digits, without a remainder?

The given divifors are 1, 2, 3, 4, 5, 6, 7, 8, 9.

Now $\sqrt{4} = 2$; 6 may be cancelled, being compofed of 2×3; and 3, 5 and 7 are prime numbers; and $\sqrt[3]{8} = 2$. Alfo $\sqrt[2]{9} = 3$.

Then per rule, $1 \times 2 \times 3 \times 2 \times 5 \times 7 \times 2 \times 3 = 2520$, the number required.

E. 3. Required the leaft number which being divided by 7, 6, 5, 4, 3 and 2, fhall leave 6, 5, 4, 3, 2 and 1 refpectively?

Firft the divifors are 7, 6, 5, 4, 3, 2.

Now $\sqrt[2]{4} = 2$, and 3, 5 and 7 are prime numbers, 6 may be cancelled, being a compofite number.

Then per rule, $2 \times 3 \times 2 \times 5 \times 7 = 420$, the leaft number that can be divided by the given divifors without a remainder. Therefore $420 - 1 = 419$, the number required.

E. 4. John the gardener counting fome apples into a bafket, found that when he counted them in by two at a time, three at a time, and four at a time, there remained one; but when he counted them in by five at a time, there remained none; query, the number of apples?

Firft 2, 3 and 4, are the divifors; now 2 and 3 are prime numbers, and $\sqrt[2]{4} = 2$.

Then per rule, $2 \times 3 \times 2 = 12$; then $12 + 1 = 13$, which divided by 2, 3 and 4, leaves 1, according to the queftion; but divided by 5 will leave 3, which is 2 fhort of 5. ∴ To twice 12 add 1, and the fum will be 25, the number fought.

E. 5.

E. 5. *A country girl to town did go,*
Some walnuts there to sell;
A gentleman she chanc'd to meet,
And thus it her befell:
My pretty maid, says he to her,
What number have you here?
I can't tell, Sir, says she to him,
But this I'll make appear;

I told them o'er, e'er I came out,
By six's, fives, four's, three's, two's,
And every time I numbered them,
One remained overplus:
I told them o'er by seven's at last,
And there were no remains;
If you can find the number out,
Pray take it for your pains.

First, the least number that can be divided by 1, 2, 3, 4, 5, 6, without a remainder, will (per rule) be $1 \times 2 \times 3 \times 2 \times 5 = 60$. Then $60 + 1 = 61$, which divided by 2, 3, 4, 5, 6, will leave 1 according to the question; but divided by 7, will leave 5; $\because \overline{60 \times 5} + 1 = 301$, the least number which admits of the conditions of the question. Then to find the next least number which admits of the same conditions, by proceeding as above we shall find to be $\overline{60 \times 12} + 1 = 721$.

Also $721 - 301 = 420$, the common difference of all the numbers answering the conditions of the question. Therefore 301, 721, 1141, 1561, 1981, &c. ad infinitum, will answer the conditions of this question.

LXV. DUODECIMALS;

OR,

CROSS MULTIPLICATION.

THIS rule is called duodecimals, because the unit, or integer, is divided into 12 equal parts; and hence this way of computation is chiefly used amongst workmen in casting up the contents of superficial and solid works, the lineary dimensions being generally taken in feet, inches, and parts.

RULE. 1. Under the multiplicand, write the correspondent denominations of the multiplier.

2. Multiply each term in the multiplicand, beginning with the lowest, by the feet in the multiplier; placing each result under its respective term, remembering to carry an unit for every 12 from each lower denomination to its next superior.

3. Work in the same manner with the inches and parts, setting the result of each term one place more to the right-hand; having thus finished multiplication, the sum of all will be the product required.

Note. In multiplying feet, inches, and parts; if feet be multiplied by feet, the product is feet; and feet multiplied by inches, the product is inches; and parts multiplied by feet, the product is parts; parts multiplied by inches, the product is seconds; and parts multiplied by parts, the product is thirds.

EXAMPLE 1. What is the product of 8 feet 9 inches and 6 parts, by 5 feet 6 inches and 3 parts?

By the rule, First method.

	F.	I.	P.		
	8	9	6		
	5	6	3		
	43	11	6		
	4	4	9	0	
		2	2	4	6
Answer	48	6	5	4	6

Second method.

	8	9	6		
	5	6	3		
		2	2	4	6
	4	4	9	0	
	43	11	6		
Answer	48	6	5	4	6 as before

Third method.

	8	9	6		
	5	6	3		
	40	45	30		
		48	54	36	
			24	27	18
Answ.	48	6	5	4	6 as before

Fourth method.

	F.	I.	P.		
	8	9	6		
	5	6	3		
	40				
	3	9			
	0	2	6		
	4	0	0		
	0	4	6		
	0	0	3		
	0	2	0		
	0	0	2	3	
	0	0	0	1	6
Answer	48	6	5	4	6 as bef.

Fifth method, by practice.

In.	F.	I.	P.		
$4 = \frac{1}{3}$	8	9	6		
			5		
	43	11	6		
$2 = \frac{1}{2}$	2	11	2		
$3'' = \frac{1}{8}$	1	5	7		
	0	2	2	4	6
Answer	48	6	5	4	6

The above example is worked by five different methods, to shew the conciseness of each.

E. 2. Multiply 12 feet 8 inches by 5 feet?

	F.	I.
	12	8
	5	
Answer -	63	4

E. 3. Multiply 97 feet 8 inches by 8 feet 9 inches?

	F.	I.	
	97	8	
	8	9	
	781	4	
	73	3	0
Answer	854	7	0

E. 4. Multiply 57 feet 3 inches by 28 feet 6 inches?

	57	3	
	28	6	
	456		
	114		
	28	7	6
	7	0	0
Answer	1631	7	6

Note.

Note. As this kind of arithmetic is useful to persons concerned in building, measuring, &c. I thought proper to insert a few promiscuous examples, with an intention to give them a clear insight into this useful rule.

QUESTIONS for EXERCISE.

Q. 1. If a floor be 53 feet 6 inches long, and 47 feet 9 inches broad, how many squares are contained in that floor?

	F.	I.	
	53	6	
	47	9	
	371		
	212		
	40	1	6
	23	6	0
Answer	25,54	7	6 = 25 squares, 54 feet, 7 in. 6 parts.

Note. The reason of cutting off two figures, is, because there are 100 square feet in one square rod of 10 feet long, which is the same as dividing by 100.

Q. 2. If a house within the walls, be 44 feet 6 inches long, and 18 feet 3 inches broad; how many squares of roofing will cover that house?

	F.	I.	
	44	6	
	18	3	
	352		
	44		
	11	1	6
	9	0	0
Add {	812	1	6 Flat *
	406	0	0 Half-flat
Ans.	12,18	1	6 = 12 sq. 18 ft.

Q. 3. If a pane of glass be 4 feet $8\frac{3}{4}$ inches long, and one foot 4 inches and one quarter broad, how many feet of glass does it contain?

	F.	I.	P.		
	4	8	9		
	1	4	3		
	4	8	9		
	1	6	11	0	
		1	2	2	3
Answer	6	4	10	2	3

Q. 4. If there be 8 panes of glass, each 4 feet 7 inches and three-quarters long, and one foot $5\frac{1}{4}$ inches broad; how many feet of glass are contained in the said 8 panes?

	F.	I.	P.		
	4	7	9		
	1	5	3		
	4	7	9		
	1	11	2	9	
		1	1	11	3
	6	8	1	8	3
					8 No. of panes.
Answ.	53	5	1	6	0

* In tyling and roofing, it is customary to reckon the flat and half-flat of any building within the walls, to be the depth or width of the roof of that building, when the said roof is a true pitch; that is, when the rafters are three-fourths of the breadth of the building.

Q. 5.

Q. 5. If there are 16 panes of glass, each 4 feet $5\frac{1}{2}$ inches long, and 1 foot $4\frac{3}{4}$ inches broad; how many feet of glass are contained in them?

F.	I.	P.		
4	5	6		
1	4	9		
4	5	6		
1	5	10	0	
	3	4	1	6
6	2	8	1	6
				4 × 4 = 16
24	10	8	6	0
				4
Answ. 99	6	10	0	0

Q. 6. If a room be painted, whose height being girt over the mouldings, is 16 feet 6 inches, and the compass of the room be 67 feet 9 inches, how many yards are there in that room.

F.	I.	
67	9	
16	6	
414	0	
67		
33	10	6
9)1117	10	6
Answer 124	1 = 124 yards 1 ft.	

Note. The inches and parts in this kind of measure are generally neglected.

Q. 7. If a room of wainscot be 16 feet 3 inches high, girt over the mouldings, and the compass of the room is 137 feet 6 inches, how many yards does it contain?

	F.	I.	
	137	6	
	16	3	
	830	0	
	137		
	34	4	6
The sq. feet in 1 yd. = 9)2234		4	6
Answer	248	2	4 6 = 248 *yds.* 2 *ft.* 4 *in.* 6 *pts.*

Q. 8. If a piece of timber be 21 inches broad, 21 inches deep, and 15 feet long, how many solid feet are contained therein?

F.	I.		
1	9	Breadth	
1	9	Depth	
1	9		
1	3	9	
3	0	9	
		5 × 3 = 15 Length	
15	3	9	
		3	
Ans. 45	11	3 Solid content.	

Q. 9. If a piece of timber be 25 inches broad, 7 inches deep, and 25 feet long; how many solid feet are contained therein?

F.	I.		
2	1	Breadth	
0	7	Depth	
1	2	7	
		5 × 5 = 25 Length	
6	0	11	
		5	
Ans. 30	4	7 Solid content.	

GEO-

GEOMETRY.

PART IV.

GEOMETRY originally ſignifies the art of meaſuring the earth, or any diſtances or dimenſions thereon. But now it is uſed for the ſcience of quantity, abſtractedly conſidered, without any regard to matter.

It very probably had its firſt riſe in Egypt, where the Nile annually overflowing the country, and covering it with mud, obliged men to diſtinguiſh their lands one from another, by the conſideration of their figure; and to be able alſo to meaſure the quantity of it, and to know how to plot it, and to lay it out again in its juſt dimenſions, figure and proportion: after which it is likely a further contemplation of thoſe draughts and figures helped them to diſcover many excellent properties belonging to them, which ſpeculation has continually been improving to this day.

Before I proceed, I ſhall firſt explain the following uſeful terms.

1. *Axiom*, is a principle in any art, ſo evident, that it needs nothing but the light of reaſon to demonſtrate it.

2. *Conſtruction*, is the drawing of lines, and framing of figures, or preparing the propoſition for a demonſtration.

3. *Corollary*, is a conſequent truth gained from a preceding demonſtration.

4. *Definition*, is the unfolding or explicating of the nature and affection of a thing in a few words.

5. *Demonſtration*, is the proving of a thing by definitions and axioms, and ſo from ſeveral arguments drawing a concluſion, that it has that affection the propoſition did aſſert.

6. *Hypotheſis*, is when a thing is ſuppoſed, or given to be ſo.

7. *Lemma*, is the demonſtration of ſome premiſe, in order to ſhorten a following demonſtration.

8. *Problem*, is when ſomething is propoſed to be done.

9. *Propoſition*, is uſed promiſcuouſly, either for a problem or theorem.

10. *Poſtulate*, is a grantable requeſt, or ſuch a demand as cannot reaſonably be denied.

11. *Scholium*, is a ſhort critical expoſition, gained from a former demonſtration.

12. *Theorem*, is when ſomething is propoſed to be done.

GEO-

GEOMETRICAL DEFINITIONS.

1. A Geometrical Point is ſo infinitely ſmall, as to be void of length, breadth, and thickneſs; and therefore a point may be ſaid to have no parts.

2. A Line, is called a quantity of one dimenſion, becauſe it may have any ſuppoſed length, but no breadth nor thickneſs.

3. A Superfices, is a figure which hath length and breadth, and is bounded by lines either ſtraight or circular.

4. All three-ſided figures are called Triangles, but admit of ſeveral diſtinctions; as an Equilateral, when the ſides are equal: Iſoſceles, when only two ſides are equal: Scalene, when the three ſides are unequal; and Right-angled, when it has one right angle.

5. All four-ſided figures are called Quadrilaterals, but are divided into ſquares, parallelograms, rhombus's, and rhomboides. A Square is that where all the angles are right, and the lines equal: a Parallelogram, or oblong ſquare, is a figure that hath all its angles right, and its two oppoſite ſides equal: a Rhombus, is that which hath its four ſides equal, but no right angle.

6. A Circle, is a plane bounded by one curved line, called the circumference, to which all right lines drawn from a certain point within the figure, called its center, are equal.

7. The Diameter of a circle, is a right line drawn through the center, terminated at each end by the circumference, and divides the circle into two equal parts, each of which is called a ſemi-circle; half the diameter is called the Radius.

8. The circumference of every circle is divided into 360 equal parts called Degrees; each degree into 60 equal parts, called Minutes; and each minute into 60 equal parts, called Seconds, &c. Any part of the circumference is called an Arch.

9. The Chord of an arch, is a right line joining the extremities of an arch, and by which the circles are divided into two unequal parts, called Segments.

10. A Sector, is a figure comprehended under two radiuſes of a circle, and the arch included between thoſe radiuſes.

11. A Polygon, is a figure contained under ſeveral ſides; and called a regular polygon, if the ſides and angles are regular amongſt themſelves, but if they are not, it is called an irregular polygon.

A polygon has different names, according to its number of ſides, viz. if it has five ſides, it is called a pentagon; if ſix, a hexagon; if ſeven, a heptagon; if eight, an octagon; if nine, a nonagon; if ten, a decagon; if eleven, an undecagon; and if 12, a duodecagon.

12. The Altitude, or height of any figure, is the perpendicular, let fall from its ſummit to its baſe, or line on which the figure is ſuppoſed to ſtand.

13. The Area of any figure, is the ſuperficial content of it.

LXVI. PROBLEMS.

PROB. 1. *Upon a given right line,* A B, *to erect a perpendicular.* Plate 1st, fig. 1.

1. ON each side of the point D, take any two equal distances, D *e* and D *n*.

2. From *e* and *n*, with any radius greater than D *e* or D *n*, describe the two arches cutting each other in *c*.

3. Through the points D, *c*, draw the line D *c*, and it will be the perpendicular required.

Prob. 2. *From a given point* C, *above the given line* A B, *to let fall a perpendicular* C D. Fig. 2.

From the point C, with any radius, describe the arch *a c*, intersecting A B in *a c*; from the points *a* and *c*, with the same radius, describe two arches cutting each other in *h*; lay a ruler from C to *h*, and draw C D, and it will be the perpendicular required.

Prob. 3. *To divide a given line* A B, *into two equal parts.* Fig. 3.

From the points A and B, with any distance greater than half A B, describe the two arches cutting each other in *a* and *c*; through the points *a* and *c* draw the line *a c*, and it will divide the line A B as required.

Prob. 4. *To erect a perpendicular on the extremity* A, *of a given right line* A B. Fig. 4.

From the point A describe the arch *a d*; and with the same opening of the compasses, from *a* make the intersection *b*, and on *b*, the intersection *c*; then from *b* and *c* make the intersection *e*, and draw *e* A, the perpendicular required.

Another method, Fig. 5. Take any point *e*, and with the distance *e* C, describe the arch *m* C *n*, cutting A C in *m* and C; through the center *e*, and the point *m*, draw the line *m e n*, cutting the arch *m* C *n* in *n*; then through the points *n* C, draw the line *n* C, and it will be the perpendicular required.

Prob. 5. *To divide an angle* A B C, *into two equal parts.* Fig. 6.

From the point B, with any radius, describe the arch *a b* cutting the sides in *a* and *b*; on which points, with the same radius, describe the arches cutting each other in *e*; then draw the line B *e*, and it will bisect the angle, as required.

Prob. 6. *At the end* B *of a given right line* A B, *to make an angle equal to a given angle* C D G. Fig. 7.

From the angular point D, describe at pleasure the arch *a b*; and with the same radius upon the point B, describe the arch *c d*, on which make *c e* = *a b*, and through the points B, *e*, draw the line E B, and it will make the angle A B E = C D G.

Prob. 7. *To find the center of a circle.* Fig. 8.

Draw any chord A B, and bisect it with the chord C D; then bisect C D with the chord E F, and their intersection O, will be the center required.

Prob. 8. *To bring three points, not lying in a straight line, into the circumference of a circle.* Fig. 9.

Let A, B, and C, be the three points; upon A and B, with the same radius, make the intersections *a* and *b*, and draw the line *a b*: on the points B and C, make the intersections *d*, *e*, and draw *d e*, and it will intersect *a b* in I, the center of the circle, that runneth upon the three given points.

Note. By this problem may the center to any arch, or circle, be found.

Prob. 9. *To draw a tangent to a given circle, when the point* A *is without the circle.* Fig. 10.

From the center O, draw O A, and bisect it in *a*; and from the point *a*, with the radius *a* A, or *a* O, describe the semi-circle A B O, cutting the given circle in B; then through the points A and B, draw the line A B, and it will be the tangent required.

Prob. 10. *Between two given right lines* A *and* B, *to find a mean proportional.* Fig. 11.

Draw any right line, in which take *c b* equal to A, and *b a* equal to B; bisect *a e* in *o*, and with *o a* or *o e*, as radius, describe the semi-circle *a d e*; then from the point *b*, draw *b d* perpendicular to *a e*, and it will be the mean proportional required.

Prob. 11. *Upon a given right line* A B, *to make an equilateral triangle.* Fig. 12.

From the points A and B, with a radius equal to A B, describe arches cutting in C; then draw A C and B C, and the figure A C B is the triangle required.

Note. We have a problem directing us how to draw parallel lines, but now we have a parallel ruler, which solves this problem with accuracy and expedition; I would, therefore, advise the practitioner to make use of that instrument, before the problem.

Prob. 12. *Upon a given right line* A B, *to describe a square.* Fig. 13.

On the point B, erect the perpendicular B C = A B; with the extent A B on the points A C, describe the arches intersecting in D; draw A D and C D, and it is done.

Prob. 13. *To inscribe a circle in a given triangle* A B C. Fig. 14.

Bisect any two of the angles, as A and B, with the right lines A D and B D, meeting each other in D; then from the point of intersection D, let fall the perpendicular D E, and it will be the radius of the circle required.

Prob.

Prob. 14. *To make a triangle, whose three sides shall be equal to three given lines*, A, B, C. Fig. 15.

Draw a line D E, equal to the line A; and on the point D, with a radius equal to B, describe an arch; then on the point E, with a radius equal to C, describe another arch, cutting the former in F; lastly, draw the lines D F and E F, and D F E will be the triangle required.

Prob. 15. *To make an angle of any proposed number of degrees.* Fig. 16.

Take the first 60 degrees from the scale of chords, and from the point A; with this radius describe the arch *a b*, and take the chord of the proposed number of degrees from the same scale, and apply it from *a* to *b*; then from the point A, draw the lines A *a* and A *b*, and they will form the angle required. In this example $ab = 60^\circ$.

Note. Angles greater than 90°, are usually taken at twice.

Prob. 16. *About a given triangle* A B C, *to circumscribe a circle.* Fig. 17.

Bisect the two sides A B and B C, with the perpendiculars *n o* and *b o*, then from the point of intersection *o*, with the distance *o* A or *o* B, describe the circle A C B, and it is done.

Prob. 17. *On a given line* A B, *to make a regular pentagon.* Fig. 18.

On the points A and B, with the distance A B, describe two circles cutting each other in *m* and *n*; draw the line *m n*, and from the point *n*, with the same radius as before, describe the arch *r* A *o* B *s*, cutting the two circles in the points *r* and *s*, and the line *m n* in the point *o*; draw the lines *r o* and *s o*, and produce them till they meet the circumferences in E and C; then from the points E and C, with the radius A B, describe arches crossing in D. Lastly, join the points A E, E D, D C, and C B, and A E D C B will be the pentagon required.

Prob. 18. *To draw a helex, or spiral line, with a pair of compasses.* Fig. 19.

Let the two centers be *a* and *o*, through which draw a right line what length you please, set one foot of the compasses in *a*, and extend the other to *o*, and draw the first semi-circle; remove that point of the compasses from *a* to *o*, and extend the other to join the semi-circle now drawn, and draw another semi-circle; remove the point of the compasses from *o* again to *a*, and extend the other point to the last semi-circle, and join it, and draw another semi-circle; do thus as long as you please, and you will have a spiral line, rolling in several circles, as per figure.

Prob. 19. *To reduce a circle to a square.* Fig. 20.

Divide the diameter A B, into 14 equal parts, and at 11 of those parts erect the perpendicular C D, and draw A D, so is A D the side of the square, nearly equal in content to the given circle.

Note. This problem is grounded upon *Archimedes's* proportion of the diameter of a circle to the circumference, being as 7 to 22; and although this proportion is not true, yet it is the nearest in whole numbers, and may serve very well for common purposes.

Prob. 20. *To reduce a square to a circle.* Fig. 21.

Divide the side of the given square into 11 equal parts; at 5½ of those parts, draw the semi-circle A B C, and at 8 of those parts, on the side of the square, erect the perpendicular D B, draw A B continued to the side of the square at E, so is A E the diameter of the circle, nearly equal in content to the given square.

Prob. 21. *Two points within a circle being given, to describe the arch of another circle, which shall pass through those two points, and also divide the circumference of the given circle into two equal parts.* Plate 1. Fig. 22.

Let the two points be *e* and *c*, within the circle. First, through either of them (as through *e*) draw the right-line *e* D, passing through the center of the circle at O. Then at right angles thereto, draw the line A C. Lastly, draw the line *e* A, and upon the point A, erect the perpendicular A G, cutting the line B D (produced) in the point G; so have you three points, *e*, *c*, G, through which (by probl. 8th) you may draw the arch P *e c* N G, whose center will be at *k*. Now, if you lay a rule upon the points P and N, and it passes over the center of the given circle at O, the circle is truly drawn.

Prob. 22. *To divide a circle into any number of equal parts.* Fig. 23.

1. Draw a circle of any radius, and draw the diameter A B; this divides the circle into two equal parts.

2. Erect the perpendicular F C, and that is the side of an hexagon, or the sixth part of the circle = A D.

3. Set F C from A to D, and from D to E, draw A E, for the side of an equilateral triangle.

4. Draw A C for the side of the square inscribed, or the fourth part of the circle.

5. Bisect F B in G, and draw C G; make G H = G C, and draw C H for the side of the pentagon, or fifth part of the circle.

6. Join E G for the side of an heptagon, or one-seventh part of the circle.

7. Bisect the arch A C in I, and draw A I for the side of an octagon, or one eighth part of the circle.

8. Divide the arch A D E, into three equal parts, in K, and draw A K, for the ninth part of the circle.

9. The line H F, is the side of a decagon, or a ten-sided figure.

10. The line F L is the endecagon, or eleventh-sided figure.

11. The line D C, is the twelfth part of the circle; and by doubling and tripling these lines, the circle may be geometrically divided into any number of equal parts at pleasure.

Prob. 23. *In a given circle, to inscribe any regular polygon.* Fig. 24.

1. Draw the diameter A B, on which make the equilateral triangle, A C B.

2. Divide the diameter A B into as many equal parts as the required polygon has sides.

3. From

3. From the point C, through the second division of the diameter, draw the line C D.

4. Join the points A and D, and the line A D will be the side of the polygon required; (in this construction A D is the side of a heptagon) and so of any other.

Note. This construction is the invention of *Renaldinus*. See his second book, *De Resol. &c. Comp. Mathem.* page 367.

Prob. 24. *To draw an oval, by having the two diameters given.* Fig. 25.

Divide each diameter into four equal parts, and through those parts draw the lines *a b c d*, then set one foot of the compasses in *d*, and extend the other foot to F, and draw the arch E F G; with this extent of the compasses, set one foot in *b*, and draw the arch H I K. Lastly, set one foot of the compasses in *a* and *c* severally, and draw the arch G H and E K; and the oval is compleated.

Prob. 25. *To draw an oval by the help of a parallelogram, or two geometrical squares.* Fig. 26.

First, draw the line A C, and make C F = C B; then draw D F parallel to A C; draw also A D and B E, and you will have two squares A B D E, and B C E F; then draw the diagonal lines A E and B F, and opening the compasses with the extent of A E or C E, place one foot in E, and draw the arch A C; then with the former extent, one foot placed in B, describes the arch D F; then set one foot in *a*, and with the distance *a* A sweep the arch D A; with the same extent from *c* sweep the arch C F, and the oval is compleated.

Prob. 26. *Having a line equal to the length of an oval, to make thereof a true oval.* Fig. 27.

Let A B be the given line; divide it into three equal parts A *b* B; then from the point *b*, with the distance *b* B, describe the circle B *a* G *c*; and upon the other division at *a*, draw the circle A *b* G *c*; these two circles will intersect one another in the center of each, and also at the points *d c*, draw C *b e* and F *a d* parallel, also G *a c* and H *b d* parallel; then from *c*, with the distance G *c*, sweep the arch G N G, and from *d* with the same extent, sweep F K H, and you have a true oval.

Prob. 27. *To draw an oval from three circles.* Fig. 28.

Draw the line A B, and divide it into four equal parts, and on the three points *d*, *c*, *e*, describe three circles; draw M G and O F parallel thereto, and also draw F N and L G parallel thereto; then on G, with the extent G L or G M, describe the arch L M; and upon F, with the same extent, describe O N. Lastly, upon the point *d* describe O A L, and upon *e* describe M B N, and the oval is finished.

Prob. 28. *To lay down an ellipsis by the line of sines on the sector, having the transverse and conjugate diameters given.* Fig. 29.

First, take A E or E B in your compasses; then open the sector at 90, 90 on the line of sines; and as the sector now stands, take off the sines 10, 20, 30, 40, 50, 60, 70, 80, and set them from E, each way towards A and B; draw lines through those points in the transverse

sector

fector on the fines 90, 90, to the radius C E, and take in your compaffes the fine of 80, and fet 10 to 80; take the fine 70, and fet from 70 to 20 on each fide the conjugate diameter; the fine 60, fet from 30 to 60; the fine 50, fet from 40 to 50; the fine 40, fet from 50 to 40; the fine 30, fet from 60 to 30; the fine 20, fet from 70 to 20; the fine 10, fet from 80 to 10; fo will the points 10, 20, 30, 40, 50, 60, 70, 80, C B D A, be in the ellipfis, through which points draw the curve, and you will have a true mathematical ellipfis.

Prob. 29. *Any angle being given, to find the number of degrees it contains.* Fig. 16.

1. Take 60° out of your line of chords, and fet one foot of your compaffes in A, with the other defcribe the arch *a b*.

2. Take the diftance *a b* in your compaffes, and fet one foot in the beginning of the line of chords, and the other will reach to 60 upon the fame line, the meafure of the angle required.

Prob. 30. *In a given circle, to infcribe a polygon of any propofed number of fides.* Fig. 30.

1. Divide 360° by the number of fides, and make an angle A *c* B, at the center, whofe meafure fhall be equal to the degrees in the quotient.

2. Join the points A and B, and apply the chord A B to the circumference, the given number of times, and you will have the polygon required.

Prob. 31. *To defcribe a lune in a quadrant.* Fig. 31.

Draw the triangle A B C, and on the center B, defcribe the quadrantal arch A C; upon the middle of the hypothenufe A C, draw the other femi-circle, and you will have the lune A F C D required.

LXVII. MENSURATION OF SUPERFICIES.

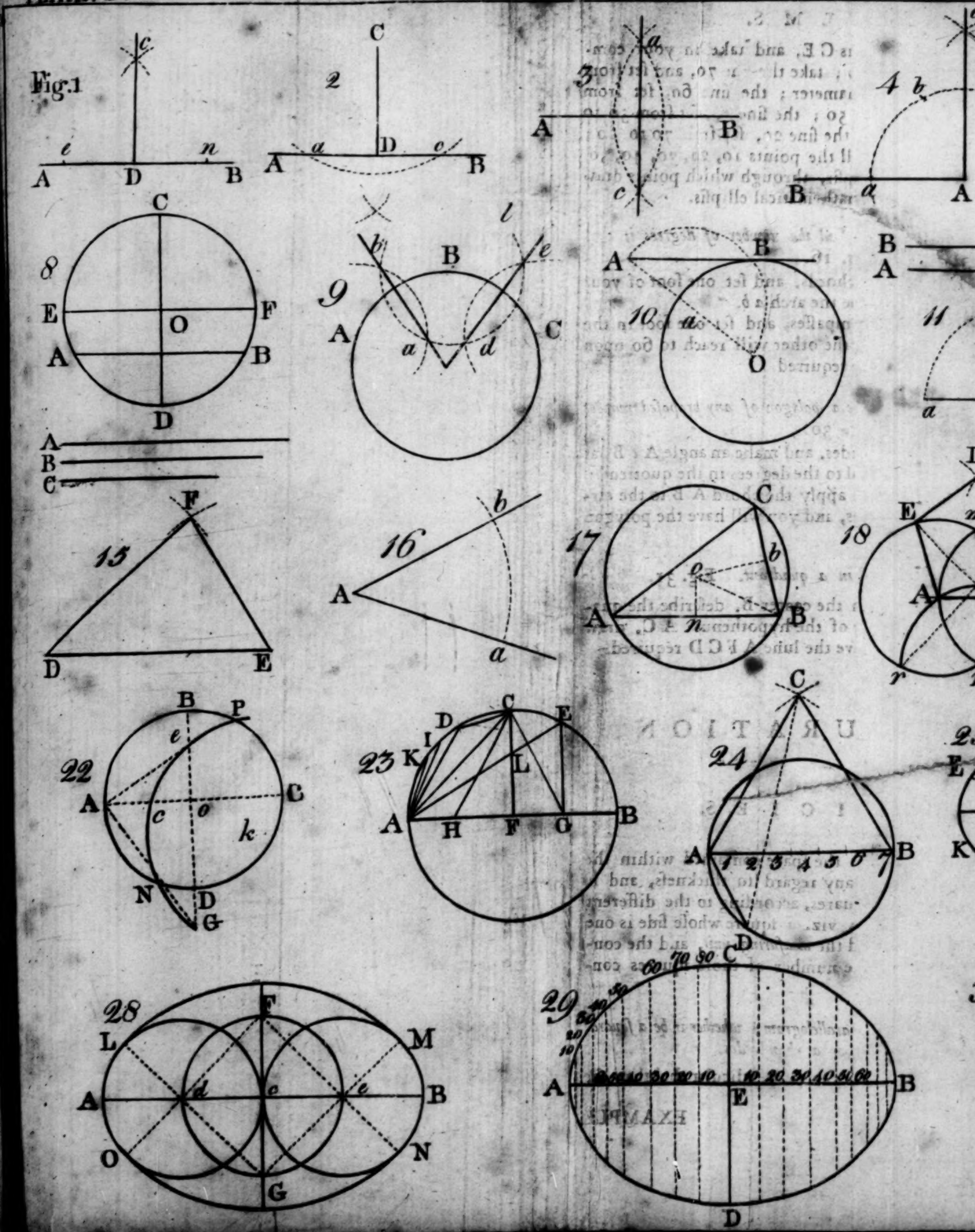
THE area of any plain furface is the fpace contained within the bounds of that furface, without any regard to thicknefs, and is made up of fome certain number of fquares, according to the different meafures the dimenfions are taken in, viz. a fquare whofe fide is one inch, one foot, one yard, &c. is called the *meafuring unit*, and the content of any figure is computed by the number of thofe fquares contained in that figure.

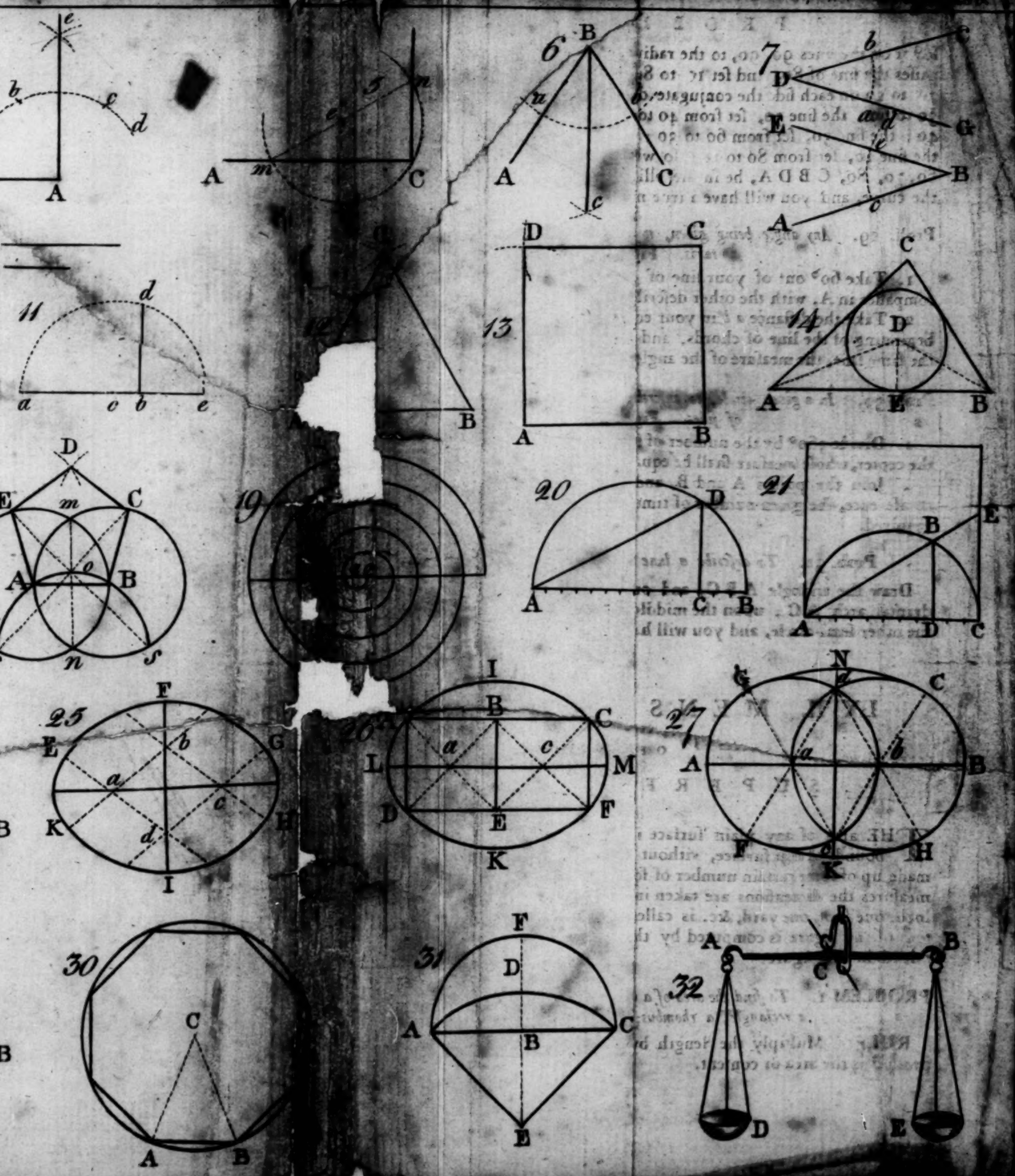
PROBLEM 1. *To find the area of a parallelogram; whether it be a fquare, a rectangle, a rhombus, or a rhomboides.*

RULE. Multiply the length by the perpendicular height, the product is the area or content.

EXAMPLE

PLATE. 1.




6
7
11
12
13
14
19
20
21
25
26
27
30
31
32

EXAMPLE 1. What is the area of the ſquare S S, whoſe ſide is 5 feet 6 inches ?

By decimals.

```
   F.
  5,5
  5,5
 ----
  275
 275
 ----
30,25
   12
 ----
 3,00
   F. I.
Anſwer 30 3
```

By croſs multiplication.

```
        F.  In.
        5   6
        5   6
       -------
       27   6
        2   9
       -------
Anſwer 30   3
```

By practice.

```
6 in. = ½) 5   6
               5
          ------
          27   6
           2   9
          ------
Anſwer    30   3
```

E. 2. What is the area in acres of a ſquare, whoſe ſide is 35,25 chains ?

```
    35,25 = Length of the ſide
    35,25
    -----
    17625
    7050
   17625
  10575
 ----------
 124,25625
         4
 ----------
   1,02500
        40
   -------
     1,00
```

Anſwer 124 acres, 1 rood, 1 p.

E. 3. Required the area of a ſquare, whoſe ſide is 9 feet 3 inches ?

By decimals.

```
    9,25
    9,25
   -----
    4625
   1850
  8325
 -------
 85,5625
      12
 -------
  6,7500
      12
  ------
  9,00
```

By croſs multiplication.

```
      F.  I.
      9   3
      9   3
     --------
     83   3
      2   3   9
     --------
Anſ. 85   6   9
```

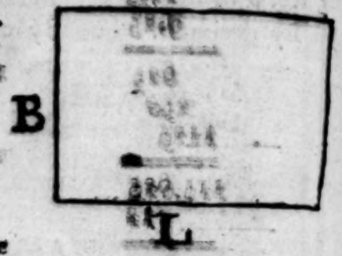
By practice.

```
In.         F.  I.
3 = ¼)      9   3
                9
           -------
           83   3
            2   3   9
           ----------
Anſwer     85   6   9
```

Anſ. 85 feet, 6 in. 9″.

E. 4.

E. 4. What is the area of the rectangle B L, whose length is 18 feet 6 inches, and breadth 12 feet 6 inches?



By decimals.

F.
18,5
12,5
925
370
185
231,25
12
3,00

Answer 231 feet 3 inches.

By cross multiplication.

F.	I.
18	6
12	6
222	0
9	3

Answer 231 3 as before

E. 5. What is the superficial content of a parallelogram, whose length is 68 feet, and breadth 16 feet?

F.
68
16

Answer 1088 feet.

E. 6. If one side of a parallelogram is 130 feet, and the other 50 feet, what is the superficial content?

130
50

Answer 6500 feet.

E. 7. How many feet are contained in a floor 45 feet 6 inches long, and 9 feet 3 inches broad?

By Decimals.

45,5
9,25
2275
910
4095
420,875
12
10,500
12
6,0

Answer 420 feet, 10 inches, 6″.

By cross multiplication.

F.	I.	
45	6	
9	3	
409	6	
11	4	6
Ans. 420	10	6

By practice.

In.	F.	I.	
3 = ¼)	45	6	
		9	
	409	6	
	11	4	6
Answer	420	10	6

E. 8. Required the superficial content of a rhombus, whose length is 12 feet 6 inches, and perpendicular height 9 feet 3 inches?

By

By decimals.	By cross multiplication.	By practice.
12,5	12 6	3 = ¼) 12 6
9,25	9 3	9
625	112 6	112 6
250	3 1 6	3 1 6
1125	115 7 6	115 7 6
115,625		
12		
7,500		
12		
6,000		

Answer 115 feet, 7 inches, 6 parts.

E. 9. What is the area of a rhombus, the length of whose side is 81 feet, and height 9 feet 6 inches?

By decimals.	By cross multiplication.	By practice.
81	F. I.	I. F.
9,5	81 0	6 = ½) 81
405	9 6	9
729	729 0	729
769,5	40 6 0	40 6
12	769 6 0	769 6
6,0		

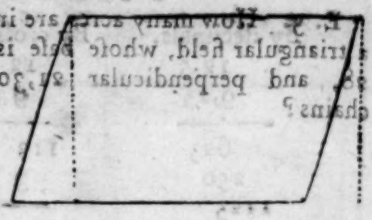
Answer 769 feet, 6 inches.

E. 10. How many square feet of paving are there in a court yard, in the form of a rhombus, whose length is 64 feet 6 inches, and perpendicular breadth 47 feet 8 inches?

By decimals.	By cross multiplication.
47,666	F. I.
64,5	64 6
238330	47 8
190664	448
285996	256
3074,4570	43 6
12	43 0
5,4840	3074 6 Answer
12	
5,8080	

E. 11. What is the area of a rhomboides, whose length is 26,5 feet, and perpendicular height 20,2 feet?

	26,5
	20,2
	530
	530



Answer 535,30 = 535 ft. 3 in. 7''.

E. 12. What is the area of a rhomboides, whose length is 36 feet 9 inches, and perpendicular breadth 18 feet 6 inches?

By decimals.

36,75
18,5
18375
29400
3675

Answ. 679,875 = 679 ft. 10 in. 6''.

By cross multiplication.

36	9	
18	6	
648	0	
13	6	
18	4	6

Answ. 679 10 6 as before.

Prob. 2. *To find the area of a triangle.*

RULE. Multiply the base by half the perpendicular height, the product will be the area; or multiply the base by the perpendicular, and half the product will be the area.

EXAMPLE 1. What is the area of a triangle, whose base B is 36 feet, and perpendicular height H 16 feet?

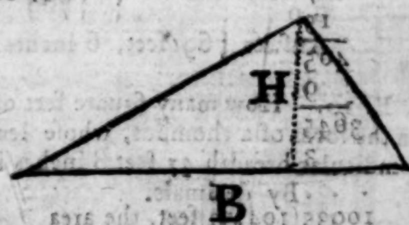
36
16
2) 576

Ans. 288

Or thus

36
8

Answer 288



E. 2. Required the area of a triangle, whose base is 6 feet 6 inches, and perpendicular height 4 feet 3 inches?

By decimals.

4,25
6,5
2125
2550
2)27,625
13,8125
12
9,7500
12
9,0000

By cross multiplication.

	F.	I.	
	6	6	
	4	3	
	26	0	
	1	7	6
2)	27	7	6
	13	9	9

By practice.

I.		F.	I.	
3 =	$\frac{1}{4}$)	6	6	
			4	
		26	0	
		1	7	6
	2)	27	7	6
		13	9	9

Answer 13 feet, 9 inches, 9''.

E. 3.

E. 3. How many acres are in a triangular field, whose base is 28, and perpendicular 21,30 chains?

21,30	Perpendicular
14	Half base
,29820	
4	
1,19280	
40	
7,71200	

Answer 1 rood, 7 poles, ,712.

Note. The reason of pointing off 5 figures to the right, is the same as dividing by 100000, the number of square links in an acre.

Prob. 3. *To find the area of a triangle, whose three sides only are given.*

RULE. From half the sum of the three sides, subtract each side severally; multiply the half sum, and the three remainders continually together, and the square root of the product will be the area of the triangle.

E. 1. What is the area of a triangle, whose three sides measure 12, 18, and 24 feet respectively?

12		27			
18		12			
24					
54	Sum	15	First difference.		
27	Half sum	27		27	
15		18		24	
405		9	Second diff.	3	Third diff.
9					
3645					
3					

10935(104,57 feet, the area
1
204) 935
816
2085) 11900
10425
20907) 147500
146349
1151 Remains

E. 2. Required the area of a triangle, whose three sides measure 50, 60, and 70?

First, $50 + 60 + 70 = \frac{180}{2} = 90.$

$$\text{And } 90 - \left\{ \begin{array}{l} 50 = 40 \\ 60 = 30 \\ 70 = 20 \end{array} \right\} \text{The three remainders.}$$

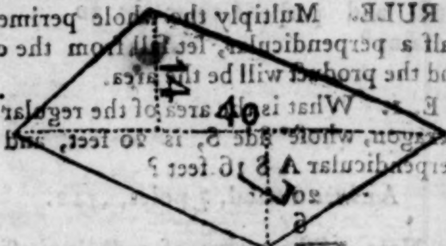
Then $90 \times 40 \times 30 \times 20 = 2160000$. $\therefore \sqrt{2160000} = 1469,69$ the answer.

Prob. 4. *To find the area of a trapezium.*

RULE. Multiply the diagonal by the sum of the two perpendiculars falling upon it from the opposite angles, and half the product will be the area.

E. 1.

E. 1. What is the area of a trapezium, whose diagonal is 40, and the two perpendiculars 14 and 7 feet?



14
7

21 Sum of the perpendicul.
40 Diagonal

2) 840

420 Feet, answer.

E. 2. How many square yards of paving are there in a trapezium, whose diagonal line is 102 feet, and perpendiculars 30 feet and 24 feet?

30
24

54 Sum of the perpendiculars
102 Diagonal

108
54

2)5508

9)2754

Answer 306 Yards.

Prob. 5. *To find the area of any irregular polygon.*

RULE. Divide it into triangles, in the manner you judge most convenient; then the sum of the areas of those triangles, calculated by problem 2, will be the area of the irregular polygon.

E. 1. What is the area of an irregular polygon, whose two diagonals measure 49 and 39 feet, and three perpendiculars, 19, 9 and 14 feet respectively?

19
9

28
49

252
112

2) 1372

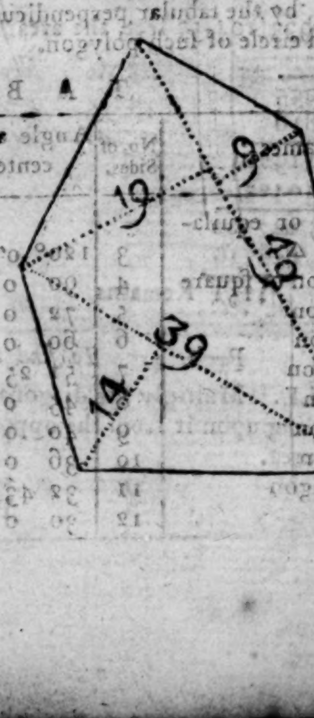
686 Area of the trapezium
273 Area of the triangle

Ans. 959 Area of the polygon

39
14

2)546

273



Prob

Prob. 6. *To find the area of a regular polygon.*

RULE. Multiply the whole perimeter, or sum of the sides, by half a perpendicular, let fall from the center, upon one of the sides, and the product will be the area.

E. 1. What is the area of the regular hexagon, whose side S, is 20 feet, and perpendicular A S 16 feet?

20
6
120 Perimeter
8 Half perpendicular

Answer 960 Square feet.

E. 2. What is the area of a pentagon, whose side is 14,6 feet, and perpendicular 12,64 feet?

14,6
5
73,0 Perimeter
6,32 Half perpendicular
1460
2190
4380
461,360 Square feet, answer.

E. 3. What is the area of an octagon, whose side is 9,5, and perpendicular 12?

9,5
8
76,0 Perimeter
6 Half perpendicular
456,0 Answer.

Prob. 7. *To find the area of a regular Polygon, when the side only is given.*

RULE. Multiply the square of the side of the polygon, by the multiplier standing opposite to its name in the following table, and the product will be the answer; and if you multiply the side of any polygon, by the tabular perpendicular, it will give the radius of the inscribed circle of such polygon.

T A B L E.

Names.	No. of Sides.	Angle at the center.			Perpendiculars.	Multipliers.
Trigon or equilateral △	3	120°	0′	0″	0,2886751	0,4330127
Tetragon or square	4	90	0	0	0,5000000	1,0000000
Pentagon	5	72	0	0	0,688191	1,7204774
Hexagon	6	60	0	0	0,866025	2,5980762
Heptagon	7	51	25	$42\frac{6}{7}$	1,038260	3,6339124
Octagon	8	45	0	0	1,2071068	4,8284272
Nonagon	9	40	0	0	1,373739	6,1818241
Decagon	10	36	0	0	1,5388418	7,6942087
Undecagon	11	32	43	$38\frac{2}{11}$	1,702844	9,3656412
Duodecagon	12	30	0	0	1,866026	11,1961524

E. 1.

E. 1. What is the area of a pentagon, whose side is 10? First 10 × 10 = 100 square of the side. Then 1,7204774

100

Answer 172,0477400 the area.

E. 2. Required the area and radius of the inscribed circle of a hexagon, whose side is 20?

First 20 × 20 × 2,5980762 = 1039,23 the area.

Again, ,866025 × 20 = 17,32 the radius of the inscribed circle,

E. 3. What is the area of an octagon whose side is 10?

First 10 × 10 = 100 square of the side.

Then 4,8284272 × 100 = 482,84272 the area.

Prob. 8. *The diameter of a circle being given, to find the circumference; or the circumference being given, to find the diameter.*

RULE 1. As 7 is to 22, so is the diameter to the circumference. Or, as 22 is to 7, so is the circumference to the diameter.

E. 1. If the diameter of a circle be 7, what is the circumference;

As 7 : 22 :: 7

7

7) 154

Answer 22 per rule.

E. 2. If the circumference of a circle be 22, what is the diameter?

As 22 : 7 :: 22

7

22) 154(7 Ans.

154

...

E. 3. If the diameter of a circle be 4, what is the circumference?

As 7 : 22 :: 4

4

7)88,00

Answer 12,57 the circumference.

E. 4. If the circumference of a circle be 12,57, what is the diameter? As 22 : 7 :: 12,57

7

22) 87,99(3,999 = 4, nearly the diam.

RULE

RULE 2. As 113 is to 355, so is the diameter to the circumference. Or, as 355 is to 113, so is the circumference to the diameter.

E. 1. If the diameter of a circle be 5 feet, what is the circumference?

As 113 : 355 :: 5

```
       355
         5
113)1775(15,7 Feet, the
    113      (circumference.
    ---
     645
     565
     ---
      800
      791
      ---
        9
```

E. 2. What is the diameter of that circle, whose circumference is 15,7?

As 355 : 113 :: 15,7

```
        15,7
        ----
         791
        565
        113
     --------
355)1774,1(4,99 = 5, near-
    1420        (ly the diam.
    ----
     3541
     3195
     ----
      3460
      3195
      ----
       265
```

RULE 3. Multiply the diameter by 3,1416, and the product will be the circumference. Or, divide the circumference by 3,1416, and the quotient will be the diameter.

E 1. If the diameter of a circle be 10, what is the circumference?

```
          3,1416
              10
          ------
Answer    31,416 = the circumference.
```

E. 2. If the circumference of a circle be 31,416, what is the diameter?

```
3,1416)31,4160(10 = the diameter.
       31416
       -----
       .....0
```

E. 3. If the diameter of a circle be 100 inches, what is the circumference?

```
          3,1416
             100
          ------
Answer    314,16 the circumference.
```

E. 4. The circumference of the earth is known to be 25020 miles, what is the diameter?

```
3,1416)25020,0000(7964 Miles, the diameter.
       219912
       ------
        302880
        282744
        ------
         201360
         188496
         ------
          128640
          125664
          ------
            2976
```

Prob.

Prob. 9. *To find the area of a circle.*

RULES. 1. Multiply half the circumference by half the diameter, and the product will be the area.

2. Multiply the square of the diameter by ,7854, the product will be the area.

3. Multiply the square of the circumference by ,079574, the product is the area.

4. Multiply the square of the semi-diameter by 3,1416, the product will be the area.

5. Multiply the circumference by the diameter, and a fourth part of the product will be the area.

Note. ,7854, and 3,1416, are areas of circles, whose diameters are 1 and 2, and ,079577, is the area of a circle, whose circumference is 1, likewise 452, and 1,273939, are squares of the diameters of circles, whose areas are 355, and 1, and 1,12831 is the diameter of a circle, whose area is equal to a square, whose side is 1.

E. 1. What is the area of a circle, whose diameter is 100 inches, and circumference 314,16?

By rule 2,
```
   100
   100
 -----
 10000
 ,7854
 -----
Answer 7854 area.
```

Or thus, by rule 4, $50 \times 50 = 2500$. Then $3,1416 \times 2500 = 7854$, the area, as before.

By rule 1,
```
        2)314,16
         -------
          15708 = Half circumference
             50 = Half diameter
         -------
Answer  7854,00 Area, as before.
```

E. 2. What is the area of a circle, whose diameter is 17, and circumference 53,4072 inches?

By rule 1,
```
        2)53,4072
         --------
          26,7036 = Half circumference
              8,5 = Half diameter
         --------
          1335180
         2136288
         --------
Answer  226,98060 Inches.
```

E. 3. What is the area of a circle whose diameter is 3 feet?

By rule 2,
```
        ,7854
            9 = Square of the diameter
       ------
Answer 7,0686 = 7 Feet, 0 inches, 9 parts.
```

E. 4. If the diameter of a circle be 4 inches, what is the area?

By rule 2,
```
         ,7854
            16 = Square of the diameter
       -------
Answer 12,5664 Inches.
```

E. 5. What is the area of a circle, whose diameter is 4, and circumference 12,5664 inches?

By rule 1. 2)12,5664

6,2832 = Half the circumference.
2

Answer 12,5664 Inches, the area.

Note. In this example it may be observed that when the diameter is 4, the circumference and area are equal.

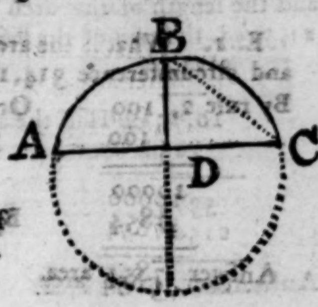
Prob. 10. *To find the length of any arch of a circle.*

RULE 1. From 8 times the chord of half the arch, subtract the chord of the whole arch, and one-third of the remainder will be the length of the arch nearly.

E. 1. The chord of the whole arch A C is 60, and the versed sine D B of half the arch is 10; what is the length of the arch A B C?

30
30

900 = Square of D C, or half A C
100 = Square of B D



1000,00000000(31,6227 = BC, chord of
9 8 (half the arch.

61)100 252,9816
61 — 60

626)3900 3)192,9816
3756

64,3272 = Length of the arch required, answer.

6322)14400
12644

63242)175600
126484

63244) 4911600
4427129

484471

E. 2. The chord of the whole arch is 48, and the versed sine of half the arch is 18, what is the length of the arch?

First, $\overline{18 \times 18} + \overline{24 \times 24} = 900$; and $\sqrt{900} = 30 =$ chord of half the arch.

Then $\frac{30 \times 8 - 48}{3} = 64$, length of the arch required.

E. 3. The chord of the whole arch is 50,8; and the chord of half the arch is 30,6; what is the length of the arch?

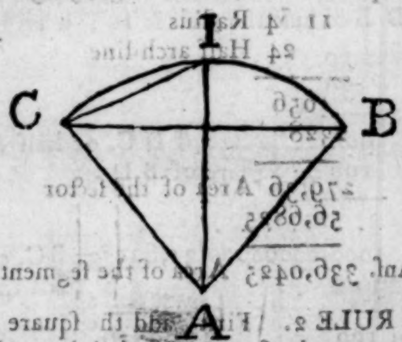
30,6
8
―――
244,8
— 50,8
―――
3)194,0
Answer 64,6 length of the arch.

Prob. 11. *To find the area of a sector, or that part of a circle which is bounded by any two radii, and their included arch.*

RULE. Multiply the radius by half the arch of the sector, found by the last problem, and the product will be the area, or superficial content.

Note. A sector may be either less or greater than a semi-circle.

E. 1. The radius A B is 25, and the length of the arch C B 21,5 req. the area of the sector?



2)21,5
―――
10,75 = Half the arch
25
―――
5375
2150
―――
Answ. 268,75 the area.

E. 2. What is the area of a sector, whose radius is 45, and the length of the arch 49?

2)49
―――
24,5 = Half the arch.
45
―――
1225
980
―――
Answer 1102,5 the area.

Prob. 12. *To find the area of a segment of a circle.* (See fig. to prob. 11.)

RULE 1. Find the area of a sector, having the same arch with the segment, by the last problem.

2. Find the area of the triangle formed by the chord of the segment, and the radii of the sector.

3. The sum, or difference of these areas, according as the segment is greater or less than a semi-circle, will be the area required.

Note. If the segment is greater than a semi-circle, the area of the triangle must be added to that of the sector, and the product will be the area of the segment.

E. 1.

E. 1. If the semi-diameter A B, of a circle, be 24,5, the arch line 45,6, and the chord upon which the triangle is formed 30,5, and the perpendicular of the triangle 16,2; what is the area of the segment?

```
 24,5 = Semi-diameter
 22,8 = Half the arch line
 ----
 1960
 490
490
------
558,60 Area of the sector
247,05
------
311,55 = Area of the segm. Ans.
```

```
30,5 Chord
 8,1 = Half perpendicular
----
 305
2440
247,05 = Ar. of the triangle.
```

E. 2. What is the area of a segment whose radius is 11,64, arch line 48, the chord upon which the triangle is formed 20,5, and the perpendicular of the triangle 5,53?

```
11,64 Radius
   24 Half arch line
 -----
 4656
2328
 -----
279,36 Area of the sector
 56,6825
 -------
```

```
 2,765 = Half perpendicular
 20,5  = chord line
 -----
 13825
5530
56,6825 = Area of the triangle
```

Ans. 336,0425 Area of the segment.

RULE 2. First, add the square of half the chord of the segment to the square of its height, and multiply the square root of the sum by 4.

2. To $\frac{1}{3}$ of the number last found, add the whole chord of the segment, this sum multiplied by $\frac{2}{3}$ of the height, will give the area.

E. 1. If the chord of a segment be 20, and its height or versed sine 5, what is the area of the segment?

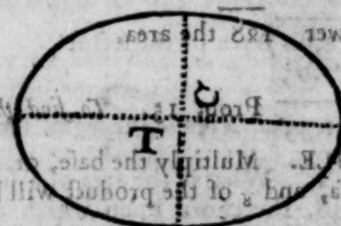
```
 10
 10
---
100 Square of half chord
 25 Square of the height
---
125(11,18
  1
21) 25
    21
221) 400
     221
2228) 17900
      17824
         76
```

```
   11,18
       4
 -------
 3)44,72
  14,906
+ 20
  ------
  34,906
       2 = 2/3 of the height
Answer 69,812 Area of the segment.
```

Prob. 13. *To find the area of an ellipsis, or oval.*

RULE. Multiply the transverse diameter by the conjugate, then multiply that product by ,7854, this last product is the area of the oval.

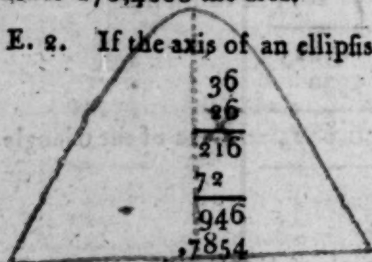
E. 1. What is the area of an ellipsis, whose transverse diameter T is 22, and conjugate C 16?




```
   22
   16
  ---
  352
,7854
-----
 1408
1760
2816
2464
------
```

Answer 276,4608 the area.

E. 2. If the axis of an ellipsis be 36 and 26, what is the area?



```
   36
   26
  ---
  216
  72
  ---
  946
,7854
-----
 3784
4730
7568
6622
------
```

Answer 742,9884 the area.

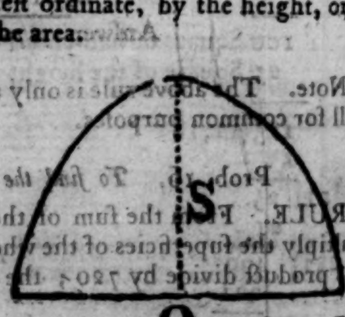
E. 3. What is the area of an ellipsis, whose greatest diameter is 100, and least diameter 70?

$,7854 \times 100 \times 70 = 5497,8$, the area.

Prob. 14. *To find the area of a parabola.*

RULE. Multiply the base, or greatest ordinate, by the height, or abscissa; and $\frac{2}{3}$ of the product will be the area.

E. 1. What is the area of the parabola, whose height S is 12, and the base, or greatest ordinate O, 36?




```
    36
    12
   ---
   432
     2
   ---
 3)864
   ---
```

Answer 288 Area required.

E. 2.

E. 2. What is the area of a parabola, whose base or greatest ordinate is 24, and the abscissa 8?

$$\begin{array}{r} 24 \\ 8 \\ \hline 192 \\ 2 \\ \hline 3)384 \\ \hline \end{array}$$

Answer 128 the area.

E. 3. The abscissa is 39,25, and the greatest ordinate, or base, 53,75; what is the area?

$$\frac{53,75 \times 39,25 \times 2}{3} =$$

1406,4583, the area, answer.

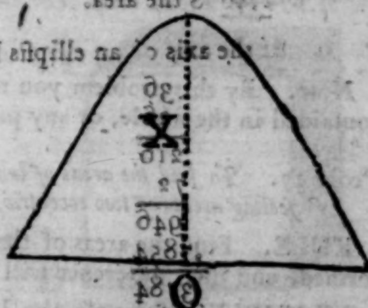
Prob. 15. *To find the area of an hyperbola.*

RULE. Multiply the base, or greatest ordinate, by the height, or abscissa, and $\frac{5}{8}$ of the product will be the area, nearly.

E. 1. What is the area of an hyperbola, whose base, or greatest ordinate O is 24, and the abscissa or height X, 10?

$$\begin{array}{r} 24 \\ 10 \\ \hline 240 \\ 5 \\ \hline 8)1200 \\ \hline \end{array}$$

Answer 150 the area.



E. 2. Required the area of an hyperbola, whose base, or greatest ordinate, is 36, and the perpendicular height, or abscissa, 12?

$$\begin{array}{r} 36 \\ 12 \\ \hline 432 \\ 5 \\ \hline 8)\ 2160 \\ \end{array}$$

Answer 270 the area.

Note. The above rule is only an approximation, but will serve very well for common purposes.

Prob. 16. *To find the area of a spherical triangle.*

RULE. From the sum of the three angles, subtract 180 degrees; multiply the superficies of the whole sphere, or globe, by the remainder, this product divide by 720; the quotient is the area of the triangle.

E. 1. Suppose the angle at A = 36°; at B 148°; at C 32°, and the diameter of the globe 29; what is the area of the triangle A B C?

First

First 36 + 148 + 32 = 216
And 216 — 180 = 36
Then 2642 surface of the (sphere*
36

15852
7926

720) 95112(132,1 = area of (the triangle
720

2311
2160

1512
1440

720
720

0

Note. By this problem you may find the number of miles or acres contained in the whole, or any part of the surface of the globe.

Prob. 17. *To find the areas of lunes, or the spaces included between the intersecting arches of two eccentric circles.* See plate 1, fig. 31.

RULE. Find the areas of the two segments from which the lune is formed, and their difference will be the area required.

EXAMPLE. Suppose the length of the chord A C is 40, the height E F 10, and E D 4; what is the area of the lune A F C D A?

First 400 = Square of ½ A C
100 = Square of E F

500 Then $\sqrt{500}$ = 22,36

22,36
4

3)89,44

29,81
+40

69,81
4 = ⅔ of the height

279,24 = Area of the segment (A C F A.

Again 400 = Square of ½ A C
16 = Square of E D

416 And $\sqrt{416}$ =
20,396
4

3)81,584

27,194
+40

67,194
1,6 = ⅔ of the height

403164
67194

107,5104 = Area of the segment (A C D A.

279,24
107,5104

Answer 171 7296 the area of the lune required. 68

* The surface of a sphere may be found by problem 9, sect. ~~70~~.

LXVIII.

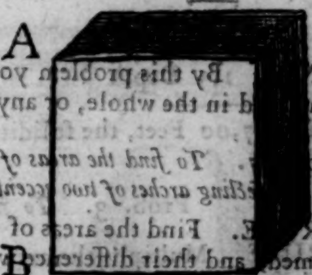
LXVIII. MENSURATION OF SOLIDS,

TEACHETH how to measure all solid bodies, which consist of length, breadth, and thickness.

Prob. 1. *To find the solidity of a cube.*

RULE. Multiply the side of the cube into itself, and that product again by the side, and it will give the solidity.

E. 1. The side A B of the cube is 6,5, what is the solidity?




```
  6,5
  6,5
 ----
  325
 390
 ----
 42,25
   6,5
 -----
 21125
25350
```

Answer 274,625 the solidity.

E. 2. The side of a cube is 12 inches, what is the solidity?

```
  12
  12
 ---
 144
  12
 ---
```

Answer 1728 inches.

E. 3. What is the solidity of a cube, whose side is 21,5?

```
    21,5
    21,5
   -----
    1075
    215
   430
  ------
  462,25
    21,5
 -------
  231125
  46225
 92450
```

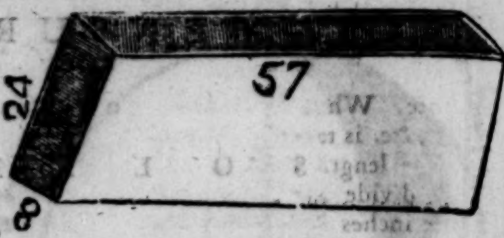
Answ. 9938,375 the solidity.

Note. If the answer be in inches, you must divide by 1728 (the solid inches in a foot) to bring them into feet.

Prob. 2. *To find the solidity of a parallelopipedon.*

RULE. Multiply the length by the breadth, and that product again by the depth or altitude, and it will give the solidity.

E. 1. Required the solidity of the parallelopipedon, whose length is 57 feet, breadth 24, and depth 8?



57 Length
24 Breadth
228
114
1368
8

Answ. 10944 the solidity.

E. 2. The length of a parallelopipedon is 12 feet, and each side of its square base 1 foot 6 inches; what is the solidity?

12 Length
1,5 Breadth
60
12
18,0
1,5 Depth

Answ. 27,00 Feet, the solidity.

E. 3. What is the solid content of a block of marble, whose length is 8 feet, breadth 4, and depth $2\frac{1}{2}$ feet?

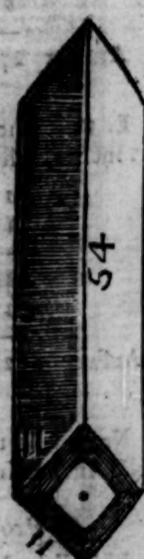
8
4
32
2,5
160
64

Answ. 80,0 Feet, the solidity.

Prob. 3. *To find the solidity of a prism.*

RULE. Multiply the area of the base into the height, and the product will be the solidity.

E. 1. What is the solidity of a square prism, whose length is 54 feet, and each of the equal sides 11 feet?



11
11
121 Area of the base
54 Length
484
605

Answer 6534 Feet, the solidity.

Note. To find the convex surface of a cube, parallelopipedon, or prism, you must find the area of each side and end separately, which areas added together, will give the whole surface.

E. 2.

E. 2. What is the solidity of a triangular prism, each side of the base of which is 16 inches, the perpendicular of the base 10 inches, and the length of the solid 12 feet?

Note. When the breadth, depth, &c. is taken in inches, and the length in feet, you must divide by 144 (the square inches in a foot) for the answer, as in this example.

```
     16
      5
    ----
     80 Area of the base
     12 Length
    ----
144)960(6,6 = 6 feet, 7½ inches, answ.
    864
    ---
     960
     864
     ---
      96
```

E. 3. What is the solidity of a prism, whose base is a hexagon, supposing each of the equal sides to be 12 inches, the perpendicular from the center to one of the sides 10,5, and the length of the prism 53 inches?

```
      12
       6
      --
      72 = Perimeter
    5,25 = ½ Perpendicular
    ----
     360
    144
   360
   ------
   378,00 = Area of the base
    53    = Length
   ----
   1134
  1890
  -----
Ans. 20034 the solidity.
```

Prob. 4. *To find the solidity of a cylinder.*

RULE. Multiply the area of the base by the height, and the product will be the solidity.

E. 1. What is the solidity of a cylinder, the diameter of whose base is 3 feet, and length 53?

,7854
9 = Square of the base
7,0686 = Area of the base
53
212058
354430
375,6358

E. 2. What is the solidity of a cylinder, whose height is 6 feet, and diameter of the end 2 feet? First ,7854 × 2 × 2 = 3,1416, the area of the base. ∴ 3,1416 × 6 = 18,8496, the solidity.

Note. To find the convex surface of a cylinder, multiply the periphery of the end by the height of the cylinder, the product will be the convex surface.

EXAMPLE. What is the convex surface of a cylinder, whose length is 53 feet, and the diameter of its base 3 feet?

3,1416
3
9,4248 = Periphery of the base
53 = Length
282744
471240
Answer 499,5144 convex surface.

Prob. 5. *To find the solidity of a cone or pyramid.*

RULE. Multiply the area of the base by ⅓ of the height, and the product will be the solidity.

E. 1. What is the solidity of a cone, whose diameter at the base is 10, and its altitude 60?

,7854
100 = Square of the diameter
78,5400 = Area of the base
20 = ⅓ of the height
Answer 1570,80 the solidity.

E. 2.

E. 2. What is the ſolidity of a cone, whoſe diameter is 25, and its perpendicular height 76?

,7854
625 = Square of the diameter
―――――
39270
15708
47124
―――――
490,8750 = Area of the baſe
25,3 = $\frac{1}{3}$ of the height
―――――
1472625
2454375
981750
―――――
Anſwer 12419,1375 the ſolidity.

E. 3. Required the ſolidity of a hexagonal pyramid, each of whoſe equal ſides of its baſe being 14, and the perpendicular height 67?

2,5980762 = Tabular multiplier
196 = Square of the diameter
―――――
155884572
233826858
25980762
―――――
509,2229352 = Area of the baſe
67 = Height
―――――
35645605464
30553376112
―――――
3)34117,9366584
Anſw. 11372,6455528 the ſolidity.

E. 4. What is the solidity of a square pyramid, each side of whose base is 17, and the perpendicular height 65?

$$\begin{array}{r l} 17 & \\ 17 & \\ \hline 289 & = \text{Area of the base} \\ 65 & = \text{Height} \\ \hline 1445 & \\ 1734 & \\ \hline 3)18785 & \\ \hline \end{array}$$

Answer $6261\frac{2}{3}$ the solidity.

Note. To find the convex surface of a cone, or pyramid, multiply the circumference of the base or perimeter by the slant height, and half the product will be the surface required.

E. 1. What is the convex surface of a cone, whose base is 32 feet in circumference, and slant side 10 feet in length?

$$\begin{array}{r} 32 \\ 10 \\ \hline 2)320 \\ \hline \end{array}$$

Answer 160 the convex surface.

E. 2 What is the surface of a square pyramid, each of whose equal sides is 17, and the slant sleight 64?

$$\begin{array}{r} 17 \\ 4 \\ \hline 68 \\ 64 \\ \hline 272 \\ 408 \\ \hline \end{array}$$

Answ. 4352 the convex surface.

Prob. 6. *To find the solidity of the frustum of a cone or pyramid.*

RULE 1. For the frustum of a cone, divide the difference of the cubes of the diameters of the two ends, by the difference of the diameters; this quotient being multiplied by ,7854, and again by $\frac{1}{3}$ of the the height, will give the solidity.

2. For the frustum of a pyramid: To the areas of the two ends of the frustum, add the square root of their products; this sum being multiplied by $\frac{1}{3}$ of the height, will give the solidity.

E. 1.

E. 1. What is the content of the frustum of a cone, the diameter of whose greater end D is 4 feet, that of the lesser end d 2 feet, and the perpendicular height D d 9 feet?

```
 4        2        4
 4        2        2
---      ---      ---
16        4        2 = Difference of D and d.
 4        2
---      ---
64 = Cube of D    8 = Cube of d
- 8
---
2)56
---
28
 3 = 1/3 of Dd
---
84
```


```
     ,7854
       84
    ------
     31416
    62832
    ------
Answer 65,9736 the solidity.
```

E. 2. What is the content of the frustum of a cone, 18 feet high, the diameter of its ends being 10 and 6 feet?

```
  10                             6
  10                             6
 ----                           ---
 100                            36
  10                             6
 ----                           ---
1000 = Cube of the greater base  216 = Cube of the lesser base
 216
 ----          ,7854        10
4) 784          1176         6
 ----          ------       ---
 196           47124         4 = Difference of the ends
   6 = 1/3 of the height  54978
 ----           7854
 1176          7854
               ------
Answer  923,6304 = solidity required.
```

E. 3. What is the solidity of the frustum of a square pyramid, one side of the greater end D being 8 inches, that of the lesserend d 4 inches, and the height 60 inches?

```
 8         4
 8         4
---       ---
64 = Area D   16 = Area d
16
---
384
64
---
 . .
1024 (32 = Square root of the products of
 9          the area of the two ends
---
62) 124       64
    124       16
    ---       32
    . . .     ---
              112
               20 = 1/3 of the height
              ---
             2240 the solidity.
```

E. 4. Required the solidity of the frustum of a hexagonal pyramid, the side D of whose greater end is 3 feet, and d of the lesser end 2 feet, and the height 6 feet?


```
 2,598 = Tabular multiplier
     4 = Square of d
 ------
10,392 = Area of the polygon d.

 2,598 = Tabular multiplier
     9 = Square of D
 ------
23,382 = Area of the polygon D.
10,392
------
  46764
 210438
 70146
23382
---------
 .  .  .  .
242,985744(15,588 = Square root of the product of the
 1                    areas of the two ends
---
25) 142
    125
    ---
305) 1798                 10,392
     1525                 23,382
     ----                 15,588
3108) 27357               ------
      24864               49,362
      -----                    2 = 1/3 of the height
31168) 249344        Answer 98,724 = solidity required.
       249344
       ------
```

Note.

Note. To find the convex ſurface of the fruſtum of a pyramid or cone, you muſt multiply the ſum of the perimeters or circumference of the ends, by the ſlant height, and half the product will be the ſurface required.

E. 1. How many ſquare feet are in the ſurface of a fruſtum of a ſquare pyramid, whoſe ſlant height is 10 feet, each ſide of the greater baſe being 3 feet 4 inches, and each ſide of the leſs 2 feet 2 inches?

f.	i.	
3	4	
2	2	
5	6	
	4	
22	0	= Sum of perimeters
10		
2) 220		

Anſw. 110 Feet, the content.

E. 2. How many ſquare feet are in the ſurface of a fruſtum of a cone, whoſe circumference of its ends are 64 and 16 feet, and ſlant ſide 14 feet?

64
+ 16
80
14
2) 1120

Anſwer 560 Feet the content.

Prob. 7. *To find the ſolidity of a cuneus or wedge.*

RULE. Multiply the ſum of twice the length of the baſe, and the length of the edge, by the product of the height of the wedge, and the breadth of the baſe, and $\frac{1}{6}$ of the laſt product will be the ſolidity.

E. 1. What is the ſolidity of a wedge, whoſe baſe is 4 inches long, and 2 inches broad; the length of the edge being 3 inches, and the perpendicular height 8 inches?

4
2
8 = Twice the length of the baſe L
3 = Length of the edge *l*
11 = Sum.

8 = Height of the wedge *h*.
2 = Breadth of the wedge B
16 = Product
11
6) 176

Anſwer $29\frac{2}{6}$ inches the ſolidity.

When the length of the baſe is equal to that of the edge, the wedge is evidently equal to half a priſm of the ſame baſe and altitude, and may be meaſured by the following

RULE.

RULE. Multiply the area of the base by half the altitude of the wedge, and the product will give the solidity.

EXAMPLE. What is the solidity of a wedge, whose base measures 15 feet by 8, and perpendicular height 6 feet?

15
8

120 = Area of the base
3 = $\frac{1}{2}$ the altitude

Answer 360 Feet the solidity.

Prob. 8. *To find the solidity of a prismoid.*

RULE. To the sum of the areas of the two ends, add four times the area of a section parallel to, and equally distant from both ends, and this last sum multiplied by $\frac{1}{6}$ of the heighth will give the solidity.

EXAMPLE 1. How many solid feet are there in a tree whose ends are rectangles, the length and breadth of the greater end measures 12 inches by 8, and the lesser end 8 inches by 6; and the length 60 inches?

12 | 8
8 | 6

96 = Area of the greater L B. | 48 = Area of the lesser base l. b.
12 | 8
8 | 6

2)20 | 2)14

10 = length of the middle rectangle. | 7 = breadth of the middle rectangle.
7

70
4

280 = 4 times the area of the middle rectangle.
96
48

424
10 = $\frac{1}{6}$ of the length.

4240

1728)4240(2,45 solid feet the answer.
3456

7840
6912

9280
8640

640

Note.

Note. The length of the middle rectangle is equal to half the ſum of the lengths of the rectangles of the two ends, and its breadth equal to half the ſum of the breadths of thoſe rectangles.

Prob. 9. *To find the convex ſurface of a ſphere or globe, or any ſegment or zone of it.*

RULE. Multiply the circumference of the ſphere by its diameter or height of the part required, and the product will be the convex ſurface.

Note. The height of the whole ſphere is its diameter.

E. 1. What is the convex ſurface of a globe, whoſe diameter D is 3 inches?


```
    3,1416
         3
    9,4248 = Circumference
         3 = Diameter
Anſw. 28,2744 the ſurface required.
```

E. 2. What is the convex ſurface of a globe, whoſe diameter is 9 inches?

```
    3,1416
         9
   28,2744 = Circumference
         9 = Diameter
Anſw. 254,4696 the ſurface.
```

E. 3. If the diameter of the earth be 7964 miles, what is the whole ſurface, ſuppoſing it to be a perfect ſphere?

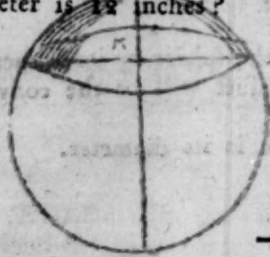
```
          3,1416
            7964
          125664
         188496
        282744
       219912
      25019,7024 = Circumference
            7964
      1000788096
     1501182144
    2251773216
   1751379168
Anſwer 199256909,9136 ſquare miles.
```

 Prob. 10.

Prob. 10. *To find the solidity of a sphere or globe.*

RULE. Multiply the cube of the diameter by ,5236, and the product will be the solidity.

E. 1. How many solid ~~feet~~ inches are there in a globe, whose diameter is 12 inches?



12
12
144
12
1728 = Cube of the diameter
,5236
10368
5184
3456
8640
Answer 904,7808 inches the solidity.

E. 2. What is the solidity of the earth, supposing it to be perfectly spherical, and its diameter 7964 miles?

7964
7964
31856
47784
71676
55748
63425296
7964
253701184
380551776
570827664
443977072
505119057344 = Cube of the diameter
5236
3030714344064
1515357172032
1010238114688
2525595286720
Answer 264480338425,3184 miles the solidity.

Prob. 11. *To find the solidity of the segment of a sphere.*

RULE. To three times the square of the radius of its base, add the square of its height; this sum multiplied by the height, and the product again by ,5236, will give the solidity.

E. 1.

E. 1. The radius An of the base of the segment of A B C, is 12 inches, and the height Bn 9 inches; what is its solidity?

12
12
144
3
432 = 3 times square of An
81 = Square of Bn
513
9 = height Bn
4617
,5236
27702
13851
9234
23085
Anf. 2417,4612 solid inches.

E. 2. What is the solidity of a segment of a sphere, the base of whose diameter is 14 inches, and its height 5?

7
7
49
3
147
25 = Square of the height
172
5 = Height
860

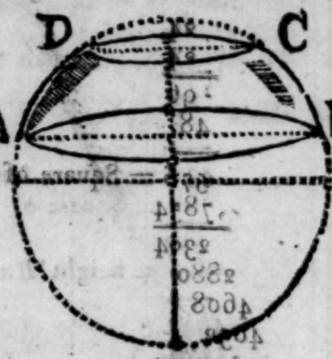
,5236
860
314160
41888
Anf. 450,2960 solid inches.

Prob. 12. *To find the solidity of a frustum or zone of a sphere.*

RULE. To the sum of the squares of the radii of the two ends, add $\frac{1}{3}$ of the square of their distance, or the breadth of the zone; and this sum multiplied by the said breadth, and the product again by 1,5708, will give the solidity.

E. 1. What is the solid content of the frustum of the sphere A B C D, whose greater diameter A B is 48 inches, the lesser diameter D C 40 inches, and the distance of the ends 8 inches?

576 = Square of $\frac{1}{2}$ AB
400 = Square of $\frac{1}{2}$ DC



976
21,33 = $\frac{1}{3}$ the square of the dif. of the ends

997,33
8 = breadth or distance of the ends

7978,64
1,5708

6382912
5585048o
3989320
797864

12532,8477712 = Solid inches the answer.

E. 2. Required the solidity of the middle zone of a sphere, whose top and bottom diameters are each 3 feet, and the breadth of the zone EF 4 feet?

2,25 = DF²
2,25

4,50
5,33 = $\frac{1}{3}$ square of EF

9,83
4 = Breadth EF

39,32
1,5708

31456
275240
19660
3932

Answer 61,763856 solid feet.

Prob. 13. *To find the solid content of a parabolic conoid.*

RULE. Multiply the area of the base by half the altitude, and the product will be the content.

E. 1. Required the solidity of a parabolic conoid, whose height is 24, and the diameter of its base 34?

34
34

136
102

1156 = Square of the base.

,7854
1156

47124
39270
7854
7854

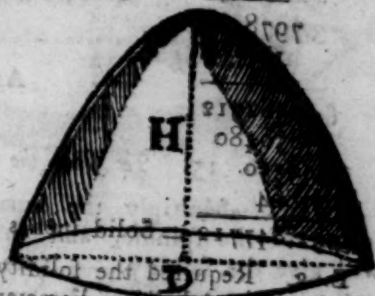
907,9224 = Area of the base
12 = $\frac{1}{2}$ the height

Answer 10895,0808 the solidity.

E. 2.

E. 2. What is the solidity of the paraboloid H D, whose height is 8 inches, and the diameter D of its circular base 24 ?

24
24
———
96
48
———
576 = Square of the diameter
,7854
———
2304
2880
4608
4032
———
452,3904 = Area of the base
4 = $\frac{1}{2}$ the height
———
Answ. 1809,5616 the solidity.



Prob. 14. *To find the solidity of the frustum of a parabolic conoid.*

RULE. Multiply the sum of the squares of the diameters of the two ends, by the height of the frustum, and the product again by ,3927, and it will give the solidity.

E. 1. What is the solidity of the parabolic frustum A B C D, the diameter A B of the greater end being 32, that of the lesser end C D 24, and the height *c d* 20 ?

32
32
———
64
96
———
AB^2 = 1024
CD^2 = 576
———
1600
20 = height *c d*
———
32000
,3927
———
224000
64000
288000
96000
———
Answer 12566,4000 the solidity.

E. 2.

E. 2. What is the solidity of the frustum of a parabolic conoid, the diameter of the greater end being 50, that of the lesser end 38, and the distance of the ends 16?

50
50
———
2500 = □ of the greater base
1444 = □ of the lesser base
———
3944
16 = height
———
63104

63104
,3927
———
441728
126208
567936
189312
———
Answer 24780,9408 the solidity.

Prob. 15. *To find the solidity of a parabolic spindle.*

RULE. Multiply the square of the middle diameter by the length of the spindle, and the product again by ,41888, and it will give the solidity.——Note. ,4188 = $\frac{8}{15}$ of ,7854.

E. 1. The length of the parabolic spindle A B C D is 30, and the middle diameter A B 17; what is the solidity?

17
17
———
AB square = 289
CD = 30
———
8670
,41888
———
69360
69360
69360
8670
34680
———

Answ. 3631,68960 the solidity.

E. 2. The length of a parabolic spindle is 6 feet, and the middle diameter 2 feet; what is the solidity?

2
2
———
4 = Square of the diameter
6 = Length
———
24

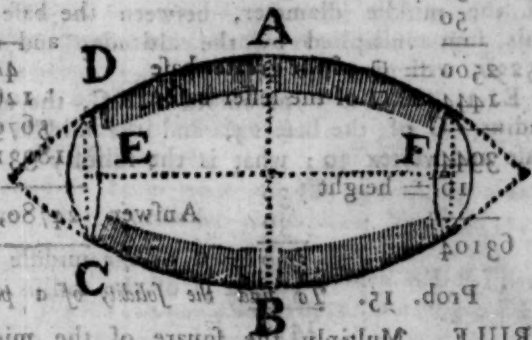
,41888
24
———
167552
83776
———
Answer 10,05312 the solidity.

Prob. 16. *To find the solidity of the middle frustum of a parabolic spindle.*

RULE. Add 8 times the square of the middle diameter, 3 times the square of the less, and 4 times the product of those diameters into one sum; then this sum being multiplied by the length, and the product again by ,05236, will give the solidity.

E. 1.

E. 1. In the middle frustum A B E F of the parabolic spindle, the middle diameter A B is 72, the diameter of the end D C is 40, and the length E F 72; what is the solidity?



72
72

144
504

5184
8

41472 = 8 times the □ of A B
4800 = 3 times the □ of D C
11520 = 4 times the product of A B and D C

57792
72 = Length E F

115584
404544

4161024
,05236

24966144
12483072
8322048
20805120

217871,21664 solidity, the Answer.

E. 2. What is the solidity of the middle frustum of a parabolic spindle, the middle diameter being 16, the diameter at the ends 12, and the length 20?

16
16

256
8

2048 = 8 times the □ of the middle diameter
432 = 3 times the □ of the less diameter
768 = 4 times the product of the two ends.

3248
20 = Length

64960

64960
,05236

389760
194880
129920
324800

Answer 3401,30560 the solidity.

Prob. 17.

Prob. 17. *To find the solidity of an hyperbolic conoid.*

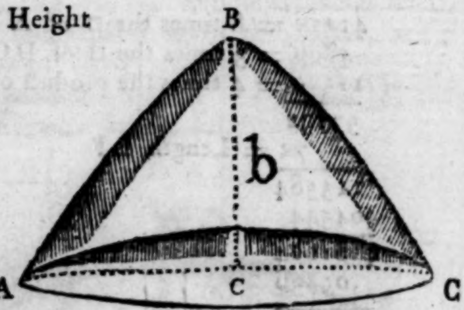
RULE. To the square of the radius of the base, add the square of the middle diameter, between the base and the vertex; and this sum multiplied by the altitude, and the product again by ,5236, will give the solidity.

E. 1. In the hyperboloid A B C, the altitude *b* is 20, the radius *c* C of the base 24, and the middle diameter between the base and vertex 30; what is the solidity?

30
30
900 = □ of the middle diameter
576 = □ of the radius *c* C
1476
20 = Height

29520
,5236

177120
88560
59040
147600



Answer 15456,6720 the solidity.

E. 2. In an hyperbolic conoid, the altitude is 20, the radius of the base 32, and the middle diameter 48; what is the solidity?

32
32
64
96
1024

48
48
384
192

2304 = Square of the middle diameter
1024 = Square of the radius
3328
20 = Height
66560
,5236
399360
199680
133120
332800

Answer 34850,8160 the solidity.

Prob. 18. *To find the solidity of the middle frustum of a circular spindle.*

RULE 1. Divide the square of half the length of the frustum by half the difference of the middle diameter, and that of either of

of the two ends, and $\frac{1}{2}$ this quotient added to $\frac{1}{4}$ of the said difference, will give the radius of the circle.

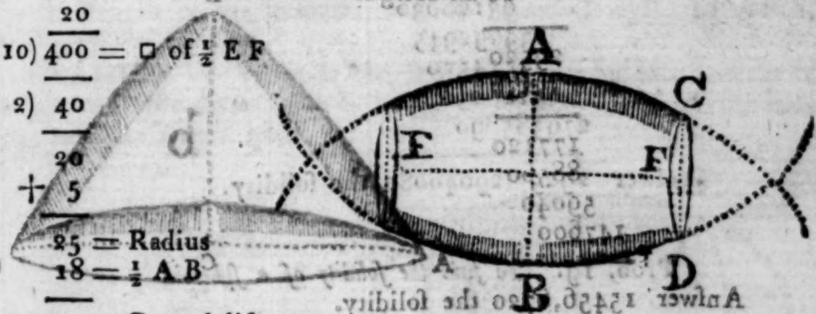
2. Find the central distance, by taking half the middle diameter from the radius of the circle.

3. From the square of the radius, take the square of the central distance, and the square root of the remainder will give half the length of the spindle.

4. From the square of half the length of the spindle, take $\frac{1}{3}$ of the square of half the length of the frustum, and multiply the remainder into the said half length.

5. Take this product from that of the generating area and central distance, and the remainder multiplied by 6,2832, will give the content of the frustum.

EXAMPLE. What is the solidity of the frustum A B E F, whose middle diameter A B is 36, the diameter C D 16, and the length E F 40?



20
20
10) 400 = □ of ½ E F
2) 40
20
+ 5
25 = Radius
18 = ½ A B
7 = Central distance.

36 = A B
16 = C D
2) 20
10 = ½ Difference and
5 = ¼ Difference.
40 = E F
8 = C F
320 = Area of E A C F.

50) 10,0
,2 = Tabular versed sine
,111823 = Tabular segment
2500 = Square of the diameter
55911500
223646
279,557500 = Area of the segm. E A C
320
599,557500 = generating area E A C F
7 = Central distance
4196,9025

25
25
125
50
625 = □ of the radius
49 = □ of central dis.
576 (24 = ½ length of the spindle
4
44) 176
176

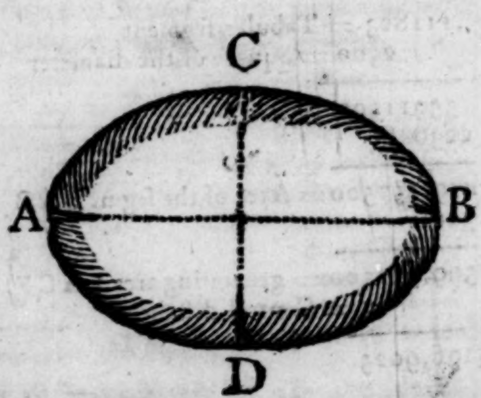
24
24
―――
96
48
―――
576 = □ of $\frac{1}{2}$ the length of the spindle
133,3333 = $\frac{1}{3}$ square of $\frac{1}{2}$ E F
―――
442,6667
20 = $\frac{1}{2}$ E F
―――
8853,3340
4196,9025
―――
4656,4315
6,2832
―――
93128630
139692945
37251452 0
93128630
279385890
―――
Anſwer 29257,29040080 the ſolidity.

Prob. 19. *To find the ſolidity of a ſpheroid*

RULE. Multiply the ſquare of the revolving axe by the fixed axe, and this product again by ,5236, and it will give the ſolidity required.

E. 1. What is the ſolidity of a ſpheroid A B C D, the tranſverſe or fixed axe A B is 40, and the conjugate or revolving axe C D is 30; what is the ſolidity?

30
30
―――
900 = □ of CD
40 = A B
―――
36000
,5236
―――
216000
108000
72000
180000
―――
18849,6000 ſolidity, the Anſwer.



E. 2.

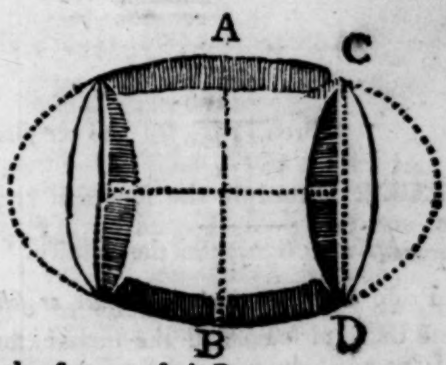
E. 2. What is the ſolidity of a ſpheroid, whoſe fixed axe is 90, and its revolving axe 50?

50	,5236
50	225000
2500	26180
90	10472
	10472
225000	Anſwer 117810,0000

Prob. 20. *To find the content of the middle fruſtum of a ſpheroid.*

RULE. To twice the ſquare of the middle diameter, add the ſquare of the diameter of either of the ends; this ſum multiplied by the length of the fruſtum, and the product again by ,2618, will give the ſolidity

EXAMPLE 1. What is the ſolidity of the middle fruſtum of a ſpheroid, the diameter A B being 32, that of either of the ends 24, and the length 40?



32
32
―
64
96
―
1024
2
―
2048 = Twice the ſquare of A B
576 = Square of C D
2624
40 = Length
―
104960
,2618
―
839680
104960
629760
209920
―
Anſwer 27478,5280 the ſolidity.

Prob. 21. *To find the ſolidity of the ſegment of a ſpheroid.*

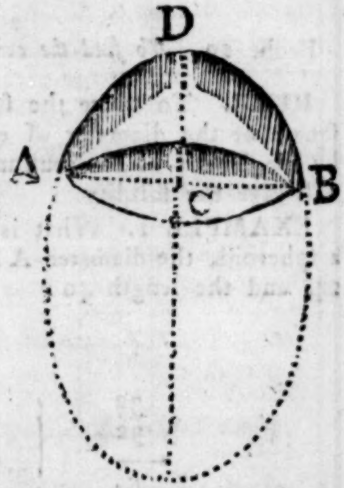
RULE 1. Divide the ſquare of the revolving axis by the ſquare of the fixed axe, and multiply the quotient by the difference between three times the fixed axe, and twice the height of the ſegment.

2. Multiply the product thus found by the square of the height of the segment, and this product again by ,5236, and it will give the solidity required.

EXAMPLE. The axes of a spheroid are 50 and 30; what is the solidity of that segment, whose height is 5, and its base perpendicular to the fixed axe?

```
                                   30
                                   30
□ of fixed axe = 2500) 900 = □ of the revolving axe
                                  ,36
                                  140
                                 1440
                                  36
                                 50,40
                                    25 = □ of DC
                                 25200
                                10080
                               1260,00

                              ,5236
                               1260
                             314160
                            10472
                            5236
Answer                     659,7360 the solidity.
```



Prob. 22. *To find the surface or solidity of the five regular bodies.*

RULE 1. To find the surface, multiply the square of the side of the given body by the tabular superficial multiplier against the given name, and the product will be the superficies.

2. To find the solid content, multiply the tabular number against the given name by the cube of the side, and the product will be the solidity.

A TABLE of the Superficies and Solidity of each Body, whose Side is 1.

No. of Sides.	Names.	Superficies.	Solidity.
4	Tetaredron	1,732051	0,1178511
6	Heaxaedron	6,000000	1,0000000
8	Octaedron	3,464102	0,4714045
12	Dodecaedron	20,645729	7,663119
20	Icosaedron	8,660254	2,181695

E. 1. The side of a tetraedron is 4; what is the superficies?

1,732051 = Tabular multiplier
16 = Square of the side

27,712816 the superficies.

E. 2. The side of a tetraedron is 4; what is the solidity?

,1178511 = Tabular multiplier
64 = Cube of the side

4714044
7071066

7,5424704 the solidity.

E. 3. What is the superficies of an hexaedron, whose linear side is 3?

6 = Tabular multiplier
9 = Square of the side

54 the superficies.

E. 4. The side of a hexaedron is 3; what is the solidity?

27 = Cube of the side
1 = Tabular multiplier

27 the solidity.

E. 5. What is the solidity of an octaedron, whose side is 4?

,4714045 = Tabular multiplier
64 = Cube of the side

18856180
28284270

30,1698880 the solidity.

E. 6. What is the superficies of an octaedron, whose side is 4?

3,464102 = Tabular multiplier
16 = Square of the side

55,425632 the superficies.

E. 7. Required the superficies of a dodecaedron, whose linear side is 3?

20,645729 = Tabular multiplier
9 = Square of the side

185,811561 the superficies.

E. 8. The linear side of a dodecaedron is 3; what is the solidity?

7,663119 = Tabular multiplier
27 = Cube of the side

53641833
15326238

206,904213 the solidity.

E. 9. What is the superficies of an icosaedron, whose linear side is 3?

8,660254 = Tabular multiplier
9 = Square of the side

77,942286 the superficies.

E. 10. The linear side of an icosaedron is 3; what is the solidity?

2,181695 = Tabular multiplier
27 = Cube of the side

15271865
4363390

58,905765 the solidity.

Note. The solidity of any irregular body may be determined by immersing the same in a vessel of water; for the solid content of the additional space, occupied by the fluid on account of the immersed body, will be equal to the solidity of that body.

Prob. 23.

Prob. 23. *To find the ſolidity of timber.*

The menſuration of timber (ſuch as cylinders, pyramids, cones, &c. and their fruſtums) being very troubleſome by the exact rules given in this ſection, an approximation has taken place, and the contents of ſuch trees, or pieces of timber, are generally computed by the following

RULES 1. Multiply the ſquare of the quarter girt (or $\frac{1}{4}$ of the circumference) in inches by the length in feet, and divide that product by 144; the quotient will be the content in feet.

2. Multiply the ſquare of $\frac{1}{5}$ of the girt, or circumference, by twice the length; the product will be the content.

E. 1. What is the content of a tree, whoſe girt is 40 inches, and length 6 feet?

By Rule 1.

4) 40

10 = $\frac{1}{4}$ of the girt
10

100
6 = Length in feet

144 { 12) 600
12) 50

Anſwer 4,16 feet.

By Rule 2.

5) 40

8 = $\frac{1}{5}$ of the girt
8

64
12 = Twice length

144 { 12) 768
12) 64

Anſwer 5,33 feet.

Note. By the above example it appears, that the firſt rule is erroneous, by above $\frac{1}{5}$ part of the true content. The ſecond rule differs from the truth only 1 foot in 190, and is full as eaſy in practice; therefore I think it ought to be brought into general uſe among the practitioners in this art, ſince the eaſe of the other method is the only argument alledged for employing it.

E. 2. How many feet of timber are there in a tree, whoſe girt is 48 inches, and length 9 feet?

By Rule 1.

4) 48

12
12

144
9

144) 1296 (9 feet the Anſwer.

By Rule 2.

5) 48

9,6
9,6

576
864

92,16
18

144) 1658,88 (11,52 feet Anſw.

ARTIFICERS

LXIX. ARTIFICERS WORK.

1. Of BRICKLAYERS WORK.

BRICKLAYERS compute, or value their work, at the rate of a brick and a half thick; and, if a wall be more or leſs than this ſtandard, it muſt be reduced to it by the following

RULE. Multiply the ſuperficial content of the wall in feet, by the number of half bricks in the thickneſs, and $\frac{1}{3}$ of the product will be the content required.

E. 1. How many ſquare rods are there in a wall 52 feet long, 12 feet high, and $2\frac{1}{2}$ bricks thick?

```
          52 = Length
          12 = Height
         ----
272) 624,00 (2,29
     544        5
     ---    -----
      800 3)11,45
      544   -----  rods.  ft.  inch.
      ---   3,816 = 3    221   11 the Anſwer.
      2560
      2448
      ----
       112
```

Note. In practice it is uſual to divide the ſquare feet by 272, omitting the $\frac{1}{4}$ in favour of the workmen.

The uſual way to take the dimenſions of a building, is to meaſure half round its middle, on the outſide, and half round on the inſide; and this will give the true compaſs, in which the thickneſs of the wall is included.

When the height of the building is unequal, take ſeveral different altitudes, and their ſum being divided by the number you have taken, may be conſidered as the mean height.

To meaſure a chimney ſtanding by itſelf, without any party wall adjoining, girt it about for the length, and reckon the height of the ſtory for the breadth; but if it ſtands againſt a wall, you muſt meaſure it round to the wall for the girt, and take the height as before.

When the chimney is wrought upright from the mantle-tree to the cieling, the thickneſs muſt always be the ſame with the jambs; and nothing is deducted for the vacancy between the floor and the mantle-tree, becauſe of the gathering of the breaſt and wings, to make room for the hearth in the next ſtory.

To meaſure chimney ſhafts, or that part which appears above the roof, girt them with a line, about the leaſt place for the length, and take the height for the breadth; and if they be 4 inches thick, ſet down the thickneſs at one brick-work; but if they be 9 inches thick, reckon it at a brick and a half, in conſideration of the plaſtering and ſcaffolding.

All windows, doors, &c. are to be deducted out of the contents of the walls in which they are placed. But this deduction is made only with regard to materials; for the value of their workmanship is added to the bill, at the stated rate agreed on.

E. 2. A gentleman built a wall round his garden, which is 942 feet long, 8 feet high, and 2 ½ bricks thick; how many rods doth it contain?

942 = Length
8 = Height

272) 7536 (27,7
544 — 5
2096 — 3) 138,5
1904 — rods. ft. inch.
46,16 = 46 43 6 the Answer.
1920
1904
16

E. 3. How many rods are there in a wall 82 ½ feet long, 12 ½ feet high, and 2 bricks thick?

82,5
12,5
4125
1650
825
272) 103125 (3,79
816 — 4
2152 — 3) 15,16
1904 — rods. ft. inch.
5,05 = 5 13 7 the Answer.
2485
2448
37

2. Of MASONS WORK.

To masons work belongs all sorts of stone work, and the measure made use of is a foot, either superficial or solid.

RULE 1. For solid measure, multiply continually into one sum the length, breadth, and thickness, and the product will be the solidity.

2. For superficial measure, multiply the length and breadth of every part of the projection together, and the product will be the content.

E. 1. What is the solid content of a wall, whose length is 32 feet 6 inches, its height 5 feet 3 inches, and thickness 2 feet?

By

By Decimals.
32,5
5,25
1625
650
1625
170,625
2
Anſwer 341,250

By Croſs Multiplication.		
ft.	*in.*	
32	6	
5	3	
162	6	
8	1	6
170	7	6
		2
Anſwer 341	3	0 Same as before.

E. 2. Required the ſolid content of a wall, whoſe length is 107 feet, its height 24 feet 6 inches, and its thickneſs 4 feet?

107
24,5
535
428
214
2621,5
4
Anſwer 10486,0 feet.

E. 3. What is a marble ſlab worth, whoſe length is 8 feet 6 inches, and breadth 2 feet 6 inches, at 5*s.* per foot?

8,5		*in.* *s.*
2,5		3 = ¼) 5
425		7 × 3 = 21
170		1 15
		3
		5 5
		0 1 3
21,25 = 21 3 cont.	*f.* *in.*	£.5 6 3 Anſwer.

3. Of CARPENTERS and JOINERS WORK.

Carpenters and joiners work is that of flooring, partitioning, roofing, &c. and is meaſured by the ſquare of 100 feet.

RULE. Multiply the length by the breadth, and divide this product by 100 for the content.

E. 1. If a floor be 25 feet 3 inches long, and 12 feet 6 inches broad, how many ſquares will it contain?

25,25
12,5
12625
5050
2525
Anſwer 3|15,625 = 3 ſquares, 15 feet.

E. 2. How many oaken planks will floor a room, 60½ feet long, and 32½ wide; ſuppoſing the planks each 12 feet long, and 1 foot wide?

Firſt 1 × 12 = 12, the area of one plank
And 60,5 × 32,5 = 1966,25, the area of the room.
∴ 12) 1966,25 (163,854 planks, the Anſwer.

E. 3. If a house within the walls be 22 feet 6 inches long, and 10 feet 3 inches broad, how many squares of roofing will cover it?

2) 10,25 = breadth

5,125 = ½ breadth

15,375

22,5 = Length

76875

30750

30750

Answer 3|45,9375 = 3 squares, 45 feet.

E. 4. If a house measure within the walls 52 feet 8 inches in length, and 30 feet 6 inches in breadth, and the roof be of a true pitch, what will it cost roofing, at 10s. 6d. per square?

First 30,5 + 15,25 = 45,75, width of the roof.

Then 52,66 × 45,75 = 2409 feet = 24 squares 9 feet, the content of the room.

24|09

,525 of a pound = 10s. 6d.

12045

4818

12045

Answer 12,64725 = £.12 12s. 11¼d.

Note. In measuring joiners work, the string is made to ply close to every part of the work over which it passes.

4. Of SLATERS and TILERS WORK.

Slaters and tilers work is measured by the square of 100 feet.

RULE. Multiply the length in feet of the ridge, by the girt from eave to eave, and the product will be the content in feet, which must be reduced to squares as taught in carpenters work.

E. 1. The length of a slated roof is 50 feet 6 inches, and its girt 32 feet 3 inches; how many squares will it contain?

50,5 = Length

32,25 = Girt

2525

1010

1010

1515

Answer 16|28,625 = 16 squares, 28 feet.

Note. In slating it is common to reckon the breadth of the roof 2 or 3 inches broader than what it measures, because the first row is almost covered by the second: this is sometimes practised in tiling.

E. 2. How many squares are contained in a roof, whose length is 70 feet, and depth 30 feet?

First 30 × 2 = 60; and 70 × 60 = 4200 feet = 42 squares, the Answer.

5. PLASTERERS WORK.

Plasterers work is of two kinds; viz. plastering upon laths, called cieling, and plastering upon walls, called rendering, and is measured by the square yard, which is 9 square feet.

RULE. Divide the square feet by 9, and the quotient will be the number of square yards.

E. 1. If a cieling be 60 feet 9 inches long, and 22 feet 6 inches broad, how many yards does it contain?

By Decimals.

```
      60,75
      22,5
    -------
     30375
    12150
   12150
   --------
 9)1366,875
   --------
    151,875
          9
    -------
      7,875
         12
    -------
     10,500
         12
    -------
        6,0
```

Answer 151 yards, 7 feet, 10 inches, 6 parts.

By cross multiplication.

```
     ft.  in.
     60   9
     22   6
   ----------
   1320
     16   6
     30   4   6
   ------------
 9)1366  10   6
   ------------
```

Answer 151 7 10 6 the same as before.

E. 2. There is a quantity of partitioning, that measures 260 feet about, and 18 feet high, and is rendered between quarters; the lathing and plastering of which will be 8*d*. per yard, and the whitening 2*d*. per yard; what will the whole come to?

```
        260
         18
     ------
   9) 4680
     ------
  ⅕) 520 the whole content
     104
     ------
```

			£.	s.	d.
Plastered	416 at 8*d*. per yard =		13	17	4
Whitened	624 at 2*d*. per yard =		5	4	0
		Answer	19	1	4

Note.

Note. In measuring between quarters, there is commonly $\frac{1}{5}$ part of the whole area deducted; but when rendering between quarters is whitened or coloured, there is $\frac{1}{5}$ part to be added to the whole, for the sides of the quarters and braces.

E. 3. What will plastering a cieling, at 10*d.* per yard come to, supposing the length 26 feet, and the breadth 15 feet?

First $26 \times 15 = 390$, and $390 \div 9 = 43$ yds. 3 feet, the content.

By Multiplication.

F.	*d.*
$3 = \frac{1}{3}$)	10
	$10 \times 4 + 3 = 43$
8	4
	4
1 13	4 = 40
2	6 = 3
	$3\frac{1}{3} = \frac{1}{3}$ of 10*d.*
Ans. 1 16	$1\frac{1}{3}$

By the Rule of Three.

yd. *d.*	*yds.* *f.*
1 : 10 ::	43 3
9	9
9	390
	10
9)	3900
12)	433 $3 = \frac{1}{3}$
2|0)	3|6 1

Answer £. 1 16 $1\frac{1}{3}$ same as before.

6. PAINTERS WORK.

Painters work is measured the same as plasterers, and in taking the dimensions, the line must be forced into all the mouldings and cornets.

E. 1. If a room be painted, whose height is 9 feet 6 inches, and its compass 40 feet 3 inches; how many yards does it contain?

By Decimals.

40,25
9,5
20125
36225
9) 382,375
Answer 42,486

By Cross Multiplication.

F.	*I.*	
40	3	
9	6	
382	3	
20	1	6
9) 382	4	6
Answer 42	4 4	6 as before.

E. 2. How many square yards are there in a room, whose height is 12 feet 6 inches, and the circumference 98 feet 9 inches?

98,75
12,5
49375
19750
9875
9) 1234,375

Note. Windows are done at so much a piece, and in carved mouldings, &c. it is customary to allow double the usual measure.

Answer 137,152 = 137 yards, 1 foot, 4 inches, 4 parts.

7. GLAZIERS WORK.

Glaziers take their dimensions in feet, inches, and parts, and estimate their work by the square foot.

E. 1. If a pane of glass be 3 feet, 8 inches, and 3 quarters long, and 1 foot, 4 inches, 1 quarter broad; how many feet does it contain?

By Decimals.	By Cross Multiplication.
3,729	F. I. P.
1,354	3 8 9
14916	1 4 3
18645	3 8 9
11187	1 2 11 0
3729	11 2 3
Answer 5,049066	Answer 5 0 7 2 3 same as before.

Note. In taking the length and breadth of a window, the cross bars between the panes are always included; and no allowance is ever made for round or oval windows, as the trouble of cutting them to those shapes, is more than the value of the glass omitted.

Windows are sometimes measured by taking the dimensions of one pane, and multiplying it continually by the number of panes.

E. 2. There is a window with 16 panes of glass, each 3 feet, 7 inches, 3 quarters long, and 1 foot, 5 inches, 1 quarter broad; how many feet of glass are contained in the said window?

By Decimals.	By Cross Multiplication.
3,643	F. I. P.
1,437	3 7 9
25501	1 5 3
10929	3 7 9
14572	1 6 2 9
3643	10 11 3
5,234991	5 2 10 8 3
4	$4 \times 4 = 16$
20,939964	20 11 6 9 0
4	4
Answer 83,759856	Answer 83 10 3 0 0 same as before.

8. PAVIOURS WORK.

Paviours work is done by the square yard, and the content is found by dividing the area in feet by 9.

E. 1. If a ſtreet be 665 ½ feet long, and 52 ¾ feet wide, how many ſquare yards are contained therein?

By Croſs Multiplication.

F.	I.	
665	6	
52	9	
1330		
3325		
26		
499	1	6
9)35105	1	6
Anſwer 3900	5	1 6

By Decimals.

665,5
52,75
33275
46585
13310
33275
9)35105,125
Anſwer 3900,569 ſame as before.

E. 2. There is a rectangular court-yard, whoſe length is 86 feet 3 inches, and breadth 40 feet 6 inches; how many ſquare yards are contained therein?

86,25
40,5
43125
345000
9)3493,125
Anſwer 388,125

F.	I.	
86	3	
40	6	
3450	0	
43	1	6
9)3493	1	6
Anſwer 388	1	1 6 ſame as before.

VAULTED and ARCHED ROOFS.

Vaulted Roofs are formed by arches ſpringing from the oppoſite walls, and meeting in a line at the top.

Domes are made by the arches ſpringing from a circular or polygon baſe, and meeting in a point at the top.

Saloons are formed by arches connecting the ſide walls to a flat roof, or cieling, in the middle.

Groins are formed by the interſection of vaults with each other.

Prob. 1. *To find the ſolid content of circular, elliptic, or gothic vaulted roofs.*

RULE. Multiply the area of one end by the length of the roof, and the product will give the ſolidity.

EXAMPLE. What is the ſolid content of a ſemi-circular vault, whoſe ſpan is 30 feet, and its length 100 feet.

,7854
900 = Square of 30
2)706,8600
353,4300 = Area of the end
100
Anſwer 35343 Feet the ſolidity.

Prob. 2.

Prob. 2. *To find the concave or convex surface of circular, elliptic, or gothic roofs.*

RULE. Multiply the length of the arch by the length of the vault, and the product will be the superficies.

EXAMPLE. What is the concave surface of a semi-circular vault, whose span is 30 feet, and its length 100?

3,1416
30
2)94,2480
47,1240 = Length of the arch.
100
Answer 4712,4 Feet the concave surface.

Prob. 3. *To find the solid content of a dome; its height, and the dimensions of its base, being known.*

RULE. Multiply the area of the base by $\frac{2}{3}$ of the height, and the product will be the solidity.

EXAMPLE. What is the solid content of a spherical dome, the diameter of whose cicular base is 50 feet, and height 30?

,7854
2500 = Square of 50.
3927000
15708
1963,5000
20 = $\frac{2}{3}$ of the height
Answer 39270,0 Feet the solidity.

Prob. 4. *To find the superficial content of a spherical dome.*

RULE. Multiply the area of the base by 2, and the product will be the superficial content.

EXAMPLE. What is the superfical content of an hexagonal spherical dome, each side of whose base being 20 feet?

2,598076 = Area of an hexagon, whose side is 1.
400 = Square of 20
1039,230400 = Area of the base
2
Answer 2078,460800 Feet the superficial content.

Elliptical domes are measured by the following Rule: Add the height to half the diameter of the base; this sum multiplied by 1,5708, will give the superficial content nearly.

Prob. 5.

Prob. 5. *To find the solid content of the vacuity formed by a groin arch, either circular or elliptical.*

RULE. Multiply the area of the base by the height, and the product again by ,904, and it will give the solidity.

E. 1. What is the solid content of the vacuity formed by a circular groin, one side of its square base being 10 feet, and the height 5?

```
      10
      10
     100 = Area of the base
       5 = Height
     500
    ,904
    2000
   4500
```

Anf. 452,000 feet the solidity.

E. 2. What is the solid content of the vacuity formed by an elliptical groin, one side of its square base being 40 feet, and the height 12?

```
        40
        40
      1600
        12
     19200
      ,904
     76800
   1728000
```

Answer 17356,800

Prob. 6. *To find the concave superficies of a circular groin.*

RULE. Multiply the area of the base by 1,1416, and the product will be the superficies.

E. 1. What is the curve surface of a circular groin arch, one side of its square base being 10 feet?

```
      10
      10
     100 = Area of the base
  1,1416
```

Anf. 114,16 the superficies.

E. 2. What is the concave superficies of a circular groin arch, one side of its square base being 8 feet?

```
        8
        8
       64 = Area of the base
   1,1416
    45664
   68496
```

Anf. 73,0624 the superficies.

Note. Elliptical groins may be measured by the above Rule, the error being too small to be regarded in practice.

The general rule for measuring all arches is this: From the content of the whole, considered as solid, from the springing of the arch to the outside of it, deduct the vacuity contained between the said springing and the under side of it, and the remainder will be the content of the solid part.

QUESTIONS

QUESTIONS in MENSURATION.

Q. 1. What difference is there between a floor 30 feet long, and 20 broad, and two others of half the dimensions? And what will they all three come to at 2*l*. 10*s*. per square?

First $30 \times 20 = 600$ } 300 the difference
And $15 \times 10 \times 2 = 300$ }

1|00) 9|00 Sum

9 squares, at 2*l*. 10*s*. = 2,5*l*.

Then $2,5 \times 9 = 22,5 =$ 22*l*. 10*s*. the answer.

Q. 2. There is a street, whose length is 21,5 feet, and the breadth 12,5 feet, is to be paved with stones, each 15 inches square; how many will it take?

```
   21,5               1,25
   12,5               1,25
  ----               ----
  1075                625
  430                250
 215                125
 ------             ------
1,5625) 268,7500 (172 stones Answ.   1,5625 = Area of one stone
        15625
        ------
        112500
        109375
        ------
          31250
          31250
          -----
              0
```

Q. 3. What is the difference between a solid half foot, and half a foot solid?

First $1728 \div 2 = 864$ solid inches in a solid half foot.
And $6 \times 6 \times 6 = 216$ solid inches in half a foot solid.

Difference 648 solid inches the answer.

Q. 4. Suppose the ball at the top of St. Paul's Church is 6 feet in diameter; what did the gilding of it come to, at $3\frac{1}{2}$ per square inch?

First $6 \times 12 = 72$ inches; then $3,1416 \times 72 = 226,1952$, the circumference.

And $226,1952 \times 72 = 16286,0544$ inches, superficial content.

$\therefore 16286,0544 \times 3,5 = 57001,1904$*d*. = 237*l*. 10*s*. $1\frac{1}{4}$*d*. Answ.

Q. 5. What is the diameter of a circle, whose area is 9 times as much as one of 21 inches diameter?

21
3 = Square root of 9

Answer 63 inches.

Q. 6. The diameter of a circle is 63 inches; the diameter of another circle is required, whose area is 9 times less.

$$\sqrt{9} = 3)\,\underline{63}$$

Answer 21 inches.

Q. 7. The tranverse diameter of an ellipsis is 57, and conjugate 41; the diameter of a circle is required, whose area is equal to that of the ellipsis.

First $57 \times 41 = 2337$, and $\sqrt{2337} = 48,34$, the diameter required.

Q. 8. Our satellite, the Moon, is a globe, in diameter 2170 miles: I would know how many quarters of wheat she would contain, if hollow, $2150\frac{1}{10}$ solid inches being the bushel; and how much yard-wide stuff would make her a waistcoat, was she to be cloathed?

First $2170 \times 2170 \times ,5236 = 5350308686,8$ solid miles. Then a mile $= 1760 \times 1760 \times 1760 = 5451776000$ solid yards. $\therefore 5350308686,8 \times 5451776000 = 29168684491287756800$ solid yards in the moon. Now in a solid yard are $36 \times 36 \times 36 = 46656$ solid inches. $\therefore 29168684491287756800 \times 46656 = 1360894143625521581260800$ solid inches in the moon:—— And a quarter $= 2150,425 \times 8 = 17203,4$. Consequently, $17203,4)1360894143625521581260800(79106115281021285 40,06$ quarters of wheat, the moon would hold if hollow.

Again, $2170 \times 3,1416 = 6817,272$, the circumference of the moon. Then $6817,272 \times 2170 = 14793480,24$ square miles the surface of the moon. And a mile $= 1760 \times 1760 = 3097600$ square yards: Therefore $14793480,24 \times 3097600 = 45824284391424$ square yards of stuff. Q. E. F.

LXX. GAUGING.

GAUGING in general, is nothing more than the application of the foregoing rules in Mensuration of Solids, to particular vessels used by brewers, wine-merchants, &c. because the contents of all sorts of vessels, used for liquors, &c. are computed as though they were really solid bodies. But on account of the different capacities of gallons, bushels, and feet, it is necessary to find factors and divisors for each different denomination; because the dimensions are all taken in inches.

Prob. 1. *To find divisors and gauge points for circles.*

RULE. Divide the solid capacities of each gallon, bushel, &c. by ,7854, the several quotients will be proper divisors for the square of any diameter, to reduce the area into ale, wine, and malt gallons, or bushels; and the square roots of those divisors will be the gauge points for circles.

EXAMPLES.

EXAMPLES.

Divisors.	*Dividends.*	*Quotients.*	*Sq. Rts.*	
,7854)	282,0000	(359,05	18,95	Ale gallons
,7854)	231,0000	(294,12	17,15	Wine gallons
,7854)	268,8000	(342,24	18,5	Malt gallons
,7854)	2150,4200	(737,92	52,32	Malt bushels
,7854)	227,0000	(289,	17	Mash tun gallons.

In like manner any other divisor, or gauge point, may be found, when the solid capacity of the integer is given, whether it be a gallon, bushel, or a foot, &c. And in this manner was the following table computed.

Prob. 2. *To find factors for circles.*

RULE. Divide ,7854 by the solid capacites of each gallon, bushel, &c. and the quotients will be proper multipliers for the square of the diameter of any circle, to reduce the area of that circle into ale, wine, malt gallons, or bushels, &c.

EXAMPLES.

Divisor.	*Divid.*	*Quotients.*	
282,)	,7854	(,002785	the multiplier for Ale gallons
231,)	,7854	(,003389	——— Wine gallons
268,8)	,7854	(,002922	——— Malt gallons
2150,42)	,7854	(,000365	——— Malt bushels
227,)	,7854	(,00346	——— Mash tun gallons.

And in this manner were the other factors found for circles, in the following table.

Prob. 3. *To find factors for squares.*

RULE. Divide unity by the solid capacity of each gallon, bushel, foot, &c. and the quotients will be the proper factors or multipliers.

EXAMPLES.

282,)	1,000000	(,003546	factors for Ale gallons
231,)	1,000000	(,004329	——— Wine gallons
268,8)	1,000000	(,003720	——— Malt gallons
2150,42)	1,000000	(,000465	——— Malt bushels
227,)	1,000000	(,0044	——— Mash tun gallons.

Prob. 4. *To find gauge points for squares.*

RULE. Extract the square roots of the solid capacities of each gallon, bushel, &c. in inches, and it is done.

EXAMPLES.

$\sqrt{282}$	$= 16,79$	the square gauge point for Ale gallons
$\sqrt{231}$	$= 15,19$	——— Wine gallons
$\sqrt{268,8}$	$= 16,39$	——— Malt gallons
$\sqrt{2150,42}$	$= 46,37$	——— Malt bushels
$\sqrt{227}$	$= 15$	——— Mash tun gallons.

A TABLE of FACTORS.

Multipliers, Divisors, and Gauge Points, for Squares and Circles.	Factors for		Divisors for		Gauge Pts. for	
	Squares.	Circles.	Squares.	Circles.	Squ.	Circ.
Inches the area of unity	1	,785398	1	1,27324	1	1,128
A superficial foot	,006944	,005454	144	183,34	12	13,54
A solid foot	,000578	,000454	1728	2200,16	41,57	46,9
Ale gallon	~~,003546~~	~~,002785~~	~~282~~ 277.274	~~359,05~~	~~16,79~~	~~18,95~~
Wine gallon	,004329	,003399	231	294,12	15,19	17,15
Malt or corn bushel	~~,000465~~	~~,000365~~	~~2150,42~~ 2218.192	~~2737,92~~ 2824.29	~~46,37~~	~~52,~~
Malt gallon	,003720	,002922	268,8	342,24	16,39	18,5
Mash tun gallon	,004405	,00346	227	289	15,1	17
Pound of hard soap, cold	,036845	,028939	27,14	34,56	5,21	5,88
Pound of hot soap	,035714	,028050	28,0	35,65	5,29	5,97
Pound of green soap	,038956	,0306	25,67	32,68	5,06	5,72
Pound of white soft soap	,039123	,030731	25,56	32,54	5,05	5,7
Pound of tallow, net	,031844	,025101	31,4	39,98	5,6	6,32
Pound of green starch	,028736	,022565	34,8	44,32	5,9	6,66
Pound of dry starch	,024813	,019491	40,3	51,3	6,35	7,16
Pound of flint glass	,094697	,074405	10,56	13,44	3,25	3,69
Pound of white glass	,071123	,05586	14,06	17,9	3,74	4,34
Pound of green glass	,082102	,064516	12,18	15,5	3,48	3,94
Imperial gallons	.003605	.002833	277.274	353.04	16.65	18.79
Imperial bushel	.000451	.000356	2218.192	2824.29	47.10	53.14

Prob. 5. *To find the area of a square tun, back, or cooler, &c.*

RULE. Multiply one side of the square by itself, and that product multiply or divide by the factor, &c. for squares, and the product, or quotient, will be equal to the area of the same kind as the factor or divisor made use of.

EXAMPLE. What is the area in ale gallons of a square, each of whose equal sides is 30 inches?

By Division.

```
       30
       30
      ----
282)  900 (3,19 Area
      846
      ----
       540
       282
      ----
       2580
       2538
      ----
         42
```

By Multiplication.

Square factor = ,003546
Square of side = 900

Area 3,1914 as before.

Note. In gauging all superficies, the areas are always understood to be one inch deep.

To find the same by the Rule.

Set the proper divisor on A to a side of the square on B; then against the other side of the square on A is the area on B, thus:

A B A B
As 282 : 30 :: 30 : 3,19, the area in ale gallons as before.

Or thus: Set unity on C to the square gauge point on D, and against any side of a square on D is the area on C; thus:

D C D C
As 16,79 : 1 :: 30 : 3,19, as before.

In like manner may the area of the square be found in any of the other denominations mentioned in the Table, by making use of the respective factors, &c. For if, instead of 282, I had divided by 231, 2150,42, the area, would have been found in wine gallons, or malt bushels.

Prob. 6. *To find the area of a parallelogram.*

RULE. Multiply the longest side by the shortest, and that product multiply or divide by the factors or divisors in the table, and the product or quotient will be the area required.

EXAMPLE. The longest side of a parallelogram is 40 inches, and the shortest side 20 inches; what is the area in ale gallons?

By Division.

```
    40
    20
   ---
282)800(2,83 gallons the area
    564
    ---
    2360
    2256
    ----
    1040
     846
    ----
     194
```

By Multiplication.

```
,003546 = Square factor
    800 = Squr. of the two sides
  -----
2,836800 gallons as before. Answ.
```

The same by the Rule.

A B A B
As 282 : 40 :: 20 : 2,83, the same as before.

Prob. 7. *To find the area of a rhombus.*

RULE. Multiply the perpendicular by one of the sides, and that product multiply or divide by the factors, &c. for squares, and the product or quotient will be the area required.

EXAMPLE.

EXAMPLE. The side of a rhombus is 37 inches, and the perpendicular 30 inches; what is its area in ale gallons?

37 = Side
30 = Perpendicular

282) 1110 (3,93 Area in ale gallons.
846
2640
2538
1020
846
174

The same by the Rule.

	A		B		A		B	
As	282	:	37	::	30	:	3,93,	area as before.

Prob. 8. *To find the area of a rhomboides.*

RULE. Multiply the longest side by the perpendicular, and that product multiply or divide by the proper factors in the table, and the product or quotient will be the area required.

EXAMPLE. Required the area in ale gallons of a rhomboides, whose longest side is 60 inches, and perpendicular 37 inches?

37
60

282) 2220 (7,87 Area in ale gallons
1974
2460
2256
2040
1974
66

The same by the Rule.

	A		B		A		B	
As	282	:	60	::	37	:	7,87,	Area as before.

Prob. 9. *To find the area of a plain triangle.*

RULE. Multiply half the longest side by the perpendicular, and that product multiply or divide by the factors in the table, and the product or quotient will be the area required.

EXAMPLE.

EXAMPLE. The length of the base of a triangle is 50 inches, and its perpendicular height 30 inches; what is its area in ale gallons?

30 = Perpendicular
25 = Half base

282) 750 (2,65 Area in ale gallons.
564
1860
1692
1680
1410
270

The same by the Rule.

A B A B
As 282 : 30 :: 25 : 2,65 the area as before.

Prob. 10. *To find the area of a trapezium.*

RULE 1. Divide the trapezium into two triangles, then let fall a perpendicular from each of the angles upon the diagonal, which is a common base to both triangles.

2. Multiply half the diagonal by the perpendiculars, or half the sum of the perpendiculars by the whole diagonal, and that product multiply or divide by the factors in the table, and the product or quotient will be the area required.

EXAMPLE. Required the area in ale gallons of a trapezium, whose diagonal is 60 inches, and the two perpendiculars 15 and 27 inches?

$\left.\begin{array}{l}15\\27\end{array}\right\}$ = Perpendiculars

42
30 = ½ Diagonal

282) 1260 (4,47 Area in ale gallons.
1128
1320
1128
1920
1974
46

Note. The area above found is not quite 4,47, but it is nearer to 4,47 than 4,46, and in the practice of gauging, the officers use but two decimal places of figures; therefore they take the nighest to the second place, whether it be more or less.

The

The same by the Rule.

A B A B
As 282 : 42 :: 30 : 4,47, the area as before.

Prob. 11. *To find a mean geometrical proportion between two given numbers.*

RULE. Multiply the two given numbers together, and extract the square root from their product, which will be the mean required.

E. 1. What is the mean proportional between 36 and 64?

```
     64
     36
    ---
    384
   192
    ---
   2304 (48 the mean required.
   16
    ---
88) 704
    704
    ---
    ...
```

By the Rule.

Set one of the given numbers upon C to the same number upon D; then against the other given number upon C is the number sought upon D; thus:

C D C D
As 36 : 36 :: 64 : 48, mean as before.

E. 2. *by the Rule.* What is the mean between 42 and 30?

C D C D
As 42 : 42 :: 30 : 35,5 the mean required.

Note. This mean last found is a mean proportional between the sum of the perpendiculars and half the diagonal of the trapezium, in Prob. 10. by which the area may be found by the lines C and D, as follows; thus:

D C D C
As 16,79 : 1 :: 35,5 : 4,47, the area as before, found by the lines A and B.

And in like manner may the area of the parallelogram, rhombus, rhomboides, and triangle be found.

Prob. 12. *To find the area of any regular polygon.*

RULE. Divide it into triangles, then find the area of one triangle, as in Prob 9. and because there are as many triangles as there are sides in the polygon, multiply the area of the triangle by the number of sides, and the product will be the area of the polygon.

EXAMPLE.

EXAMPLE. What is the area in ale gallons of a pentagon, whose side is 50 inches, and the perpendicular 34,2 inches?

```
        43,2 = Perpendicular
          25 = ½ one of the sides
        ----
        2160
        864
        ----
282) 1080,0 (3,032 the area of the triangle
                  5
Answer      15,160 Area of the polygon in ale gallons.
```

Prob. 13. *To find the area of a polygon, when the side only is given.*

RULE. Multiply the square of the side of any regular polygon, mentioned in the following table, by the common factor belonging to that polygon, and the product will be the area in inches, ale gallons, wine gallons, or malt bushels respectively.

A TABLE of REGULAR POLYGONS.

No. of Sides.	Names of the Polygons.	Area. Sq. inches.	Area. Ale gallons.	Area. Winegals.	Area. Malt bush.
5	Pentagon	1,72	,006099	,007445	,00078
6	Hexagon	2,598	,009212	,011246	,001208
7	Heptagon	3,633	,012883	,015727	,001689
8	Octagon	4,828	,01712	,0209	,002245
9	Nonagon	6,183	,021925	,026726	,002875
10	Decagon	7,695	,027287	,033311	,003579
11	Undecagon	9,361	,033195	,040523	,004353
12	Duodecagon	11,196	,039702	,048467	,005203

EXAMPLE. Required the area of a pentagon in ale gallons, whose side is 50 inches?

```
        ,006099 = Tabular multiplier
           2500 = Square of side
        -------
        3049500
        12198
        -------
Answer 15,247500 Area in ale gallons, same as before, nearly.
```

In like manner may the area of any other regular polygon mentioned in the table be found, in any of the denominations there mentioned.

Prob. 14. *To find the area of a circle.*

RULE. Square the diameter of the circle, and multiply or divide that ſquare by the factors in the table for circles (page 340), and the product, or quotient, will be the area required.

EXAMPLE. The diameter of a circle is 80 inches; what is its area in ale gallons?

```
                 80
                 80
359,05) 6400,00 (17,82 Area in ale gallons
        35905
        280950
        251335
         296150
         287240
           89100
           71810
           17290
```

By the Rule.

	D		C		D		C	
As	18,95	:	1	::	80	:	17,82,	Area as before.

The Rule being thus ſet, it is like a table, for againſt any diameter on D, is the area in ale gallons on C.

Prob. 15. *To find the area of an ellipſis, or oval.*

RULE. Multiply the tranverſe and conjugate diameters together, and that product multiply or divide by the factors in the table for circles (p. 340), then that product, or quotient, will be the area of the ellipſis.

EXAMPLE. The tranſverſe diameter of an ellipſis is 72 inches, and the conjugate 50 inches; what is its area in ale gallons?

```
                 72 = Tranſverſe diameter
                 50 = Conjugate diameter
359,05) 3600,00 (10,02 Area in ale gallons
        35905
           95000
           71810
           23190
```

By the Rule.

	A		B		A		B	
As	359	:	72	::	50	:	10,02	Area as before.

Note. If a mean proportion between the transverse and conjugate diameters is found, the proportion will be the same as a circle, and may be found on the lines C and D.

LXXI. Of SOLID BODIES.

SOLIDS are comprehended under length, breadth and depth: now by having the area given at one inch deep, it will be easy to find the content of any solid body, at any depth; for if the content at one inch deep be multiplied by the whole depth, the product will be the solid content of the body.

Prob. 1. *To find the content of a solid, whose bases are either squares or parallelograms.*

RULE 1. Multiply the length of the base by the breadth, and that product by the depth; and this last product multiply or divide by the factors, &c. for squares in the table of factors (p. 340), and the product, or quotient, will be the content required.

2. Or find the area of the base by Problems 5, and 6. of the last Section, and multiply that area by the depth, and the product will be the content.

E. 1. There is a cube, each of whose equal sides are 30 inches? what is the content in ale gallons?

```
       30
       30
      ---
      900
       30
      ---
282) 27000 (95,7 Content in ale gallons.
      2538
      ----
      1620
      1410
      ----
       2100
       1974
       ----
        126
```

The area of the base of this solid was found in Problem 5. of the last section to be 3,19 ale gallons.

$\therefore$ 3,19 $\times$ 30 = 95,7, the content in ale gallons, as before.

By the Rule.

	D		C		D		C	
As	16,79	:	30	::	30	:	95,7	the content, as before.

E. 2. Required the content of a prism in ale gallons, whose length is 45,6 Inches, breadth 27,5, and depth 21,5 inches.

```
        45,6 = Length
        27,5 = Breadth
        ----
        2280
       3192
       912
       --------
       1254,00
         21,5 = Depth
        ----
        6270
       1254
      2508
      ----
282) 26961,0 (95,6 Content in ale gallons.
      2538
      ----
      1581
      1410
      ----
      1710
      1692
      ----
        18
```

The proportion by the rule is the same as that for a square base, when there is a mean proportional found between the longest and shortest sides of the base.

1. *For the mean proportional by the Rule.*

$$\begin{array}{ccccccc} & C & & D & & C & & D \\ \text{As} & 27{,}5 & : & 27{,}5 & :: & 45{,}6 & : & 35{,}4, \text{ the mean.}\end{array}$$

2. *For the content by the Rule.*

$$\begin{array}{ccccccc} & D & & C & & D & & C \\ \text{As} & 16{,}79 & : & 21{,}5 & :: & 35{,}4 & : & 95{,}6, \text{ Content as before.}\end{array}$$

In like manner the content of any right-lined solid may be found, either regular or irregular, if by the foregoing Rules you find the area of its base, and multiply that area by its depth.

Prob. 2. *To find the content of a cylinder, by having the depth and diameter given.*

RULE. Multiply the square of the diameter by the depth, and divide by the circular divisors; the quotient will be the content required.

EXAMPLE.

EXAMPLE. There is a cylindrical vessel, whose diameter is 45 inches, and the depth 12 inches; what is the content in ale gallons?

```
          45
          45
        ----
         225
        180
        ----
        2025
          12
        ----
359,05) 24300,00 (67,67 Content in ale gallons.
        215430
        ------
         275700
         251335
         ------
          243650
          215430
          ------
           282200
           251335
           ------
            30865
```

By the Rule.

$$\begin{array}{cccc} D & C & D & C \\ \text{As } 18,95 : & 12 :: & 45 : & 67,67, \end{array} \text{ Content as before.}$$

Prob. 3. *To find the content of a solid, that has two equal ellipsis for its bases, called a cylindroid, by having the diameters and depths given.*

RULE. Multiply the transverse and conjugate diameters together and that product by the depths; and this last product divide by the circular divisor in the table of factors, the quotient will be the content required.

EXAMPLE. What is the content of a cylindroid in ale gallons, the transverse diameter of whose base is 72 inches, the conjugate 50, and depth 12 inches?

```
          72
          50
        ----
        3600
          12
        ----
359,05) 43200,00 (120,3 Content in ale gallons.
        35905
        -----
         72950
         71810
         ------
          114000
          107715
          ------
            6285
```

The

The proportion by the rule is the same as for a cylinder, when there is a mean proportional found between the transverse and conjugate diameters.

1. *For the mean proportional by the Rule.*

C D C D
As 72 : 72 :: 50 : 60, the mean diameter.

2. *For the content by the Rule.*

D C D C
As 18,95 : 12 :: 60 : 120,3, Content as before.

Prob. 4. *To find the content of a pyramid or cone.*

RULE. Find the area of the base by the foregoing rules, which area multiply by one-third part of the height, and the product will be the content.

E. 1. The side of the base of a square pyramid is 27 inches, and the perpendicular altitude 45 inches; what is the content in ale gallons?

27 = Side of the base
27
189
54
282) 729 (2,585 = Area
564 15 = $\frac{1}{3}$ of the height
1650 38,775 Content in ale gallons.
1410
2400
2256
1440
1410
30

D C D C
By the Rule. As 16,79 : 15 :: 27 : 38,77, Content as before.

E. 2. Required the content of a cone in ale gallons, the diameter of whose base is 38 inches, and altitude 45 inches.

38 = Diameter of the base
38
304
114
359,05) 1444,00 (4,022 = Area
15 = $\frac{1}{3}$ of the height
60,330 = Content in ale gallons.

D C D C
By the Rule. As 18,95 : 15 :: 38 : 60,33, Content as before.

Prob. 5.

Prob. 5. *To find the content of the frustum of a pyramid or cone.*

RULE. Find the area of each base by the foregoing rules, and a mean proportional between them; then multiply the sum of those three by $\frac{1}{3}$ part of the depth, and the product is the content required.

E. 1. What is the content in ale gallons of a square pyramid, one side of the greater base being 40 inches, that of the lesser base 30 inches, and the height 60 inches?

First $40 \times 40 = 1600$; and $1600 \div 282 = 5{,}67$, Greater area
Then $30 \times 30 = 900$; and $900 \div 282 = 3{,}18$, Lesser area
$\therefore \sqrt{5{,}67 \times 3{,}18} = \sqrt{18{,}0306} = 4{,}24$ Mean

Sum 13,09
$\frac{1}{3}$ of the height = 20

Content in ale gallons 261,80

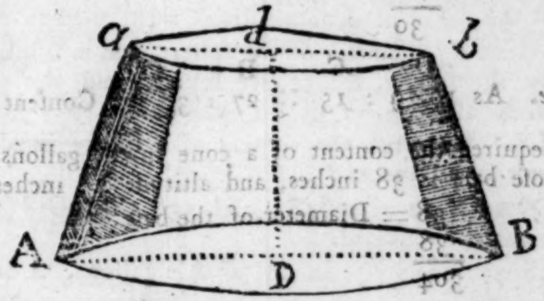
By the Rule.

	D		C		D		C	
Thus as	16,79	:	1	::	40	:	5,67,	Greater area
	16,79	:	1	::	30	:	3,18,	Lesser area.
	C		D		C		D	
Then as	5,67	:	5,67	::	3,18	:	4,24,	Mean

Sum 13,09
$\frac{1}{3}$ of the height = 20

Answer 261,80 Ale gallons as before.

E. 2. What is the content in ale gallons of the lower frustum of a cone, whose diameter at the greater base A B is 38 inches, the diameter of the lesser base *a b* 20,2 inches, and the depth or height *D d* 21 inches?



First $38 \times 38 \div 359{,}05 = 4{,}02$ Greater area
And $20{,}2 \times 20{,}2 \div 359{,}05 = 1{,}13$ Lesser area
$\therefore \sqrt{4{,}02 \times 1{,}13} = \sqrt{4{,}5426} = 2{,}13$ Mean

7,28 Sum
7 = $\frac{1}{3}$ of the depth

Answer 50,96 Content in ale gallons.

By

By the Rule.

		D		C		D		C	
Thus as	{	18,94	:	1	::	38	:	4,02,	Greater area.
		18,94	:	1	::	20,2	:	1,13,	Lesser Area.

	C		D		C		D	
Then as	4,02	:	4,02	::	1,13	:	2,13,	Mean.
							7,28	Sum
							7	= $\frac{1}{3}$ of the height
					Answer		50,96	Same as before.

Prob. 6. *To find the content of a ſphere or globe.*

Every ſphere is two-thirds of its circumſcribing cylinder; therefore, if the cube of the diameter of a ſphere be multiplied by $\frac{2}{3}$ parts of the circular factors, &c. in page 340, or if it be divided by 1, and half of the circular diviſors, then the product or quotient will be equal to the content of that ſphere. From hence may be found factors and diviſors for the cube of the diameter of any ſphere for all the denominations in page 340.

1. For FACTORS.

		Factors for a cylinder.		*Factors for a ſquare.*	
$\frac{1}{2}$ of	{	,000365	=	,000243	Malt buſhels.
		,002785	=	,0018567	Ale gallons.
		,003399	=	,002267	Wine gallons.

2. For DIVISORS the proportion is:

1	:	1,5	::	2737,9	:	4106,99,	Diviſor for malt buſhels.
1	:	1,5	::	359,05	:	538,58,	Diviſor for ale gallons.
1	:	1,5	::	294,12	:	441,18,	Diviſor for wine gallons.

And in this manner may factors and diviſors for a ſphere be found in all the different denominations mentioned in page 340.

RULE. Cube the diameter of the ſphere, and multiply or divide that cube by the factors, &c. above found, and the product or quotient will be the content of the ſphere.

EXAMPLE. Required the content of a ſphere in ale gallons, whoſe diameter is 22 inches?

Firſt 22 × 22 × 22 = 10648; and 538,58) 10648 (19,76 Ale gallons, the content required.

Or 10648 × ,0018567 = 19,76 Ale gallons, as before.

The ſame by the lines D *and* E *on the Rule.*

	D		E		D		E	
As	1	:	,001856	::	22	:	19,76	the content, as before.

The ſame may be performed by the lines C and D on the Rule; for if the ſquare roots of the ſeveral diviſors be extracted, theſe roots will be the gauge points for a ſphere; thus:

The square root of $\begin{cases} 4106,99 = 64,1 \text{ the gauge point for malt bushels.} \\ 538,58 = 23,2 \text{ ditto for ale gallons.} \\ 441,18 = 21,0 \text{ ditto for wine gallons.} \end{cases}$

Then as $\overset{D}{23,2}$, : $\overset{C}{22}$ :: $\overset{D}{22}$: $\overset{C}{19,76}$, Ale gallons as before.

Prob. 7. *To find the content of the lesser frustum of a sphere, by having its diameter and altitude given.*

RULE. To three times the square of half the diameter of its base, add the square of its altitude; this sum multiply by its altitude, and the product multiply or divide by the proper factors for a sphere, in page 352, and the product or quotient will be the content required.

EXAMPLE. What is the content of the frustum of a sphere in ale gallons, the diameter of whose base is 30 inches, and the altitude 9 inches?

225 = Square of ½ the diameter
3
675 = Three times the square of ½ of the diameter
81 = Square of the altitude
756
9 = Altitude

538,58) 6804,00 (12,6 the content in ale gallons.

Prob. 8. *To find the content of the greater frustum of a sphere.*

RULE. Multiply the square of half the diameter by three times the altitude of the frustum, and to this sum add the cube of the altitude: and this last sum multiply or divide by the factors, &c. for a sphere, in page 352; the product or quotient will be the content required.

EXAMPLE. Required the content of the frustum of a sphere in ale gallons, the diameter of whose base is 30 inches, and altitude 25 inches?

225 = Square of ½ the diameter
75 = Three times the altitude
1125
1575
16875
15625 = Cube of the altitude

538,58) 32500,00 (60,3 the content in ale gallons.

Prob. 9. *To gauge a mash tun in the form of the frustum of a cone.*

RULE 1. With some convenient instrument, find the top and bottom diameters, and also the depth of the tun; then add the two diameters together, and take half that sum for a mean diameter, which will be near enough the truth in practice.

2. To find the area at one inch deep, square the mean diameter, and divide that sum by the circular divisor for a mash tun gallon in p. 340, and the quotient will be the area at one inch deep, which serves as a mean for the whole depth.

EXAMPLE. There is a mash tun, whose top diameter is 70 inches, the bottom diameter 50,4 inches and depth 40 inches; what is the area at one inch deep?

70 = Top diameter
50,4 = Bottom diameter

Sum 120,4
Half sum 60,2 = Mean diameter
× 60,2

1204
36120

289) 3624,04 (12,54 Mean area in gallons.

And in this manner was the table of mash tun areas, in section 75, constructed.

By the Rule.

D		C		D		C	
As 17	:	1	::	60,2	:	12,54,	the area as before.

Prob. 10. *To gauge a square mash tun, whose sides are every where straight, from base to base.*

RULE 1. With some convenient instrument measure the length and breadth of the base, and also the depth of the tun.

2. Multiply the length of the base by the breadth, and the product multiply or divide by the factors, &c. for a square mash tun, in page 340, and the product or quotient will be the area at one inch deep.

EXAMPLE. There is a mash tun, whose base is a square, each side measuring 50 inches, and the depth 20 inches; what is the area in mash tun gallons?

50
50

227) 2500 (11 Area in gallons.
227

230
227

3

By the Rule.

D		C		D		C	
As 15,1	:	1	::	50	:	11,	Area as before.

Prob. 11.

Prob. 11. *To deduct the heat out of warm worts, in casting up the gauges.*

RULE 1. By subtraction: Set down the number of warm gallons twice; but the under number must be set one figure farther to the right hand, or, which is the same, remove the dot of the subtrahend one place farther to the left hand, and it is divided by 10; this tenth part subtracted from the number of warm gallons, will leave the quantity of wort to be charged.

2. By multiplication: Multiply the number of warm gallons by ,9, a decimal, and the product will be neat gallons.

E. 1. Suppose a gauge of warm wort were 35 gallons, what must be charged?

By Subtraction.	By Multiplication.
35	35
3,5	,9
Answer 31,5 Neat gallons.	Answer 31,5 Neat gallons.

If the warm worts are in circular bye-tubs, and the depth under diameters, then, instead of the gauge point 18,95, on the line D, make use of 20, to which set the depth on C, and against any diameter on D is the content in neat gallons on C.

E. 2. Suppose the diameter of a bye-tub is 30 inches, and the depth of warm wort in it 10 inches; how many neat gallons does it contain?

By the Sliding Rule.

D C D C
As 20 : 10 :: 30 : 22,5 Neat gallons, the content.

Prob. 12. *To find new gauge points.*

In business it sometimes happens, that when one of the given numbers is set to the gauge point, the other given number will fall off the rule: In this case there must be new gauge points found.—Thus, set unity on C to the old gauge point on D, and against the other 1 on C is the new gauge point on D.

E. 1. Suppose it were required to find a new gauge point to 18,95, the old ale gauge point?

Set unity on C to 18,95 on D, and against the other 1 on C is 59,92, the new gauge point on D for circular ale gallons.

E. 2. To find a new gauge point for wine measure.

Set unity on C to 17,15 on D, and against the other 1 on C is 54,22 on D, the new gauge point for circular wine gallons.

And in like manner may any other new gauge point be found.

Note. These second gauge points are the square roots of the divisors, multipled by 10.—Thus, the circular divisor for ale gallons is 359,05, which multiplied by 10 = 3590,5, whose square root is 59,92, the new gauge point for ale gallons, the same as before found; and after the same manner may any other new gauge point be found.

Prob. 13. *To gauge and inch a mash tun, whose bases are equal squares or parallelograms.*

RULE 1. Find the area, at one inch deep, by the foregoing rules, and multiply that area by the depth for the content in gallons.

2. Reduce both the area and the whole content of the tun into quarters, bushels, and gallons.

3. To find the content upon every inch of the tun's depth, add the area of the first inch reduced, to itself, and the sum will be the content at 2 inches deep; to this sum add the area of the first inch, and you will have the content at 3 inches deep; and in this manner keep continually adding, till you arrive at the depth of the whole tun; which last will be equal to the content of the whole tun, if the work be right.

EXAMPLE. The depth of a square mash tun is 30 inches, the length of the bases 119 inches, and the breadth 102 inches; what is the content of the tun, and also the content at every inch of its depth?

```
        119
        102
        ———
        238
       1190
      —————
227) 12138 (53,47  Area at 1 inch deep
               30 = Depth
          ——————
       8) 1604,10 = Content in gallons
          ——————
       8)  200,5 12 = 200   4,1 = Content in bushels
          ——————
            25,064 = 25q.  0b.  4,1 = Content in quarters

8) 53,47.
   ——————
   6,68 = 6 b.  5 g.  47 = Area at 1 inch in bushels.
               Q.  B.   G.
Then per rule  0   6   5,47 = Content at 1 inch
               0   6   5,47
               ————————————
               1   5   2,94 = Content at 2 inches
               0   6   5,47
               ————————————
               2   4   0,44 = Content at 3 inches.
```

And in this manner proceed till you come to 30 inches, the whole depth of the tun.

The several contents thus found, are placed in the following table against their respective inches through the whole depth.

Note. In filling up the table, I have rejected the odd tenths, if they were under 5; and when they exceed 5, I have added one more to the even gallons.

TABLE.

T A B L E.

Wet Inch.	Q.	B.	G.	Wet Inch.	Q.	B.	G.	Wet Inch.	Q.	B.	G.
1	0	6	5	11	9	1	4	21	17	4	3
2	1	5	3	12	10	0	2	22	18	5	1
3	2	4	1	13	10	6	7	23	19	1	6
4	3	2	6	14	11	5	5	24	20	0	3
5	4	1	4	15	12	4	2	25	20	7	1
6	5	0	1	16	13	3	0	26	21	5	6
7	5	6	6	17	14	1	5	27	22	4	4
8	6	5	4	18	15	0	3	28	23	3	1
9	7	4	1	19	15	7	0	29	24	1	7
10	8	2	7	20	16	5	6	30	25	0	4

The U S E of the T A B L E.

EXAMPLE. Suppose the depth of the goods or grains were 20 inches in the mash tun, how many quarters, &c. would it contain at that depth?

I seek for 20 in the column, under wet inches, and against it are 16 quarters, 5 bushels, 6 gallons, the content.

Prob. 14. *To gauge and inch a copper with a rising crown, and make an allowance for the same.*

EXAMPLE. Let the figure A B C D represent the copper to be gauged and inched.

RULE 1. Take a small cord and fasten one end of it at A, and extend the other end to the opposite side of the copper at B, where make it fast; then with some convenient instrument find the greatest depth of it, that is, the nearest distance from the thread to the bottom at C, and suppose its equal 30 inches: in like manner take the least depth of the copper, which is the nearest distance from the thread to the top of the crown at H, which suppose to be 27 inches; then 30—27 = 3 inches, the height of the crown.

2. To

2. To find the diameter at the bottom of the crown, measure A B the top diameter, which suppose 90 inches; then hold a thread with a plummet at the end of it, so that it may hang over D or C; then measure the distance between the edge of the copper and the place where the threads cut each other, which suppose 5 inches; then 5 + 5 = 10, subtracted from the top diameter 90, leaves 80 = D C, the diameter at the bottom of the crown.

3. To find the content of the liquor required to cover the crown, measure the diameter qq, which touches the top of the crown, and suppose it 82 inches; then by having the top and bottom diameters, and the altitude of the frustum of a cone, or that part qq C D, the content of that part may be found by Prob. 6, Sect. 68, from which the content of the crown must be subtracted.

4. The content of the crown may be found by Prob. 11, Sect. 68, as the segment of a sphere; or thus: multiply the area of the bottom diameter D C by half the altitude of the crown, and the product will be nearly equal to the content of the crown; which subtracted from the part qq C D, will leave the quantity of liquor required to cover the crown.

Ex. To find the quantity of liquor to cover the crown by the last rule.

Diameter qq = 82 } Areas { 18,727
Diameter C D = 80 } Areas { 17,825
Mean Diameter = 80,9 } Areas { 18,23

Area of diameter D C = 17,825
$\frac{1}{2}$ the altitude = 1,5

89125
17825
26,7375

The content of qq C D = 54,782
Content of the crown = 26,7375
Liquor to cover the crown 28,0445

To find the content of the copper, take mean diameters between every 6 or 10 inches, from the top downwards to the crown, viz. the part A B qq; as supposing in this example I found the first mean diameter at 5 inches, from the top 88 inches; at 15 inches, from the top 85,5 inches; and at 25 inches, from the top 82,5 inches, as they are set down in the second column of the following table: then find their respective areas, and set them down against their diameters in the third column, and the contents of the several parts of the depth in gallons in the fourth column; which several contents are reduced to barrels, &c. in the three last columns.

Parts of Depth.	Diameter.	Area.	Content in Gallons.	Content in B.	F.	G.
10	88,0	21,568	215,680	6	1	3,180
10	85,5	20,360	203,600	5	9	8,100
07	82,5	18,956	132,692	3	3	5,192
03	To cover the crown		28,044	0	3	2,044
30	Content of copper		580,016	17	0	2,516

Note.

Note. The two upper contents in gallons are found by removing the dot in the areas one place farther to the right hand; the last content is found by multiplying its correspondent area by the depth 7.

To find the content upon every inch of the copper's depth.

RULE. From the whole content of the copper reduced into barrels, &c. subtract the area of the first 10 inches, and the remainder will be the content, when one inch is dry; and so continue subtracting that area from the remainder, until 10 inches are dry, and you will gain the contents of the first 10 dry inches.—See the whole operation.

Whole	B.	F.	G.	*Contin.*	B.	F.	G.	*Contin.*	B.	F.	G.
Cont. =	17	0	2,516	10 =	10	2	7,336	20 =	4	2	7,736
Subtract.	0	2	4,568		0	2	3,360		0	2	1,956
1 =	16	1	5,948	11 =	10	0	3,976	21 =	4	0	5,780
	0	2	4,568		0	2	3,360		0	2	1,956
2 =	15	3	1,380	12 =	9	2	0,616	22 =	3	2	3,824
	0	2	4,568		0	2	3,360		0	2	1,956
3 =	15	0	5,812	13 =	8	3	5,756	23 =	3	0	1,868
	0	2	4,568		0	2	3,360		0	2	1,956
4 =	14	2	0,744	14 =	8	1	2,396	24 =	2	1	8,412
	0	2	4,568		0	2	3,360		0	2	1,956
5 =	13	3	4,676	15 =	7	2	7,536	25 =	1	3	6,456
	0	2	4,568		0	2	3,360		0	2	1,956
6 =	13	1	0,108	16 =	7	0	5,176	26 =	1	1	4,500
	0	2	4,568		0	2	3,360		0	2	1,956
7 =	12	2	4,040	17 =	6	2	0,816	27 =	0	3	2,544
	0	2	4,568		0	2	3,360	Crown	0	3	2,544
8 =	11	4	7,972	18 =	5	3	5,956	Remain.	0	0	0,000
	0	2	4,568		0	2	3,360				
9 =	11	3	3,404	19 =	5	1	2,596				
	0	2	4,568		0	2	3,360				
10 =	10	2	7,336	20 =	4	2	7,736				
	6	1	3,180		6	1	3,180				
	17	0	2,516		5	3	8,100				
					17	0	2,516				

Note. When I have finished the subtraction with the first area, which is at the 10th dry inch, I add the content of the first 10 inches to the remainder, in order to prove my work, whose sum, if right, will be equal to the content of the copper.

Then

Then I take the second area, and subtract it from the last remainder; and so continue to do till I come to the 20th dry inch; and then to prove my work, I add the contents of the first and second 10 inches to the last remainder, whose sum, if right, will be also equal to the whole content of the copper.

Lastly, I take the third area, and subtract it from the last remainder, and so proceed till I come to the 27th dry inch, whose remainder, if the work be right, will be equal to the quantity of liquor to cover the crown, as found before.—Thus having finished the subtraction, I transfer the several contents to the nearest even gallon, as in the annexed table, and it is done.

Dry Inch.	B.	F.	G.
Full.	17	0	2
1	16	1	6
2	15	3	1
3	15	0	5
4	14	2	1
5	13	3	5
6	13	1	0
7	12	2	4
8	11	3	8
9	10	1	3
10	10	2	7
11	10	0	4
12	9	2	1
13	8	3	6
14	8	1	2
15	7	2	8
16	7	0	4
17	6	2	1
18	5	3	6
19	5	1	3
20	4	2	8
21	4	0	6
22	3	2	4
23	3	0	2
24	2	1	8
25	1	3	6
26	1	1	4
27	0	3	3
To cover the crown	0	3	3
1 half inch	0	1	2,5
2 half inch	0	1	2
3 half inch	0	1	1,5

Prob. 15. *To gauge a back or cooler, and to find the content at every tenth of an inch.*

RULE 1. Find the area at one inch deep, by the foregoing rules, which reduce to barrels, firkins, and gallons; then find the one-tenth of that area by removing the dot one place farther to the left hand, which also reduce to barrels, &c.

2. To find the content at every tenth, add the area or content of the first tenth to itself, and you will have the content at two tenths; again, add the content of the first tenth to the content of the second tenth, and you will have the content of three tenths; and thus by continually adding the content of the first tenth for every tenth, you may tenth a back to what depth you please.

EXAMPLE. Suppose I found the length of a back 272 inches, and breadth 90,8 inches; what is the content at every tenth of the first 3 inches of the back's depth?

Length

Length	272
Breadth	90,8
	2176
	24480
282)	24697,6 (87,58 = Area at 1 inch in gallons.
	2256
	2137
	1974
	1636
	1410
	2260
	2256
Remains	4

B.	F.	G.	
2	2	2,58	= Area at 1 inch reduced.

The area of 1 inch being 87,58 gallons, the tenth part of that sum is 8,758 gallons, which reduced is 0 *b* 1 *f.* 2,58 *g.* and so much will the back hold on every tenth of an inch of its depth.

To find the content at every tenth of an inch, till you come to the depth required, proceed as follows.

	B.	F.	G.
1 Tenth =	0	1	0,258
	0	1	0,258
2 Tenth =	0	2	0,516
	0	1	0,258
3 Tenth =	0	3	0,774
	0	1	0,258
4 Tenth =	1	0	1,032
	0	1	0,258
5 Tenth =	1	1	1,290
	0	1	0,258
6 Tenth =	1	2	1,548

	B.	F.	G.
6 Tenth =	1	2	1,548
	0	1	0,258
7 Tenth =	1	3	1,806
	0	1	0,258
8 Tenth =	2	0	2,064
	0	1	0,258
9 Tenth =	2	1	2,322
	0	1	0,258
Content at 1 inch =	2	2	2,580

Thus, when I come to 1 inch of the back's depth, I find the content to be equal to that before found, which proves the work to be true; and in this manner I proceed to make the following table, to three inches of the back's depth, where all the contents are placed to the nearest even gallon.

Aa2 TABLE.

T A B L E.

Wet Inch. and tenths.	B.	F.	G.
,1	0	1	0
,2	0	2	1
,3	0	3	1
,4	1	0	1
,5	1	1	1
,6	1	2	2
,7	1	3	2
,8	2	0	2
,9	2	1	2
1,0	2	2	3
,1	2	3	3
,2	3	0	3
,3	3	1	3
,4	3	2	4
,5	3	3	4
,6	4	0	4
,7	4	1	4
,8	4	2	5
,9	4	3	5
2,0	5	0	5
,1	5	1	5
,2	5	2	6
,3	5	3	6
,4	6	0	6
,5	6	1	6
,6	6	2	7
,7	6	3	7
,8	7	0	7
,9	7	1	7
3.0	7	2	8

Note. In gauging you must take only even inches in the mash tun: But the copper under-back, round, and squares, must be taken to the nearest half inch, and the backs to the nearest tenth.

Likewise the wet inches are taken in the mash tun, under back and backs; but the dry inches in coppers, rounds, and squares.

Prob. 16. *To find the solidity of the ungulæ, or hoofs, of the frustum of a cone, having the length of the greatest and least diameters, and also the depth given in inches.*

RULE 1. *For the greater hoof.*

Multiply the product of the greater diameter and the frustum's height by the square of the greater diameter, made less by the product of the lesser diameter into a mean proportional between the two diameters; and this last product divide by three times the circular divisors in page 340, multiplied into the difference of the diameters, and the quotient will be the solidity of the greater hoof.

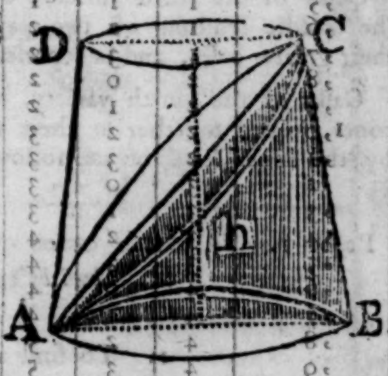
RULE 2. *For the lesser hoof.*

Multiply the product of the lesser diameter and height by the product of the greater diameter, into a mean proportional between the two diameters, made less by the square of the less diameter; and this last product divide by three times the circular divisors in page

page 340, multiplied into the difference of the diameters, and the quotient will be the solidity of the lesser hoof.

EXAMPLE. Let the following figure represent the frustum of a cone, whose greater diameter AB = 36 inches, the lesser diameter DC = 30 inches, and the height h = 20 inches; what is the solidity of the greater hoof ACB, and also the lesser hoof ADC, in ale gallons.

1. For the greater hoof ACB.



36 = AB 36 32 = Mean dia.
20 = h 36 30 = Less = DC

720 = 1st prod. 216 960 = 2*d* prod.
108

1296 = AB square
— 960 = 2*d* product

336
× 720 = 1st product

6720
2352

6462) 241920 (37,43, the content in ale gallons.

2. For the lesser hoof ADC.

30 = DC 36 = AB 30
20 = h 32 = Mean diameter 30

600 = 1st product 72 900 = □ of DC
108

1152
— 900

252
× 600

$3 \times 359 \times 6 = 6462$) 151200 (23,39 ale gallons
37,43
23,39

60,82 = Content in ale gallons of the whole frustum.

LXXII. CASK GAUGING.

IN order to have a right understanding in this matter, it is necessary for the gauger to have some idea of the conic sections, because casks are generally compared to solids, generated by one or other of those sections; which would enable him the better to distinguish to which of these solids the cask's curve bears the nearest resemblance.

 Gangers

Gaugers have reduced those different curvatures of casks to four varieties, &c.

Casks of the first variety are most curved, and are considered as the middle frustum of a spheroid; which is represented by the outer lines of the following figure.

Casks of the second variety, are supposed to be the middle frustum of a parabolic spindle, which is represented by the second lines.

Casks of the third variety, are supposed to be in the form of the middle frustum of two parabolic conoids, joined together at their greater bases, and is represented by the third lines.

Casks of the fourth variety, are formed of the frustums of two cones, joined together at their greatest bases, and are represented by the inner lines of the following figure.

Prob. I. *To find the content of a cask in any of the four varieties, both by pen and rule.*

1. To find a mean diameter.

RULE. Multiply the difference between the head and bung diameters, when it is less than 6 inches.

By { ,68 ,62 ,55 ,5 } For the { 1 2 3 4 } Variety.

Or if the difference between the head and bung exceed 6 inches.

By { ,7 ,64 ,57 ,52 } For the { 1 2 3 4 } Variety.

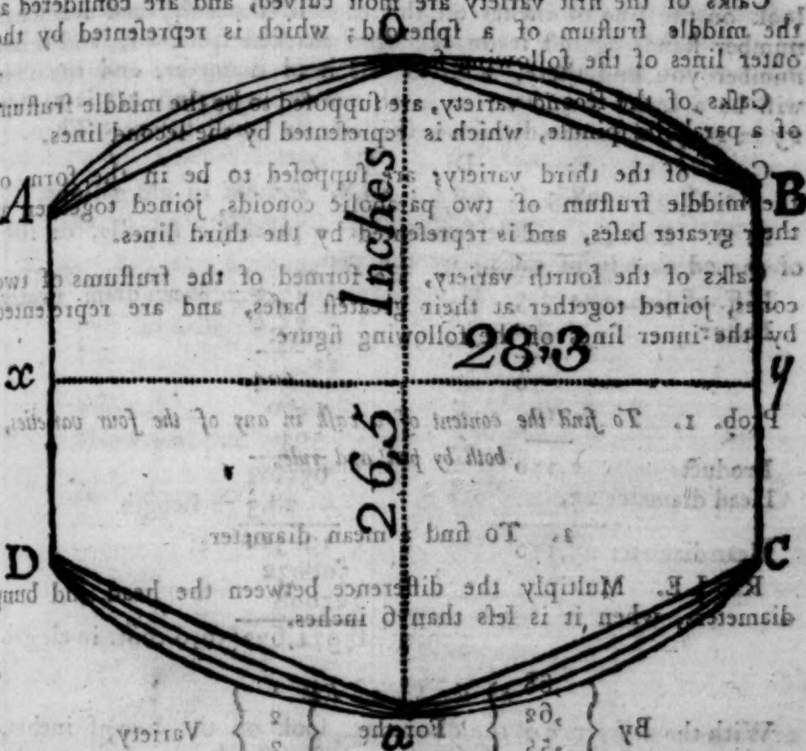
And add the product to the head diameter, then that sum will be a mean diameter.

2. To find the content.

RULE. Square the mean diameter and multiply that square by the length of the cask, and the product multiply or divide by the factors in page 340, and the product or quotient will be equal to the content of the cask.

The

The FIGURE *of the four Varieties of Casks.*



E. 1. Let the above figure represent all the four varieties of casks, whose bung diameter *O a* is 26,5 inches, head diameter A D = B C 23 inches, and length *x y* 28,3 inches; required the content in ale gallons, supposing it to belong to all the four varieties.

For the spheroid, or 1st variety, by the pen.

Bung diameter	26,5	25,4	= Mean diam. nearly
Head diameter	23,0	25,4	
Difference	3,5	1016	
Factor	,68	1270	
		508	
	280		
	210	645,16	= □ of mean diam.
		28,3	= Length
Product	2,380		
Head diameter	23,	193548	
		516128	
Mean diameter	25,38	129032	

359,05) 18258,028 (50,8 cont. in ale gal.

By

By the Sliding Rule.

Find the difference of the diameters, which in this case is 3,5; look on the line of inches on the edge of the rule, and see what number stands against it on the line marked spheroid, and what number you find there, add to the head diameter, and the sum will be a mean diameter; as in this example against 3,5 is 2,4 + 23 = 25,4, the mean diameter, the same as found before. Then,

D C D C
As 18,95 : 28,3 :: 25,4 : 56,8, content in ale gallons.

E. 2. For the middle frustum of a parabolic spindle, or that of the 2d variety of casks, by the pen.

Diff. of diameters	3,5	25,2 = Mean diam. nearly	
Factor	,62	25,2	
	70	504	
	210	1260	
		504	
Product	2,170	635,04	
Head diameter	23,	28,3 = Length	
Mean diameter	25,170	190512	
		508032	
		127008	
		359,05) 17971,632(50,0 cont. in ale gal.	

By the Sliding Rule.

With the difference of the diameters, look on the line of inches, on the edge of the rule, and what number stands against it on the line marked 2d variety, add to the head diameter, which sum will be a mean diameter. In this example the difference is 3,5, against which on the line marked 2d variety, is 2,2 nearly, which added to the head diameter is 25,2, the mean diameter. Then,

D C D C
As 18,95 : 28,3 :: 25,2 : 50, content as before.

E. 3. For the frustum of two parabolic conoids, or 3d variety of casks, by the pen.

Diff. of diameters	3,5	24,9	
Factor	,55	24,9	
	175	2241	
	175	996	
		498	
Product	1,925	620,01	
Head diameter	23,	28,3 = Length	
Mean diameter	24,925	186003	
		496008	
		124002	
		359,05) 17546,283(48,8 cont. in ale gal.	

By

By the Sliding Rule.

With the difference of the diameters, look on the line of inches on the edge of the rule, and what number you find against it on the line marked 3d variety, add to the head diameter, and that sum will be a mean diameter. In this example the difference is 3,5, against which, on the line marked 3d variety, is 1,9, which added to the head diameter, is 24,9, the mean diameter. Then,

	D	C		D	C	
As	18,95	: 28,3	::	24,9	: 48,8,	content as before.

E. 4. For the frustum of two cones, joined together at their greatest bases, called the 4th variety, by the pen.

Diff. in diameters	3,5	24,75
Factor	,5	24,75
		12375
Product	1,75	17325
Head diameter	23,	9900
		4950
Mean diameter	24,75	
		612,5625
		28,3
		18376875
		49005000
		12251250

359,05) 17335,51875 (48,281 content in ale gals.

By the Sliding Rule.

With the difference of diameters, look on the line of inches on the edge of the rule, and what number stands against it, on the line marked F C, add to the head diameter, whose sum will be a mean diameter. In this example against 3,5, on the line F C, is 1,75, which added to the head diameter = 24,75, the mean diameter. Then,

	D	C		D	C	
As	18,95	: 28,3	::	24,75	: 48,3,	content in ale gallons.

The content of the several varieties may be found in wine measure, by dividing by its proper divisor, page 340; and the proportion by the rule for wine gallons would be, as the wine gauge point on D is to the length of the cask on C, so is the mean diameter on D to the content in wine gallons on C.

EXAMPLE. What is the content of the first variety in wine gallons, see Example 1, by the rule.

	D	C		D	C	
As	17,15	: 28,3	::	25,4	: 63,5,	content in wine gallons.

And in this manner may the content of the other three varieties be found in wine gallons.

LXXIII.

LXXIII. ULLAGING of CASKS.

SEVERAL writers on this subject have shewn how to ullage a cask, by a table of segments, calculated for a cylindrical cask; but, because that requires you always to have that table ready at hand, and doth not always agree with the lines of segments, on the sliding-rule; I shall here omit it, and shew how to effect the same by pen and sliding-rule.

Prob. I. *To ullage a lying cask by the pen, having the bung diameter, wet inches, and the content of the cask given.*

RULE 1. "Divide the wet or dry inches by the bung diameter, and if the quotient be under ,500, subtract a fourth part of what that quotient wants of ,500, from the quotient, and the remainder multiply by the content of the cask, and the product will be equal to the quantity of liquor in the cask.

2. When the quotient of the wet inches divided by the bung diameter exceeds ,500, then add a fourth part of that excess to the quotient, and that sum multiplied by the content of the cask, will produce the content of the liquor in the cask; but if the dividend was dry inches, the product is what it wants to fill it up.

EXAMPLE. There is a cask, whose bung diameter is 31 inches, (wet inches 21, dry inches 10) and content 75,37 gallons; what liquor is there in the cask, and how many gallons will fill it up?

31)21,000(,677 = more than ,500
,500
4),177(,044
,677 + Quotient

The area of segment ,721 × 75,37 = 54,34177 gallons of ale, in the cask.

2. For the vacuity, or what will fill it up.

,500
31)10,000(,322 = less than ,500

4),178(,0445 } subtract
,322

Area of the segment - -	,2775
Content of the cask -	75,37
	19425
	8325
	13875
	19425
Wants to fill the cask -	20,915175
+ What is in the cask	54,34177
Content of the cask -	75,256945 ale gallons.

By

By the Sliding Rule.

1. As the bung diameter on the line of numbers on the little slider, marked N, is to 100 on the line of segment, marked S L, so is the wet or dry inches on the line of numbers N, to a segment upon S L; which reserve.

2. As 100 upon A, is to the cask's content upon B, so is the reserved segment upon A, to the quantity of liquor in the cask.

	N		SL		N		SL	
As	31	:	100	::	21	:	73,8;	which reserve.
	A		B		A		B	
And as	100	:	75,37	::	73,8	:	56,6,	ullage of liquor in the cask.

To find the vacuity by the rule, you must work in all respects as you did for the ullage, only, instead of the wet inches, you must make use of the dry inches: Thus,

	N		SL		N		SL	
As	31	:	100	::	10	:	26,2;	which reserve.
	A		B		A		B	
Then as	100	:	75,37	::	26,2	:	19,7,	the vacuity of cask.
In the cask			—		—		55,6	
Content of the cask			=				75,3,	nearly the same as before.

Prob. 2. *To find the content of the ullage of a standing cask, by the pen.*

RULE. Divide the wet or dry inches by the length of the cask, and if the quotient exceeds ,500, add to the said quotient one tenth part of the excess; but if it be under ,500, subtract one tenth part of what it wants of ,500; then let this sum or difference be multiplied by the content of the cask, and the product will be equal to the quantity of liquor therein, if the dividend was the wet inches; but if it was the dry inches, it gives the vacuity, or what it wants to fill it up.

EXAMPLE. Let us suppose a spheroidal cask posited as above, the length 32,5 inches, the bung 27, the head 23, the content of this cask will be 59,95 ale gallons: then let the wet inches be 8,5, I demand how much liquor there is in the cask, and also the vacuity?

```
                 ,5000
32,5) 8,50000 (,2615 = under ,500

        10 ),2385 (,02385 = 1/10 of the wants of ,5000
                   ,2615 = the wet quotient

        Difference  23765
        The Content  59,95
                    118825
                   213885
                  213885
                 118825

                 14,2471175 = the content of the liquor.
```

32,5) 24,00000 (,7384
,5060

10) ,2384 (,02384
+ ,73840 Dry quotient

,76224
59,95 = Content

381120
686016
686016
381120

45,6962880 = Vacuity
14,2471175 = Ullage

59,9434055 = Content of the cask.

By the Sliding Rule.

N SL N SL A
As 32,5 : 100 :: 8,5 : 24; which reserve. Then,
A B A B
As 100 : 59,95 :: 24 : 14,3, content of the liquor.

For the Vacuity.

N SL N SL
As 32,5 : 100 :: 24 : 76; which reserve. Again,
A B A B
As 100 : 59,95 :: 76 : 45,6 the vacuity
+ 14,3 the ullage

59,9 Content of the cask.

Note. The difference between the sum of the separate parts thus found, and the whole content of the cask, is occasioned by the line of segments being adapted to one particular sort of casks only; which is not material, and near enough the truth in practice.

LXXIV. MALT GAUGING.

Prob. 1. *To gauge a maltster's square, or oblong cistern.*

RULE 1.

MEASURE the length and breadth of the cistern, in several places, and in case you find any variation, add the lengths or breadths together, and divide their sum by the number of dimensions taken of each, and the quotient will be a mean length or breadth; and at the same time also, the depth of the cistern.

2. To find the area of the cistern, multiply the length by the breadth, and that product multiply or divide by the proper factors for squares malt bushels, in page 340; and the product or quotient will be equal to the area.

EXAMPLE.

EXAMPLE. There is a ciſtern, whoſe length is 114 inches, breadth 58,5 inches, and the depth 36 inches; what is the area at one inch deep?

```
            114 = Length.
            58,5 = Breadth.
            ----
            570
           912
           570
           -----
2150,42) 6669,00 (3,1 = Area in buſhels.
         645126
         ------
          217740
          215042
          ------
            2698
```

By the Rule.

	A		B		A		B	
As	2150,42	:	114	::	58,5	:	3,1	= Area as before.

Note. If any depth in inches be multiplied by the area, the product will be equal to the content of the malt in the ciſtern.

Prob. 2. *To gauge a maltſter's round ciſtern.*

RULE 1. Take mean diameters between every ſix, or ten inches of the depth, and at the ſame time take the depth.

2. Find the area of each mean diameter; then ſquare the diameters, and multiply or divide each ſquare by the circular factors for malt buſhels, in page 340, and the product or quotient will be equal to the ſeveral areas required.

EXAMPLE. Suppoſe the depth of the ciſtern be 30 inches, and the diameter at 5 inches from the baſe 29,6 inches, at 15 inches from the baſe the diameter is 33 inches, and 25 inches from the baſe the diameter is 36,2 inches; required the reſpective areas of the mean diameters in malt buſhels?

```
Mean diameter = 29,6
                29,6
                ----
                1776
               2664
               592
               ------
2737,92) 876,160 (,32 the lower area.
         821376
         ------
          547840
          547584
          ------
             256
```

And in this manner the other areas are found to be ,40, and ,48.

By the Rule.

	D		C		D		C	
As	52,32	:	1	::	29,6	:	,32	= 1st area
	52,32	:	1	::	33,0	:	,40	= 2d area
	52,32	:	1	::	36,2	:	,48	= 3d area

Note. All depths that are taken in this cistern, must be multiplied by the respective areas to which they belong.

Prob. 3. *To gauge a couch of malt, in a square or oblong frame, and find the content of the same.*

RULE. Multiply the length, breadth, and depth together, and that product divide by the square divisor for malt bushels, in page 340, and the quotient will be the content of malt in the couch.

EXAMPLE. What is the content of a couch in malt bushels, whose length is 105 inches, breadth 104 inches, and depth 20 inches?

```
   105 = Length
   104 = Breadth
  ----
   420
  1050
 -----
 10920
    20
 -----
2150,42) 218400,00 (101,5, content in bushels.
```

By the Rule.

	A		B		A		B	
As	20	:	105	::	104	:	101,5,	content as before.

Prob. 4. *To find the content of a couch, or floor of malt, having the length, breadth, and depth given in inches.*

RULE. Multiply half the length of the floor by the breadth, and that product by the depth; from this last product cut off three figures to the right hand, and it will give the content of the floor seven bushels too much in every 100; which excess may be deducted, either by subtraction or multiplication.

If the malt divisor had been 2000, this method would have answered without any deduction; but since it is 2150, the multiplying by half the length or breadth, will give too much; to remedy this, if you multiply the product of the length, breadth, and depth, by 93, you will find the true content.

Or, if you subtract seven bushels for every 100, and ,7 for every 10 bushels, or ,07 for every single bushel, you will have the content.

EXAMPLE. Suppose the length of a floor be 400 inches, breadth 215, and depth 4 inches; what is the content in malt bushels?

215 =

215 = Breadth
200 = Half the length

43,000
4 = Depth

172
,93

516
1548

159,96 Content in bushels.

By Subtraction.

172 = Product
12,04

159,96 Content as before.

For 100 deduct 7 bushels
For 70 deduct 4,9
For 2 deduct 0,14

Sum — 12,04

LXXV. MONEYING OF CHARGES.

Problem 1. *To money Goods at 1 ¼d. per pound.*

R U L E.

CUT off the right hand figure, which count so many pence and farthings, and those on the left hand will be so many shillings and half-pence; or the same may be found by the cash table, at 1 ¼*d*. per pound, at the end of this section.

EXAMPLE. What is the duty of 364 pounds of sheep-skins, at 1 ¼*d* per pound?

36|4 = 36*s*. 36 half-pence, and four times five farthings.

	£.	s.	d.
36 shillings =	1	16	0
36 half-pence =	0	1	6
4 times 1 ¼*d*. =	0	0	5
Answer	1	17	11 the duty required.

Prob. 2. *To money goods at the rate of 30l. per cent.*

RULE. Divide the value of the goods by 5, and to the quotient add its half, whose sum will be the duty required; or the same may be found by the table of 30 per cent. at the end of this section.

EXAMPLE. Suppose the value be six pounds, ten shillings, and ten-pence; what will the duty of the same amount to?

	£.	s.	d.	
5)	6	10	10	
	1	6	2	
+	0	13	1	= One half
Answer	1	19	3	the duty required.

Or the same may be found by multiplying the value of the goods by ,3.

Prob. 3.

Prob. 3. *To money goods at the rate of 15l. per cent.*

RULE. Divide the given value of the goods by 5, and from the quotient subtract one fourth part; the remainder will be the duty required: or, the same may be found by the table of 15 per cent. at the end of this section.

EXAMPLE. Suppose the value be five pounds, eight shillings, and four-pence; what is the duty?

	£.	s.	d.	
5)	5	8	4	
	1	1	8	
—	0	5	5	$= \frac{1}{4}$
Answer	0	16	3	the duty required.

Or the same may be found by multiplying the value of the goods by ,15.

Prob. 4. *To money goods at the rate of 18l. per cent.*

RULE. Divide the value of the goods by 5, and from that quotient subtract half of its one fifth; the remainder will be the duty required.—Or the same may be found by the table at 18 per cent. ad valorem, at the end of this section.

EXAMPLE. Let the value of the goods be 23l. to find the duty.

	£.	s.	d.	
5)	23	0	0	
5)	4	12	0	$= \frac{1}{5}$
2)	0	18	$4\frac{1}{4}$	$= \frac{1}{5}$
—	0	9	$4\frac{2}{5}$	$= \frac{1}{2}$ of the $\frac{1}{5}$
Answer £	4	2	$9\frac{3}{5}$	the duty required.

Or the same may be found by multiplying the value of the goods by ,18.

Prob. 4. *To find the duty of any number of barrels of victuallers strong beer, at 8s. per barrel.*

RULE. Multiply the given number of barrels by 4, and the product will be pounds, except the units figure of the product, which will be so many two shillings; to which add the money for the firkins (if any), and that sum will be equal to the duty of the whole.

EXAMPLE. What is the duty of $325\frac{1}{2}$ barrels of victuallers strong beer, at eight shillings per barrel?

	325	= number of barrels
	4	
	130,0	
+	4s.	$= \frac{1}{2}$ barrel
Answer £.	130 4	

Prob. 5. *To find the duty of any number of barrels of victuallers small beer, at 10s. 4d. per barrel.*

RULE. To the given number of barrels, add one-third of that sum, and if there be any quarters, add proportionably for them. The sum of the whole will be equal to the duty in shillings and pence, which reduce into pounds, and it is done.

EXAMPLE. What is the duty of 295½ barrels of victuallers small beer, at one shilling and four-pence per barrel?

3)	295		= number of barrels
	98	4	= ⅓ of the given number
	0	8	= ½ barrel
2|0)	39|4	0	
£.	19	14 0	Answer.

And in this manner may the amount of the duty of any other sort of goods be found: or the same may be found by the cash tables for victuallers strong and small beer, at the end of this section.

Prob. 6. *To find the drawback of any number of barrels of victuallers strong beer, at 1s. 8d. or small beer at 4d.*

RULE. The drawback of one barrel of either strong or small beer, being multiplied by the number of barrels, gives the answer.

E. 1. What is the drawback of 91 barrels of victuallers strong beer, at one shilling and eight-pence per barrel?

	s.	d.	
	1	8	
		9	× 10 + 1 = 91
	15	0	
		10	
	7	10 0	
+		1 8	
£.	7	11 8	Answer.

E. 2. Suppose a victualler be charged with 25 barrels of small beer, what must be allowed him for drawback at 4d. per barrel?

		d.	
		4	
		5	× 5 = 25
	1	8	
		5	
£.	0	8 4	Answer.

Or the same may be found by the table of drawbacks for victuallers strong and small beer, at the end of this section.

Prob. 7.

Prob. 7. *To reduce the groſs buſhels of malt, taken in the ciſtern or couch, and floor, to neat buſhels.*

RULE. It is ſuppoſed that barley, after it is firſt wetted or ſteeped in the ciſtern, and ſtood there its proper time, and from thence emptied into the couch, and lain there about 30 hours, riſes or increaſes to about $\frac{1}{5}$ part more than it was before; therefore, 4 buſhels in every 20 are to be allowed for that increaſe.

But when the malt has been out of the ciſtern above 30 hours, it is deemed to be a floor of malt; and it is ſuppoſed that a buſhel of dry barley, thus wetted and ſteeped, &c. and afterwards thrown out into the floor, and there grown according to the uſual cuſtom, will increaſe or riſe to two buſhels, or double to what it was before; therefore, 10 buſhels in every 20 are to be allowed for that increaſe.

In order to find the proper factors to reduce each of theſe buſhels to their equivalent value in neat buſhels, obſerve the following method.

1. For the ciſtern or couch buſhels.

From 20 = buſhels
Subtract 4 = $\frac{1}{5}$

Remains 16 = $\frac{4}{5}$ = ,8, factor for ciſtern or couch buſhels.

If any number of buſhels, from ciſtern or couch, be multiplied by the above factor, the product will be equal to the neat buſhels.

EXAMPLE. In 200 buſhels, from ciſtern or couch, how many neat buſhels?

200
,8

Anſwer 160,0 neat buſhels.

2. For the floor buſhels.

There being 10 buſhels in every 20 to be allowed for the increaſe of floor buſhels, therefore the floor buſhel is = $\frac{10}{20}$ = $\frac{1}{2}$ = ,5, a common factor for floor buſhels.

If any number of floor buſhels be multiplied by the above factor, the product will be equal to the neat buſhels.

EXAMPLE. In 260 buſhels from the floor, how many neat buſhels?

260
,5

Anſwer 130,0 neat buſhels.

Prob. 8. *To find factors for reducing of couch buſhels into floor buſhels; and on the contrary, for reducing floor buſhels to couch buſhels.*

RULE. The factors found in the laſt prob. are to each other as unity to the required factors; therefore, the proportions are as follows:

1ſt. As

1st. As ,5 : ,8 :: 1 : 1,6, the factor for couch bushels.
2d. As ,8 : ,5 :: 1 : ,625, the factor for floor bushels.

Or the charge may be found, by multiplying the floor bushels by ,625; and if the product be more than the bushels from the best, the charge will be from the floor, but if less, then from the best of the cistern and couch.

EXAMPLE. Suppose the content of a floor gauge of malt be 261 bushels, and the content of the best, cistern, or couch, were 200 bushels; from which will the charge arise?

Unity. Fact. Couch.
As 1 : 1,6 :: 200
200

Answer 320,0 floor bushels.

Or thus, 200
1,6

Answer 320,0 the same as before.

By the Rule.

B A B A
As 1 : 1,6 :: 200 : 320 floor bushels.

By which I find the amount of the couch is = 320 floor bushels, that is 59 bushels more than the number of floor bushels before found; therefore, the charge will arise from the couch.

Or the same may be found by this proportion:

B A B A
As 1 : ,625 :: 320 : 200, couch bushels.

From whence it also appears that the couch gauge is the best.

Prob. 9. *To find the duty of any number of bushels from the cistern or couch.*

RULE. The duty of 1 bushel of malt from cistern or couch, with the allowance of 4 in 20, or the $\frac{1}{5}$ part is 1*s.* 0 $\frac{1}{2}$*d.* ,4 parts, which reduced to the decimal of a pound sterling, is ,0525, the common factor: now if any number of bushels from cistern or couch be multiplied by the above factor, the product will be equal to the duty in pounds, and decimal parts of a pound.

E. 1. How much will the duty of 120 bushels of malt, from the cistern or couch, amount to, at 1*s.* 0 $\frac{1}{2}$*d.* ,4 tenths per bushel?

,0525 = Factor
120 = No. of bushels

10500
525

6,3000 Answ. 6*l.* 6*s.* 0*d.*
20
6,0

E. 2. How much will the duty of 260 bushels of malt from the best of the cistern or couch amount to, at 1*s.* 0 $\frac{1}{2}$*d.* ,4 per bushel?

,0525 = Factor
260 = No. of bushels

31500
1050

Ans. 13,6500 = 13*l.* 13*s.* duty.

 But

But to save the trouble of multiplying, I have inserted a table to money couch bushels, at the end of this section, which will shew all the separate duties of any number of bushels, from cistern or couch, at one view, with a total column of the whole.

Prob. 10. *To find the duty of any number of bushels from the floor.*

RULE. This may be done by a factor, which is found as follows: The duty of 1 bushel of malt from the floor, with the allowance of 10 in every 20, is 7*d*. 3 qrs. ,5 tenths, which reduced to the decimal of a shilling, is = ,65625, the factor.—Now if any number of bushels from the floor be multiplied by the above factor, the product will be equal to the duty in shillings, and decimal parts of a shilling.

EXAMPLE. What is the duty of 400 bushels of malt from the floor, at 7 ¾*d*. ,5 tenths per bushel?

```
      ,65625 = Factor
         400
    ----------
2|0) 2|62,50000
    ----------
      13,125
          20
      ------
       2,500
          12
      ------
         6,0 Answer 13l. 2s. 6d. the duty required.
```

Or thus: 7 ¾*d*. ,5 tenths, reduced to the decimal of a pound, = ,0328125, the common factor, by which if you multiply any number of bushels from the floor, the product will be the duty in pounds, and decimal parts of a pound.

EXAMPLE. What is the duty of 80 bushels of malt from the floor, at 7 ¾*d*. ,5 per bushel?

```
 ,0328125 = Factor
       80 = No. of bushels
 ---------
2,6250000
       20
 ---------
   12,500
       12
 ---------
      6,0 Answer 2l. 12s. 6d. the duty required.
```

And in this manner may the duty of any number of bushels of malt, from the floor, be found: but for the satisfaction of my readers, I have inserted a table to money floor bushels, at the end of this section, which will shew at one view all the separate duties from one tenth of a bushel to one thousand bushels, with a total column of the whole.

Prob. 11.

Prob. 11. *To find the duty of any number of barrels of common brewers strong beer, at eight shillings per barrel, with the allowance of 2 ½ in every 23 barrels.*

RULE. This may be done by the cash tables for common brewers strong beer, page 385, or by a factor, which is found thus:

	£.	s.
The duty of 23 barrels, at 8s. per barrel =	9	4
Allowance out of 23 barrels is 2½, and duty =	1	0
Duty of 23 barrels of common brewers X beer =	8	4

Then the proportion for the factor is,

Bar.		£.		Bar.		Decimal.	
As 23	:	8,2	::	1	:	,35652,	the common factor.

Therefore, if any number of barrels, and quarters of a barrel, reduced to a decimal, be multiplied by it, the product will be equal to the duty in pounds, and decimal parts of a pound.

EXAMPLE. What is the duty of 150½ barrels of common brewers strong beer, at eight shillings per barrel, with the allowance of 2½ barrels in 23?

```
   ,35652 = factor for strong beer
   150,5 = given number of barrels
  ------
   178260
  1782600
  35652
 ---------
 53,656260 = 53l. 13s. 1¼d. the duty.
```

Prob. 12. *To find the duty of any number of barrels of common brewers small beer, at one shilling and four-pence per barrel, the allowance being 2½ in every 23 barrels.*

There must be a factor found for common brewers small beer, as well as strong, in the following manner:

	£.	s.	d.
The duty of 23 barrels at 1s. 4d. per barrel =	1	10	8
The allowance of 2½ barrels — — =	0	3	4
The duty of 23 bars. of common brewers small beer =	1	7	4

Then the proportion for the factor is,

Bar.		£.		Bar.		Decim. of £.	
As 23	:	1,366	::	1	:	,05942,	the factor.

Therefore, if any number of barrels, and quarters of a barrel, reduced to a decimal, be multiplied by it, the product will be equal to the duty in pounds, and decimal parts of a pound.

EXAMPLE. What will the duty of 152¼ barrels of common brewers small beer amount to, at one shillings and four-pence per barrel?

152,5 = given number of barrels
,05942 = factor for small beer
―――――
3050
6100
13725
7625
―――――
9,061550 = 9*l.* 1*s.* 2¾*d.* ,3, the duty.

Or the same may be found by the cash table for common brewers small beer, page 386.

Prob. 13. *To find the duty of any number of barrels of common brewers table beer, at three shillings per barrel, with the allowance of 2½ in every 23 barrels.*

By reason of this allowance there will be also a fraction in the price of one barrel; so there must be a factor found for common brewers table beer, as well as for strong and small.

Or the duty may be found by the cash table for common brewers table beer, page 387.

The factor is found in the following manner:

	£.	s.	d.
The duty of 23 barrels at 3*s.* per barrel is —	3	9	0
The allowance of 2½ barrels is — —	0	7	6
The duty of 23 barrels of common brewers table beer is	3	1	6

Then the proportion for the factor is,

Bar.		*£.*		*Bar.*	*Decimal of £.*
As 23	:	3,075	::	1	,1336956521739130 + the factor.

Therefore, if any number of barrels, and quarters of a barrel, of common brewers table beer, reduced to a decimal, be multiplied by the above factor, the product will be equal to the duty in pounds, and decimal parts of a pound.

EXAMPLE. What will the duty of 40 barrels of common brewers table beer amount to, at three shillings per barrel?

,1336956521739130 = Factor for table beer
40 = given number of barrels
―――――
	£.	s.	d.	qr.	pts.	
5,3478260869565200 =	5	6	11	1	21,	the duty required.

The

The common rule by which the table was made, is thus:

	Bar.		£. s. d.		Bar.
If	23	:	3 . 1 6	::	1

20
——
23) 61 (2*s*. 8*d*. *qr*. $\frac{8}{23}$ the duty of 1 barrel; therefore, all the fractional parts in the table for common brewers table beer, are so many parts of 23 of a farthing.
46
——
15
12
——
23) 186 (8
184
——
2
4
——
8

Prob 14. *To find factors for reducing the odd gallons of one denomination to their equivalent value in that of another denomination, so that they may produce the same duty.*

RULE 1. If the odd gallons to be reduced only differ in duty, and there be the same number of gallons to the barrel, hogshead, &c. then the factor may be found by dividing the pence that one sort is charged per barrel or hogshead, &c. by the pence that the other denomination is charged at, *vice versa*, and the quotient will be the factor required.

2. When the odd gallons to be reduced not only differ in duty, but also in the number of gallons to a barrel, hogshead, &c. then the factor will be found by multiplying the pence that one sort is charged with duty, by the number of gallons in a barrel, hogshead, &c. of the other for a dividend; which divided by the product of the number of gallons to the barrel, hogshead, &c. and the duty of the other, *vice versa*, the quotient will be the factor required.

E. 1. Required to find a factor to reduce strong beer, at eight shillings per barrel, to small, at one shillings and four-pence per barrel?

By Rule 1.—First 8*s*. = 96*d*. ÷ 16 = 6, the factor required.

E. 2. It is required to find a factor for reducing small beer to strong.——See the first example.

By Rule 1.—96) 16,000 (,166, the factor required.

E. 3. It is required to find a factor to reduce odd gallons of cyder, at 18*s*. 9*d*. 0 *qrs*. ,8 per hogshead, containing 63 gallons, to its equivalent value in gallons of small beer, at 1*s*. per barrel, whose barrel contain 34 gallons?

By Rule 2.

The duty of a hogshead of cyder, 18*s*. 9*d*. 0 *qr*. ,8 = 900,8 *qrs*.
And 900,8 *qrs*. × 34 gallons = 30627,2, the dividend.
Also 48 *qrs*. × 63 gallons = 3024, the divisor.
∴ 30627,2 ÷ 3024 = 10,128, the factor required.

And

And in this manner you may find factors to reduce any number of odd gallons of one denomination, to their equivalent value in those of another.

Note. The drawback must be taken out of both strong and small beer before it be reduced, as in the above example.

Prob. 15. *To find factors for reducing ale measure to wine and corn measure; and e contra, corn to ale and wine measure.*

To perform which, observe the following proportions, which are wrought by the Rule of Three Inverse.

282	: 1 ::	231	:	1,220779	factor for	ale to wine.
231	: 1 ::	282	:	,819148	——	wine to ale.
282	: 1 ::	2150,42	:	,131137	——	ale to corn.
2150,42	: 1 ::	282	:	7,625602	——	corn to ale.
231	: 1 ::	2150,42	:	,107422	——	wine to corn.
2150,42	: 1 ::	231	:	9,309177	——	corn to wine.

The USE *of the foregoing* FACTORS

Multiply any number of gallons of ale, wine, or corn measure, by its proper factor; the product will be the number of gallons, reduced to the measure required.

EXAMPLE. Required to reduce 63 gallons of wine to ale measure?

,819148 = Factor for wine to ale
63
————
2457444
4914888
————

Answer 51,606324 gallons, ale measure.

And thus may any number of gallons of ale measure be reduced to corn or wine measure, by the help of the foregoing factors.

The way to find any FACTOR *is as follows:*

Let a = number of gallons, a hogshead, &c. b = number of gallons in a barrel, &c. c = duty of a hogshead, &c. d = duty of a barrel, &c. and x = the factor required.

Then $a : b :: c : d \times x$, and $adx = bc$, whence $x = \frac{bc}{ad}$; the factor required.

When the number of gallons in each are the same, and the difference is only in the price, then $a = b$ and $x = \frac{c}{d}$.

Prob. 16. *To find factors for salaries, both for common and leap years, at any rate per annum.*

RULE. As the number of days in a year is to the salary per annum, so is one day to its salary; which decimal of the salary, for one day, will be a proper factor to find the salary of any number of days at that rate.

EXAMPLE.

EXAMPLE. If the salary be 5*l.* per annum, what will the factors be at that rate, both for a common and leap year?

Days. *£.* *Day* *Decimal.*
As 365 : 5 :: 1 : ,013699, the factor for a common year.
Days. *£.* *Day.* *Decimal.*
As 366 : 5 :: 1 : ,013661, the factor for a leap year. And in this manner was the following table of factors computed.

A TABLE of FACTORS for SALARIES.

Salaries per ann. £.	s.	d.	Factors for a common year.	Factors for a leap year.	Salaries per ann. £.	s.	d.	Factors for a common year.	Factors for a leap year.
5	0	0	,013699	,013661	90	0	0	,246575	,245901
10	0	0	,027397	,027322	100	0	0	,273072	,273224
15	0	0	,041051	,040983	115	0	0	,316439	,315574
20	0	0	,054794	,054645	120	0	0	,328766	,327869
25	0	0	,068493	,068106	200	0	0	,547974	,546448
30	0	0	,082191	,081967	300	0	0	,821917	,819672
40	0	0	,109589	,109289	400	0	0	1,095948	1,022896
48	2	6	,131849	,131489	500	0	0	1,369863	1,366119
50	0	0	,136986	,136610	600	0	0	1,643834	1,639344
52	0	0	,142465	,142076	700	0	0	1,917808	1,912568
60	0	0	,164383	,163934	800	0	0	2,191896	2,185792
70	0	0	,191780	,191256	900	0	0	2,465811	2,459616
80	0	0	,219178	,218579	1000	0	0	2,739726	2,732240
86	12	6	,237329	,233948					

The USE *of the* TABLE *of* FACTORS.

By the above table the salary due for any number of days at any rate therein mentioned may be found, both for a common or leap year; for if you take the factor of the rate, and multiply it by the number of days, the product will be equal to the salary due in pounds, and decimal parts of a pound.

E. 1. Suppose the salary to be forty pounds per annum; how much would be due to a person for sixty days?

The factor for a common year, at 40*l.* per ann. is ,109589
Multiplied by the number of days — 60

£. s. d.
Answer 6,575340 = 6 11 6

E. 2. Suppose the salary was fifty pounds per annum; how much would be due to an officer for 80 days?

The factor for 50*l.* = ,136986
Number of Days = 80

Answer 10,958880 = 10*l.* 19*s.* 2*d.*

Note. If any of the factors in the table are reduced, they will shew the amount of a day's salary, at any rate therein mentioned.

In the foregoing examples, only the gross salary is found; but the Officers of Excise are deducted 9*d*. in the pound for tax and charity; therefore, to find the neat salary, you must multiply the gross salary by ,9625, and the product will be the neat salary.

EXAMPLE. Suppose the gross salary to be fifty pounds per annum; what is the neat money?

```
 ,9625
    50
 ------
48,1250
    20
 ------
 2,5000
    12   Answer 48l. 2s. 6d. the neat salary.
 ------
   6,0
```

Or the tax and charity may be found by the tax and charity table, at the end of this section.

The construction of the salary table for a common year is thus:

```
    Days.   £.    Day.
As  365  :  50  ::  1
            20
          -----
     365) 1000 (2s.
           730
          -----
           270
            12
          -----
     365) 3240 (8d.      s.  d.  qr.  pts.
          2920    Answer  2   8   3   37
          -----
           320
             4
          -----
     365) 1280 (3 qrs.
          1095
          -----
```

$\frac{185}{360} = \frac{37}{73}$ in its lowest terms; so that all the fractional parts in that table are so many 73 parts of a farthing.

After the same manner is the salary table for a leap year calculated; only, instead of 365, take 366 days, and work as before; and the answer will be 2*s*. 8*d*. 3*qr*. $\frac{14}{366} = \frac{22}{61}$ in its lowest terms; so that all the fractional parts in that table, are so many 61 parts of a farthing.

*** At the request of several Friends in the Excise, I have inserted the following Collection of Tables, some of which are entirely new, and the others upon a different plan from any before extant.

Table 1. CASH TABLE for Common Brewers' Strong Beer, at 8s. per Barrel, with the Allowance of 2½ in every 23 Barrels, from ¼ of a Barrel to 50,000.

Bar	l.	s.	d.	q. 23p	Bar	l.	s.	d.	q. 23p	Bar	l.	s.	d.	q. 23p	Bar.	l.	s.	d.	q. 23p
					12	4	5	6	3, 3	27	9	12	6	1, 1	75	26	14	9	1,13
¼	0	1	9	1,13	12¼	4	7	4	0,16	28	9	19	7	3, 7	76	27	1	10	3,19
½	0	3	6	3, 3	12½	4	9	1	2, 6	29	10	6	9	1,13	77	27	9	0	2, 2
¾	0	5	4	0,16	12¾	4	10	10	3,19	30	10	13	10	3,19	78	27	16	2	0, 8
1	0	7	1	2, 6	13	4	12	8	1, 9	31	11	1	0	2, 2	79	28	3	3	2,14
1¼	0	8	10	3,19	13¼	4	14	5	2,22	32	11	8	2	0, 8	80	28	10	5	0,20
1½	0	10	8	1, 9	13½	4	16	3	0,12	33	11	15	3	2,14	81	28	17	6	3, 3
1¾	0	12	5	2,22	13¾	4	18	0	2, 2	34	12	2	5	0,20	82	29	4	8	1, 9
2	0	14	3	0,12	14	4	19	9	3,15	35	12	9	6	3, 3	83	29	11	9	3,15
2¼	0	16	0	2, 2	14¼	5	1	7	1, 5	36	12	16	8	1, 9	84	29	18	11	1,21
2½	0	17	9	3,15	14½	5	3	4	2,18	37	13	3	9	3,15	85	30	6	1	0, 4
2¾	0	19	7	1, 5	14¾	5	5	2	0, 8	38	13	10	11	1,21	86	30	13	2	2,10
3	1	1	4	2,18	15	5	6	11	1,21	39	13	18	1	0, 4	87	31	0	4	0,16
3¼	1	3	2	0, 8	15¼	5	8	8	3,11	40	14	5	2	2,10	88	31	7	5	2,22
3½	1	4	11	1,21	15½	5	10	6	1, 1	41	14	12	4	0,16	89	31	14	7	1, 5
3¾	1	6	8	3,11	15¾	5	12	3	2,14	42	14	19	5	2,22	90	32	1	8	3,11
4	1	8	6	1, 1	16	5	14	1	0, 4	43	15	6	7	1, 5	91	32	8	10	1,17
4¼	1	10	3	2,14	16¼	5	15	10	1,17	44	15	13	8	3,11	92	32	16	0	0, 0
4½	1	12	1	0, 4	16½	5	17	7	3, 7	45	16	0	10	1,17	93	33	3	1	2, 6
4¾	1	13	10	1,17	16¾	5	19	5	0,20	46	16	8	0	0, 0	94	33	10	3	0,12
5	1	15	7	3, 7	17	6	1	2	2,10	47	16	15	1	2, 6	95	33	17	4	2,18
5¼	1	17	5	0,20	17¼	6	3	0	0, 0	48	17	2	3	0,12	96	34	4	6	1, 1
5½	1	19	2	2,10	17½	6	4	9	1,13	49	17	9	4	2,18	97	34	11	7	3, 7
5¾	2	1	0	0, 0	17¾	6	6	6	3, 3	50	17	16	6	1, 1	98	34	18	9	1,13
6	2	2	9	1,13	18	6	8	4	0,16	51	18	3	7	3, 7	99	35	5	10	3,19
6¼	2	4	6	3, 3	18¼	6	10	1	2, 6	52	18	10	9	1,13	100	35	13	0	2, 2
6½	2	6	4	0,16	18½	6	11	10	3,19	53	18	17	10	3,19	200	71	6	1	0, 4
6¾	2	8	1	2, 6	18¾	6	13	8	1, 9	54	19	5	0	2, 2	300	106	19	1	2, 6
7	2	9	10	3,19	19	6	15	5	2,22	55	19	12	2	0, 8	400	142	12	2	0, 8
7¼	2	11	8	1, 9	19¼	6	17	3	0,12	56	19	19	3	2,14	500	178	5	2	2,10
7½	2	13	5	2,22	19½	6	19	0	2, 2	57	20	6	5	0,20	600	213	18	3	0,12
7¾	2	15	3	0,12	19¾	7	0	9	3,15	58	20	13	6	3, 3	700	249	11	3	2,14
8	2	17	0	2, 2	20	7	2	7	1, 5	59	21	0	8	1, 9	800	285	4	4	0,16
8¼	2	18	9	3,15	20¼	7	4	4	2,18	60	21	7	9	3,15	900	320	17	4	2,18
8½	3	0	7	1, 5	20½	7	6	2	0, 8	61	21	14	11	1,21	1000	356	10	5	0,20
8¾	3	2	4	2,18	20¾	7	7	11	1,21	62	22	2	1	0, 4	2000	713	0	10	1,17
	3	4	2	0, 8	21	7	9	8	3,11	63	22	9	2	2,10	3000	1069	11	3	2,14
9¼	3	5	11	1,21	21¼	7	11	6	1, 1	64	22	16	4	0,16	4000	1426	1	8	3,11
9½	3	7	8	3,11	21½	7	13	3	2,14	65	23	3	5	2,22	5000	1782	12	2	0, 8
9¾	3	9	6	1, 1	21¾	7	15	1	0, 4	66	23	10	7	1, 5	6000	2139	2	7	1, 5
10	3	11	3	2,14	22	7	16	10	1,17	67	23	17	8	3,11	7000	2495	13	0	2, 2
10¼	3	13	1	0, 4	22¼	7	18	7	3, 7	68	24	4	10	1,17	8000	2852	3	5	2,22
10½	3	14	10	1,17	22½	8	0	5	0,20	69	24	12	0	0, 0	9000	3208	13	10	3,19
10¾	3	16	7	3, 7	22¾	8	2	2	2,10	70	24	19	1	2, 6	10000	3565	4	4	0,16
11	3	18	5	0,20	23	8	4	0	0, 0	71	25	6	3	0,12	20000	7130	8	8	1, 9
11¼	4	0	2	2,10	24	8	11	1	2, 6	72	25	13	4	2,18	30000	10695	13	0	2, 2
11½	4	2	0	0, 0	25	8	18	3	0,12	73	26	0	6	1, 1	40000	14260	17	4	2,18
11¾	4	3	9	1,13	26	9	5	4	2,18	74	26	7	7	3, 7	50000	17826	1	8	3,11

Table 2. CASH TABLE for Common Brewers Small Beer, at 1s. 4d. a Barrel with the Allowance of 2½ in every 23 Barrels, from ¼ of a Barrel to 50,000.

Bar	l.	s.	d.	q. 23p	Bar	l.	s.	d.	q. 23p	Bar	l.	s.	d.	q. 23p	Bar.	l.	s.	d.	q. 23p
					12	0	14	3	0,12	27	1	12	1	0, 4	75	4	9	1	2, 6
¼	0	0	3	2, 6	12¼	0	14	6	2,18	28	1	13	3	1, 5	76	4	10	3	3, 7
½	0	0	7	0,12	12½	0	14	10	1, 1	29	1	14	5	2, 6	77	4	11	6	0, 8
¾	0	0	10	2,18	12¾	0	15	1	3, 7	30	1	15	7	3, 7	78	4	12	8	1, 9
1	0	1	2	1, 1	13	0	15	5	1,13	31	1	16	10	0, 8	79	4	13	10	2,10
1¼	0	1	5	3, 7	13¼	0	15	8	3,19	32	1	18	0	1, 9	80	4	15	0	3,11
1½	0	1	9	1,13	13½	0	16	0	2, 2	33	1	19	2	2,10	81	4	16	3	0,12
1¾	0	2	0	3,19	13¾	0	16	4	0, 8	34	2	0	4	3,11	82	4	17	5	1,13
2	0	2	4	2, 2	14	0	16	7	2,14	35	2	1	7	0,12	83	4	18	7	2,14
2¼	0	2	8	0, 8	14¼	0	16	11	0,20	36	2	2	9	1,13	84	4	19	9	3,15
2½	0	2	11	2,14	14½	0	17	2	3, 3	37	2	3	11	2,14	85	5	1	0	0,16
2¾	0	3	3	0,20	14¾	0	17	6	1, 9	38	2	5	1	3,15	86	5	2	2	1,17
3	0	3	6	3, 3	15	0	17	9	3,15	39	2	6	4	0,16	87	5	3	4	2,18
3¼	0	3	10	1, 9	15¼	0	18	1	1,21	40	2	7	6	1,17	88	5	4	6	3,19
3½	0	4	1	3,15	15½	0	18	5	0, 4	41	2	8	8	2,18	89	5	5	9	0,20
3¾	0	4	5	1,21	15¾	0	18	8	2,10	42	2	9	10	3,19	90	5	6	11	1,21
4	0	4	9	0, 4	16	0	19	0	0,16	43	2	11	1	0,20	91	5	8	1	2,22
4¼	0	5	0	2,10	16¼	0	19	3	2,22	44	2	12	3	1,21	92	5	9	4	0, 0
4½	0	5	4	0,16	16½	0	19	7	1, 5	45	2	13	5	2,22	93	5	10	6	1, 1
4¾	0	5	7	2,22	16¾	0	19	10	3,11	46	2	14	8	0, 0	94	5	11	8	2, 2
5	0	5	11	1, 5	17	1	0	2	1,17	47	2	15	10	1, 1	95	5	12	10	3, 3
5¼	0	6	2	3,11	17¼	1	0	6	0, 0	48	2	17	0	2, 2	96	5	14	1	0, 4
5½	0	6	6	1,17	17½	1	0	9	2, 6	49	2	18	2	3, 3	97	5	15	3	1, 5
5¾	0	6	10	0, 0	17¾	1	1	1	0,12	50	2	19	5	0, 4	98	5	16	5	2, 6
6	0	7	1	2, 6	18	1	1	4	2,18	51	3	0	7	1, 5	99	5	17	7	3, 7
6¼	0	7	5	0,12	18¼	1	1	8	1, 1	52	3	1	9	2, 6	100	5	18	10	0, 8
6½	0	7	8	2,18	18½	1	1	11	3, 7	53	3	2	11	3, 7	200	11	17	8	0,16
6¾	0	8	0	1, 1	18¾	1	2	3	1,13	54	3	4	2	0, 8	300	17	16	6	1, 1
7	0	8	3	3, 7	19	1	2	6	3,19	55	3	5	4	1, 9	400	23	15	4	1, 9
7¼	0	8	7	1,13	19¼	1	2	10	2, 2	56	3	6	6	2,10	500	29	14	2	1,17
7½	0	8	10	3,19	19½	1	3	2	0, 8	57	3	7	8	3,11	600	35	13	0	2, 2
7¾	0	9	2	2, 2	19¾	1	3	5	2,14	58	3	8	11	0,12	700	41	11	10	2,10
8	0	9	6	0, 8	20	1	3	9	0,20	59	3	10	1	1,13	800	47	10	8	2,18
8¼	0	9	9	2,14	20¼	1	4	0	3, 3	60	3	11	3	2,14	900	53	9	6	3, 3
8½	0	10	1	0,20	20½	1	4	4	1, 9	61	3	12	5	3,15	1000	59	8	4	3,11
8¾	0	10	4	3, 3	20¾	1	4	7	3,15	62	3	13	8	0,16	2000	118	16	9	2,22
9	0	10	8	1, 9	21	1	4	11	1,21	63	3	14	10	1,17	3000	178	5	2	2,10
9¼	0	10	11	3,15	21¼	1	5	3	0, 4	64	3	16	0	2,18	4000	237	13	7	1,21
9½	0	11	3	1,21	21½	1	5	6	2,10	65	3	17	2	3,19	5000	297	2	0	1, 9
9¾	0	11	7	0, 4	21¾	1	5	10	0,16	66	3	18	5	0,20	6000	356	10	5	0,20
10	0	11	10	2,10	22	1	6	1	2,22	67	3	19	7	1,21	7000	415	18	10	0, 8
10¼	0	12	2	0,16	22¼	1	6	5	1, 5	68	4	0	9	2,22	8000	475	7	2	3,19
10½	0	12	5	2,22	22½	1	6	8	3,11	69	4	2	0	0, 0	9000	534	15	7	3, 7
10¾	0	12	9	1, 5	22¾	1	7	0	1,17	70	4	3	2	1, 1	10000	594	4	0	2,18
11	0	13	0	3,11	23	1	7	4	0, 0	71	4	4	4	2, 2	20000	1188	8	1	1,13
11¼	0	13	4	1,17	24	1	8	6	1, 1	72	4	5	6	3, 3	30000	1782	12	2	0, 8
11½	0	13	8	0, 0	25	1	9	8	2, 2	73	4	6	9	0, 4	40000	2376	16	2	3, 3
11¾	0	13	11	2, 6	26	1	10	10	3, 3	74	4	7	11	1, 5	50000	2971	0	3	1,21

Table 3. CASH TABLE for Com. Brewers' Table Beer, at 3s. per Barrel with the Allowance of 2 ½ in every 23 Barrels, from ¼ of a Barrel to 50,000.

Bar	l.	s.	d.	q. 23p	Bar.	l.	s.	d.	q. 23p	Bar	l.	s.	d.	q. 23p	Bar.	l.	s.	d.	q. 23p
					12	1	12	1	0, 4	27	3	12	2	1, 9	75	10	0	6	2, 2
¼	0	0	8	0, 2	12¼	1	12	9	0, 6	28	3	14	10	1,17	76	10	3	2	2,10
½	0	1	4	0, 4	12½	1	13	5	0, 8	29	3	17	6	2, 2	77	10	5	10	2,18
¾	0	2	0	0, 6	12¾	1	14	1	0,10	30	4	0	2	2,10	78	10	8	6	3, 3
1	0	2	8	0, 8	13	1	14	9	0,12	31	4	2	10	2,18	79	10	11	2	3,11
1¼	0	3	4	0,10	13¼	1	15	5	0,14	32	4	5	6	3, 3	80	10	13	10	3,19
1½	0	4	0	0,12	13½	1	16	1	0,16	33	4	8	2	3,11	81	10	16	7	0, 4
1¾	0	4	8	0,14	13¾	1	16	9	0,18	34	4	10	10	3,19	82	10	19	3	0,12
2	0	5	4	0,16	14	1	17	5	0,20	35	4	13	7	0, 4	83	11	1	11	0,20
2¼	0	6	0	0,18	14¼	1	18	1	0,22	36	4	16	3	0,12	84	11	4	7	1, 5
2½	0	6	8	0,20	14½	1	18	9	1, 1	37	4	18	11	0,20	85	11	7	3	1,13
2¾	0	7	4	0,22	14¾	1	19	5	1, 3	38	5	1	7	1, 5	86	11	9	11	1,21
3	0	8	0	1, 1	15	2	0	1	1, 5	39	5	4	3	1,13	87	11	12	7	2, 6
3¼	0	8	8	1, 3	15¼	2	0	9	1, 7	40	5	6	11	1,21	88	11	15	3	2,14
3½	0	9	4	1, 5	15½	2	1	5	1, 9	41	5	9	7	2, 6	89	11	17	11	2,22
3¾	0	10	0	1, 7	15¾	2	2	1	1,11	42	5	12	3	2,14	90	12	0	7	3, 7
4	0	10	8	1, 9	16	2	2	9	1,13	43	5	14	11	2,22	91	12	3	3	3,15
4¼	0	11	4	1,11	16¼	2	3	5	1,15	44	5	17	7	3, 7	92	12	6	0	0, 0
4½	0	12	0	1,13	16½	2	4	1	1,17	45	6	0	3	3,15	93	12	8	8	0, 8
4¾	0	12	8	1,15	16¾	2	4	9	1,19	46	6	13	0	0, 0	94	12	11	4	0,16
5	0	13	4	1,17	17	2	5	5	1,21	47	6	5	8	0, 8	95	12	14	0	1, 1
5¼	0	14	0	1,19	17¼	2	6	1	2, 0	48	6	8	4	0,16	96	12	16	8	1, 9
5½	0	14	8	1,21	17½	2	6	9	2, 2	49	6	11	0	1, 1	97	12	19	4	1,17
5¾	0	15	4	2, 0	17¾	2	7	5	2, 4	50	6	13	8	1, 9	98	13	2	0	2, 2
6	0	16	0	2, 2	18	2	8	1	2, 6	51	6	16	4	1,17	99	13	4	8	2,10
6¼	0	16	8	2, 4	18¼	2	8	9	2, 8	52	6	19	0	2, 2	100	13	7	4	2,18
6½	0	17	4	2, 6	18½	2	9	5	2,10	53	7	1	8	2,10	200	26	14	9	1,13
6¾	0	18	0	2, 8	18¾	2	10	1	2,12	54	7	4	4	2,18	300	40	2	2	0, 8
7	0	18	8	2,10	19	2	10	9	2,14	55	7	7	0	3, 3	400	53	9	6	3, 3
7¼	0	19	4	2,12	19¼	2	11	5	2,16	56	7	9	8	3,11	500	66	16	11	1,21
7½	1	0	0	2,14	19½	2	12	1	2,18	57	7	12	4	3,19	600	80	4	4	0,16
7¾	1	0	8	2,16	19¾	2	12	9	2,20	58	7	15	1	0, 4	700	93	11	8	3,11
8	1	1	4	2,18	20	2	13	5	2,22	59	7	17	9	0,12	800	106	19	1	2, 6
8¼	1	2	0	2,20	20¼	2	14	1	3, 1	60	8	0	5	0,20	900	120	6	6	1, 1
8½	1	2	8	2,22	20½	2	14	9	3, 3	61	8	3	1	1, 5	1000	133	13	10	3,19
8¾	1	3	4	3, 1	20¾	2	15	5	3, 5	62	8	5	9	1,13	2000	267	7	9	3,15
9	1	4	0	3, 3	21	2	16	1	3, 7	63	8	8	5	1,21	3000	401	1	8	3,11
9¼	1	4	8	3, 5	21¼	2	16	9	3, 9	64	8	11	1	2, 6	4000	534	15	7	3, 7
9½	1	5	4	3, 7	21½	2	17	5	3,11	65	8	13	9	2,14	5000	668	9	6	3, 3
9¾	1	6	0	3, 9	21¾	2	18	1	3,13	66	8	16	5	2,22	6000	802	3	5	2,22
10	1	6	8	3,11	22	2	18	9	3,15	67	8	19	1	3, 7	7000	935	17	4	2,18
10¼	1	7	4	3,13	22¼	2	19	5	3,17	68	9	1	9	3,15	8000	1069	11	3	2,14
10½	1	8	0	3,15	22½	3	0	1	3,19	69	9	4	6	0, 0	9000	1203	5	2	2,10
10¾	1	8	8	3,17	22¾	3	0	9	3,21	70	9	7	2	0, 8	10000	1336	19	1	2, 6
11	1	9	4	3,19	23	3	1	6	0, 0	71	9	9	10	0,16	20000	2673	18	3	0,12
11¼	1	10	0	3,21	24	3	4	2	0, 8	72	9	12	6	1, 1	30000	4010	17	4	2,18
11½	1	10	9	0, 0	25	3	6	10	0,16	73	9	15	2	1, 9	40000	5347	16	6	1, 1
11¾	1	11	5	0, 2	26	3	9	6	1, 1	74	9	17	10	1,17	50000	6684	15	7	3, 7

TABLE 4.

A CASH TABLE,

FOR

Victuallers' STRONG BEER or ALE, at 8s. per Barrel,

From ¼ of a Barrel to 50,000.

Bar.	l.	s.	d.	Bar	l.	s.	d.	Bar	l.	s.	d.	Bar.	l.	s.	d.	Bar.	l.	s.	d.
				10	4	0	0	20	8	0	0	30	12	0	0	40	16	0	0
¼	0	2	0	10¼	4	2	0	20¼	8	2	0	30¼	12	2	0	40¼	16	2	0
½	0	4	0	10½	4	4	0	20½	8	4	0	30½	12	4	0	40½	16	4	0
¾	0	6	0	10¾	4	6	0	20¾	8	6	0	30¾	12	6	0	40¾	16	6	0
1	0	8	0	11	4	8	0	21	8	8	0	31	12	8	0	41	16	8	0
1¼	0	10	0	11¼	4	10	0	21¼	8	10	0	31¼	12	10	0	41¼	16	10	0
1½	0	12	0	11½	4	12	0	21½	8	12	0	31½	12	12	0	41½	16	12	0
1¾	0	14	0	11¾	4	14	0	21¾	8	14	0	31¾	12	14	0	41¾	16	14	0
2	0	16	0	12	4	16	0	22	8	16	0	32	12	16	0	42	16	16	0
2¼	0	18	0	12¼	4	18	0	22¼	8	18	0	32¼	12	18	0	42¼	16	18	0
2½	1	0	0	12½	5	0	0	22½	9	0	0	32½	13	0	0	42½	17	0	0
2¾	1	2	0	12¾	5	2	0	22¾	9	2	0	32¾	13	2	0	42¾	17	2	0
3	1	4	0	13	5	4	0	23	9	4	0	33	13	4	0	43	17	4	0
3¼	1	6	0	13¼	5	6	0	23¼	9	6	0	33¼	13	6	0	43¼	17	6	0
3½	1	8	0	13½	5	8	0	23½	9	8	0	33½	13	8	0	43½	17	8	0
3¾	1	10	0	13¾	5	10	0	23¾	9	10	0	33¾	13	10	0	43¾	17	10	0
4	1	12	0	14	5	12	0	24	9	12	0	34	13	12	0	44	17	12	0
4¼	1	14	0	14¼	5	14	0	24¼	9	14	0	34¼	13	14	0	44¼	17	14	0
4½	1	16	0	14½	5	16	0	24½	9	16	0	34½	13	16	0	74½	17	16	0
4¾	1	18	0	14¾	5	18	0	24¾	9	18	0	34¾	13	18	0	44¾	17	18	0
5	2	0	0	15	6	0	0	25	10	0	0	35	14	0	0	45	18	0	0
5¼	2	2	0	15¼	6	2	0	25¼	10	2	0	35¼	14	2	0	45¼	18	2	0
5½	2	4	0	15½	6	4	0	25½	10	4	0	35½	14	4	0	45½	18	4	0
5¾	2	6	0	15¾	6	6	0	25¾	10	6	0	35¾	14	6	0	45¾	18	6	0
6	2	8	0	16	6	8	0	26	10	8	0	36	14	8	0	46	18	8	0
6¼	2	10	0	16¼	6	10	0	26¼	10	10	0	36¼	14	10	0	46¼	18	10	0
6½	2	12	0	16½	6	12	0	26½	10	12	0	36½	14	12	0	46½	18	12	0
6¾	2	14	0	16¾	6	14	0	26¾	10	14	0	36¾	14	14	0	46¾	18	14	0
7	2	16	0	17	6	16	0	27	10	16	0	37	14	16	0	47	18	16	0
7¼	2	18	0	17¼	6	18	0	27¼	10	18	0	37¼	14	18	0	47¼	18	18	0
7½	3	0	0	17½	7	0	0	27½	11	0	0	37½	15	0	0	47½	19	0	0
7¾	3	2	0	17¾	7	2	0	27¾	11	2	0	37¾	15	2	0	47¾	19	2	0
8	3	4	0	18	7	4	0	28	11	4	0	38	15	4	0	48	19	4	0
8¼	3	6	0	18¼	7	6	0	28¼	11	6	0	38¼	15	6	0	48¼	19	6	0
8½	3	8	0	18½	7	8	0	28½	11	8	0	38½	15	8	0	48½	19	8	0
8¾	3	10	0	18¾	7	10	0	28¾	11	10	0	38¾	15	10	0	48¾	19	10	0
9	3	12	0	19	7	12	0	29	11	12	0	39	15	12	0	49	19	12	0
9¼	3	14	0	19¼	7	14	0	29¼	11	14	0	39¼	15	14	0	49¼	19	14	0
9½	3	16	0	19½	7	16	0	29½	11	16	0	39½	15	16	0	49½	19	16	0
9¾	3	18	0	19¾	7	18	0	29¾	11	18	0	39¾	15	18	0	49¾	19	18	0

CASH TABLE for Victuallers' Strong Beer and Ale continued.

Bar.	l.	s.	d.	Bar.	l.	s.	d.	Bar.	l.	s.	d.	Bar.	l.	s.	d.	Bar.	l.	s.	d.
50	20	0	0	62	24	16	0	74	29	12	0	86	34	8	0	98	39	4	0
50¼	20	2	0	62¼	24	18	0	74¼	29	14	0	86¼	34	10	0	98¼	39	6	0
50½	20	4	0	62½	25	0	0	74½	29	16	0	86½	34	12	0	98½	39	8	0
50¾	20	6	0	62¾	25	2	0	74¾	29	18	0	86¾	34	14	0	98¾	39	10	0
51	20	8	0	63	25	4	0	75	30	0	0	87	34	16	0	99	39	12	0
41¼	20	10	0	63¼	25	6	0	75¼	30	2	0	87¼	34	18	0	99¼	39	14	0
51½	20	12	0	63½	25	8	0	75½	30	4	0	87½	35	0	0	99½	39	16	0
51¾	20	14	0	63¾	25	10	0	75¾	30	6	0	87¾	35	2	0	99¾	39	18	0
52	20	16	0	64	25	12	0	76	30	8	0	88	35	4	0	100	40	0	0
52¼	20	18	0	64¼	25	14	0	76¼	30	10	0	88¼	35	6	0	101	40	8	0
52½	21	0	0	64½	25	16	0	76½	30	12	0	88½	35	8	0	102	40	16	0
52¾	21	2	0	64¾	25	18	0	76¾	30	14	0	88¾	35	10	0	103	41	4	0
53	21	4	0	65	26	0	0	77	30	16	0	89	35	12	0	104	41	12	0
53¼	21	6	0	65¼	26	2	0	77¼	30	18	0	89¼	35	14	0	105	42	0	0
53½	21	8	0	65½	26	4	0	77½	31	0	0	89½	35	16	0	106	42	8	0
53¾	21	10	0	65¾	26	6	0	77¾	31	2	0	89¾	35	18	0	107	42	16	0
54	31	12	0	66	26	8	0	78	31	4	0	90	36	0	0	108	43	4	0
54¼	31	14	0	66¼	26	10	0	78¼	31	6	0	90¼	36	2	0	109	43	12	0
54½	21	16	0	66½	26	12	0	78½	31	8	0	90½	36	4	0	110	44	0	0
54¾	21	18	0	66¾	26	14	0	78¾	31	10	0	90¾	36	6	0	120	48	0	0
55	22	0	0	67	26	16	0	79	31	12	0	91	36	8	0	130	52	0	0
55¼	22	2	0	67¼	26	18	0	79¼	31	14	0	91¼	36	10	0	140	56	0	0
55½	22	4	0	67½	27	0	0	79½	31	16	0	91½	36	12	0	150	60	0	0
55¾	22	6	0	67¾	27	2	0	79¾	31	18	0	91¾	36	14	0	160	64	0	0
56	22	8	0	68	27	4	0	80	32	0	0	92	36	16	0	170	68	0	0
56¼	22	10	0	68¼	27	6	0	80¼	32	2	0	92¼	36	18	0	200	80	0	0
56½	22	12	0	68½	27	8	0	80½	32	4	0	92½	37	0	0	300	120	0	0
56¾	22	14	0	68¾	27	10	0	80¾	32	6	0	92¾	37	2	0	400	160	0	0
57	22	16	0	69	27	12	0	81	32	8	0	93	37	4	0	500	200	0	0
57¼	22	18	0	69¼	27	14	0	81¼	32	10	0	93¼	37	6	0	600	240	0	0
57½	23	0	0	69½	27	16	0	81½	32	12	0	93½	37	8	0	700	280	0	0
57¾	23	2	0	69¾	27	18	0	81¾	32	14	0	93¾	37	10	0	800	320	0	0
58	23	4	0	70	28	0	0	82	32	16	0	94	37	12	0	900	360	0	0
58¼	23	6	0	70¼	28	2	0	82¼	32	18	0	94¼	37	14	0	1000	400	0	0
58½	23	8	0	70½	28	4	0	82½	33	0	0	94½	37	16	0	2000	800	0	0
58¾	23	10	0	70¾	28	6	0	82¾	33	2	0	94¾	37	18	0	3000	1200	0	0
59	23	12	0	71	28	8	0	83	33	4	0	95	38	0	0	4000	1600	0	0
59¼	23	14	0	71¼	28	10	0	83¼	33	6	0	95¼	38	2	0	5000	2000	0	0
59½	23	16	0	71½	28	12	0	83½	33	8	0	95½	38	4	0	6000	2400	0	0
59¾	23	18	0	71¾	28	14	0	83¾	33	10	0	95¾	38	6	0	7000	2800	0	0
60	24	0	0	72	28	16	0	84	33	12	0	96	38	8	0	8000	3200	0	0
60¼	24	2	0	72¼	28	18	0	84¼	33	14	0	96¼	38	10	0	9000	3600	0	0
60½	24	4	0	72½	29	0	0	84½	33	16	0	96½	38	12	0	10000	4000	0	0
60¾	24	6	0	72¾	29	2	0	84¾	33	18	0	96¾	38	14	0	20000	8000	0	0
61	24	8	0	73	29	4	0	85	34	0	0	97	38	16	0	30000	12000	0	0
61¼	24	10	0	73¼	29	6	0	85¼	34	2	0	97¼	38	18	0	40000	16000	0	0
61½	24	12	0	73½	29	8	0	85½	34	4	0	97½	39	0	0	50000	20000	0	0
61¾	24	14	0	73¾	29	10	0	85¾	34	6	0	97¾	39	2	0				

A CASH TABLE, FOR Victuallers' SMALL BEER, at 1s. 4d. per Barrel,

From ¼ of a Barrel to 50,000.

Bar.	l. s. d.	Bar.	l. s. d.	Bar.	l. s. d.	Bar.	l. s. d.	Bar.	l. s. d.
		10	0 13 4	20	1 6 8	30	2 0 0	40	2 13 4
¼	0 0 4	10¼	0 13 8	20¼	1 7 0	30¼	2 0 4	40¼	2 13 8
½	0 0 8	10½	0 14 0	20½	1 7 4	30½	2 0 8	40½	2 14 0
¾	0 1 0	10¾	0 14 4	20¾	1 7 8	30¾	2 1 0	40¾	2 14 4
1	0 1 4	11	0 14 8	21	1 8 0	31	2 1 4	41	2 14 8
1¼	0 1 8	11¼	0 15 0	21¼	1 8 4	31¼	2 1 8	41¼	2 15 0
1½	0 2 0	11½	0 15 4	21½	1 8 8	31½	2 2 0	41½	2 15 4
1¾	0 2 4	11¾	0 15 8	21¾	1 9 0	31¾	2 2 4	41¾	2 15 8
2	0 2 8	12	0 16 0	22	1 9 4	32	2 2 8	42	2 16 0
2¼	0 3 0	12¼	0 16 4	22¼	1 9 8	32¼	2 3 0	42¼	2 16 4
2½	0 3 4	12½	0 16 8	22½	1 10 0	32½	2 3 4	42½	2 16 8
2¾	0 3 8	12¾	0 17 0	22¾	1 10 4	32¾	2 3 8	42¾	2 17 0
3	0 4 0	13	0 17 4	23	1 10 8	33	2 4 0	43	2 17 4
3¼	0 4 4	13¼	0 17 8	23¼	1 11 0	33¼	2 4 4	43¼	2 17 8
3½	0 4 8	13½	0 18 0	23½	1 11 4	33½	2 4 8	43½	2 18 0
3¾	0 5 0	13¾	0 18 4	23¾	1 11 8	33¾	2 5 0	43¾	2 18 4
4	0 5 4	14	0 18 8	24	1 12 0	34	2 5 4	44	2 18 8
4¼	0 5 8	14¼	0 19 0	24¼	1 12 4	34¼	2 5 8	44¼	2 19 0
4½	0 6 0	14½	0 19 4	24½	1 12 8	34½	2 6 0	44½	2 19 4
4¾	0 6 4	14¾	0 19 8	24¾	1 13 0	34¾	2 6 4	44¾	2 19 8
5	0 6 8	15	1 0 0	25	1 13 4	35	2 6 8	45	3 0 0
5¼	0 7 0	15¼	1 0 4	25¼	1 13 8	35¼	2 7 0	45¼	3 0 4
5½	0 7 4	15½	1 0 8	25½	1 14 0	35½	2 7 4	45½	3 0 8
5¾	0 7 8	15¾	1 1 0	25¾	1 14 4	35¾	2 7 8	45¾	3 1 0
6	0 8 0	16	1 1 4	26	1 14 8	36	2 8 0	46	3 1 4
6¼	0 8 4	16¼	1 1 8	26¼	1 15 0	36¼	2 8 4	46¼	3 1 8
6½	0 8 8	16½	1 2 0	26½	1 15 4	36½	2 8 8	46½	3 2 0
6¾	0 9 0	16¾	1 2 4	26¾	1 15 8	36¾	2 9 0	46¾	3 2 4
7	0 9 4	17	1 2 8	27	1 16 0	37	2 9 4	47	3 2 8
7¼	0 9 8	17¼	1 3 0	27¼	1 16 4	37¼	2 9 8	47¼	3 3 0
7½	0 10 0	17½	1 3 4	27½	1 16 8	37½	2 10 0	47½	3 3 4
7¾	0 10 4	17¾	1 3 8	27¾	1 17 0	37¾	2 10 4	47¾	3 3 8
8	0 10 8	18	1 4 0	28	1 17 4	38	2 10 8	48	3 4 0
8¼	0 11 0	18¼	1 4 4	28¼	1 17 8	38¼	2 11 0	48¼	3 4 4
8½	0 11 4	18½	1 4 8	28½	1 18 0	38½	2 11 4	48½	3 4 8
8¾	0 11 8	18¾	1 5 0	28¾	1 18 4	38¾	2 11 8	48¾	3 5 0
9	0 12 0	19	1 5 4	29	1 18 8	39	2 12 0	49	3 5 4
9¼	0 12 4	19¼	1 5 8	29¼	1 19 0	39¼	2 12 4	49¼	3 5 8
9½	0 12 8	19½	1 6 0	29½	1 19 4	39½	2 12 8	49½	3 6 0
9¾	0 13 0	19¾	1 6 4	29¾	1 19 8	39¾	2 13 0	49¾	3 6 4

CASH TABLE for Victuallers' Small Beer continued.

Bar	l. s. d.	Bar.	l. s. d.	Bar.	l. s. d.	Bar.	l. s. d.	Bar.	l. s. d.
50	3 6 8	62	4 2 8	74	4 18 8	86	5 14 8	98	6 10 8
50¼	3 7 0	62¼	4 3 0	74¼	4 19 0	86¼	5 15 0	98¼	6 11 0
50½	3 7 4	62½	4 3 4	74½	4 19 4	86½	5 15 4	98½	6 11 4
50¾	3 7 8	62¾	4 3 8	74¾	4 19 8	86¾	5 15 8	98¾	6 11 8
51	3 8 0	63	4 4 0	75	5 0 0	87	5 16 0	99	6 12 0
51¼	3 8 4	63¼	4 4 4	75¼	5 0 4	87¼	5 16 4	99¼	6 12 4
51½	3 8 8	63½	4 4 8	75½	5 0 8	87½	5 16 8	99½	6 12 8
51¾	3 9 0	63¾	4 5 0	75¾	5 1 0	87¾	5 17 0	99¾	6 13 0
52	3 9 4	64	4 5 4	76	5 1 4	88	5 17 4	100	6 13 4
52¼	3 9 8	64¼	4 5 8	76¼	5 1 8	88¼	5 17 8	101	6 14 8
52½	3 10 0	64½	4 6 0	76½	5 2 0	88½	5 18 0	102	6 16 0
52¾	3 10 4	64¾	4 6 4	76¾	5 2 4	88¾	5 18 4	103	6 17 4
53	3 10 8	65	4 6 8	77	5 2 8	89	5 18 8	104	6 18 8
53¼	3 11 0	65¼	4 7 0	77¼	5 3 0	89¼	5 19 0	105	7 0 0
53½	3 11 4	65½	4 7 4	77½	5 3 4	89½	5 19 4	106	7 1 4
53¾	3 11 8	65¾	4 7 8	77¾	5 3 8	89¾	5 19 8	107	7 2 8
54	3 12 0	66	4 8 0	78	5 4 0	90	6 0 0	108	7 4 0
54¼	3 12 4	66¼	4 8 4	78¼	5 4 4	90¼	6 0 4	109	7 5 4
54½	3 12 8	66½	4 8 8	78½	5 4 8	90½	6 0 8	110	7 6 8
54¾	3 13 0	66¾	4 9 0	78¾	5 5 0	90¾	6 1 0	120	8 0 0
55	3 13 4	67	4 9 4	79	5 5 4	91	6 1 4	130	8 13 4
55¼	3 13 8	67¼	4 9 8	79¼	5 5 8	91¼	6 1 8	140	9 6 8
55½	3 14 0	67½	4 10 0	79½	5 6 0	91½	6 2 0	150	10 0 0
55¾	3 14 4	67¾	4 10 4	79¾	5 6 4	91¾	6 2 4	160	10 13 4
56	3 14 8	68	4 10 8	80	5 6 8	92	6 2 8	170	11 6 8
56¼	3 15 0	68¼	4 11 0	80¼	5 7 0	92¼	6 3 0	200	13 6 8
56½	3 15 4	68½	4 11 4	80½	5 7 4	92½	6 3 4	300	20 0 0
56¾	3 15 8	68¾	4 11 8	80¾	5 7 8	92¾	6 3 8	400	26 13 4
57	3 16 0	69	4 12 0	81	5 8 0	93	6 4 0	500	33 6 8
57¼	3 16 4	69¼	4 12 4	81¼	5 8 4	93¼	6 4 4	600	40 0 0
57½	3 16 8	69½	4 12 8	81½	5 8 8	93½	6 4 8	700	46 13 4
57¾	3 17 0	69¾	4 13 0	81¾	5 9 0	93¾	6 5 0	800	53 6 8
58	3 17 4	70	4 13 4	82	5 9 4	94	6 5 4	900	60 0 0
58¼	3 17 8	70¼	4 13 8	82¼	5 9 8	94¼	6 5 8	1000	66 13 4
58½	3 18 0	70½	4 14 0	82½	5 10 0	94½	6 6 0	2000	133 6 8
58¾	3 18 4	70¾	4 14 4	82¾	5 10 4	94¾	6 6 4	3000	200 0 0
59	3 18 8	71	4 14 8	83	5 10 8	95	6 6 8	4000	266 13 4
59¼	3 19 0	71¼	4 15 0	83¼	5 11 0	95¼	6 7 0	5000	333 6 8
59½	3 19 4	71½	4 15 4	83½	5 11 4	95½	6 7 4	6000	400 0 0
59¾	3 19 8	71¾	4 15 8	83¾	5 11 8	95¾	6 7 8	7000	466 13 4
60	4 0 0	72	4 16 0	84	5 12 0	96	6 8 0	8000	533 6 8
60¼	4 0 4	72¼	4 16 4	84¼	5 12 4	96¼	6 8 4	9000	600 0 0
60½	4 0 8	72½	4 16 8	84½	5 12 8	96½	6 8 8	10000	666 13 4
60¾	4 1 0	72¾	4 17 0	84¾	5 13 0	96¾	6 9 0	20000	1333 6 8
61	4 1 4	73	4 17 4	85	5 13 4	97	6 9 4	30000	2000 0 0
61¼	4 1 8	73¼	4 17 8	85¼	5 13 8	97¼	6 9 8	40000	2666 13 4
61½	4 2 0	73½	4 18 0	85½	5 14 0	97½	6 10 0	50000	3333 6 8
61¾	4 2 4	73¾	4 18 4	85¾	5 14 4	97¾	6 10 4		

T A B L E 6.

For the DRAWBACK on Victuallers' Strong Beer, at 1s. 8d. per Barrel, each Barrel containing 34 gallons;

From ¼ of a Barrel to 50,000 Barrels.

Bar.	l.	s.	d.	Bar	l.	s.	d.	Bar	l.	s.	d.	Bar	l.	s.	d.	Bar	l.	s.	d.
				10	0	16	8	20	1	13	4	30	2	10	0	40	3	6	8
¼	0	0	5	10¼	0	17	1	20¼	1	13	9	30¼	2	10	5	40¼	3	7	1
½	0	0	10	10½	0	17	6	20½	1	14	2	30½	2	10	10	40½	3	7	6
¾	0	1	3	10¾	0	17	11	20¾	1	14	7	30¾	2	11	3	40¾	3	7	11
1	0	1	8	11	0	18	4	21	1	15	0	31	2	11	8	41	3	8	4
1¼	0	2	1	11¼	0	18	9	21¼	1	15	5	31¼	2	12	1	41¼	3	8	9
1½	0	2	6	11½	0	19	2	21½	1	15	10	31½	2	12	6	41½	3	9	2
1¾	0	2	11	11¾	0	19	7	21¾	1	16	3	31¾	2	12	11	41¾	3	9	7
2	0	3	4	12	1	0	0	22	1	16	8	32	2	13	4	42	3	10	0
2¼	0	3	9	12¼	1	0	5	22¼	1	17	1	32¼	2	13	9	42¼	3	10	5
2½	0	4	2	12½	1	0	10	22½	1	17	6	32½	2	14	2	42½	3	10	10
2¾	0	4	7	12¾	1	1	3	22¾	1	17	11	32¾	2	14	7	42¾	3	11	3
3	0	5	0	13	1	1	8	23	1	18	4	33	2	15	0	43	3	11	8
3¼	0	5	5	13¼	1	2	1	23¼	1	18	9	33¼	2	15	5	43¼	3	12	1
3½	0	5	10	13½	1	2	6	23½	1	19	2	33½	2	15	10	43½	3	12	6
3¾	0	6	3	13¾	1	2	11	23¾	1	19	7	33¾	2	16	3	43¾	3	12	11
4	0	6	8	14	1	3	4	24	2	0	0	34	2	16	8	44	3	13	4
4¼	0	7	1	14¼	1	3	9	24¼	2	0	5	34¼	2	17	1	44¼	3	13	9
4½	0	7	6	14½	1	4	2	24½	2	0	10	34½	2	17	6	44½	3	14	2
4¾	0	7	11	14¾	1	4	7	24¾	2	1	3	34¾	2	17	11	44¾	3	14	7
5	0	8	4	15	1	5	0	25	2	1	8	35	2	18	4	45	3	15	0
5¼	0	8	9	15¼	1	5	5	25¼	2	2	1	35¼	2	18	9	45¼	3	15	5
5½	0	9	2	15½	1	5	10	25½	2	2	6	35½	2	19	2	45½	3	15	10
5¾	0	9	7	15¾	1	6	3	25¾	2	2	11	35¾	2	19	7	45¾	3	16	3
6	0	10	0	16	1	6	8	26	2	3	4	36	3	0	0	46	3	16	8
6¼	0	10	5	16¼	1	7	1	26¼	2	3	9	36¼	3	0	5	46¼	3	17	1
6½	0	10	10	16½	1	7	6	26½	2	4	2	36½	3	0	10	46½	3	17	6
6¾	0	11	3	16¾	1	7	11	26¾	2	4	7	36¾	3	1	3	46¾	3	17	11
7	0	11	8	17	1	8	4	27	2	5	0	37	3	1	8	47	3	18	4
7¼	0	12	1	17¼	1	8	9	27¼	2	5	5	37¼	3	2	1	47¼	3	18	9
7½	0	12	6	17½	1	9	2	27½	2	5	10	37½	3	2	6	47½	3	19	2
7¾	0	12	11	17¾	1	9	7	27¾	2	6	3	37¾	3	2	11	47¾	3	19	7
8	0	13	4	18	1	10	0	28	2	6	8	38	3	3	4	48	4	0	0
8¼	0	13	9	18¼	1	10	5	28¼	2	7	1	38¼	3	3	9	48¼	4	0	5
8½	0	14	2	18½	1	10	10	28½	2	7	6	38½	3	4	2	48½	4	0	10
8¾	0	14	7	18¾	1	11	3	28¾	2	7	11	38¾	3	4	7	48¾	4	1	3
9	0	15	0	19	1	11	8	29	2	8	4	39	3	5	0	49	4	1	8
9¼	0	15	5	19¼	1	12	1	29¼	2	8	9	39¼	3	5	5	49¼	4	2	1
9½	0	15	10	19½	1	12	6	29½	2	9	2	39½	3	5	10	49½	4	2	6
9¾	0	16	3	19¾	1	12	11	29¾	2	9	7	39¾	3	6	3	49¾	4	2	11

TABLE for the Drawback on Victualers' Strong Beer continued.

Bar.	l.	s.	d.	Bar.	l.	s.	d.	Bar.	l.	s.	d.	Bar.	l.	s.	d.	Bar.	l.	s.	d.
50	4	3	4	62	5	3	4	74	6	3	4	86	7	3	4	98	8	3	4
50¼	4	3	9	62¼	5	3	9	74¼	6	3	9	86¼	7	3	9	98¼	8	3	9
50½	4	4	2	62½	5	4	2	74½	6	4	2	86½	7	4	2	98½	8	4	2
50¾	4	4	7	62¾	5	4	7	74¾	6	4	7	86¾	7	4	7	98¾	8	4	7
51	4	5	0	63	5	5	0	75	6	5	0	87	7	5	0	99	8	5	0
51¼	4	5	5	63¼	5	5	5	75¼	6	5	5	87¼	7	5	5	99¼	8	5	5
51½	4	5	10	63½	5	5	10	75½	6	5	10	87½	7	5	10	99½	8	5	10
51¾	4	6	3	63¾	5	6	3	75¾	6	6	3	87¾	7	6	3	99¾	8	6	3
52	4	6	8	64	5	6	8	76	6	6	8	88	7	6	8	100	8	6	8
52¼	4	7	1	64¼	5	7	1	76¼	6	7	1	88¼	7	7	1	101	8	8	4
52½	4	7	6	64½	5	7	6	76½	6	7	6	88½	7	7	6	102	8	10	0
52¾	4	7	11	64¾	5	7	11	76¾	6	7	11	88¾	7	7	11	103	8	11	8
53	4	8	4	65	5	8	4	77	6	8	4	89	7	8	4	104	8	13	4
53¼	4	8	9	65¼	5	8	9	77¼	6	8	9	89¼	7	8	9	105	8	15	0
53½	4	9	2	65½	5	9	2	77½	6	9	2	89½	7	9	2	106	8	16	8
53¾	4	9	7	65¾	5	9	7	77¾	6	9	7	89¾	7	9	7	107	8	18	4
54	4	10	0	66	5	10	0	78	6	10	0	90	7	10	0	108	9	0	0
54¼	4	10	5	66¼	5	10	5	78¼	6	10	5	90¼	7	10	5	109	9	1	8
54½	4	10	10	66½	5	10	10	78½	6	10	10	90½	7	10	10	110	9	3	4
54¾	4	11	3	66¾	5	11	3	78¾	6	11	3	90¾	7	11	3	120	10	0	0
55	4	11	8	67	5	11	8	79	6	11	8	91	7	11	8	130	10	16	8
55¼	4	12	1	67¼	5	12	1	79¼	6	12	1	91¼	7	12	1	140	11	13	4
55½	4	12	6	67½	5	12	6	79½	6	12	6	91½	7	12	6	150	12	10	0
55¾	4	12	11	67¾	5	12	11	79¾	6	12	11	91¾	7	12	11	160	13	6	8
56	4	13	4	68	5	13	4	80	6	13	4	92	7	13	4	170	14	3	4
56¼	4	13	9	68¼	5	13	9	80¼	6	13	9	92¼	7	13	9	200	16	13	4
56½	4	14	2	68½	5	14	2	80½	6	14	2	92½	7	14	2	300	25	0	0
56¾	4	14	7	68¾	5	14	7	80¾	6	14	7	92¾	7	14	7	400	33	6	8
57	4	15	0	69	5	15	0	81	6	15	0	93	7	15	0	500	41	13	4
57¼	4	15	5	69¼	5	15	5	81¼	6	15	5	93¼	7	15	5	600	50	0	0
57½	4	15	10	69½	5	15	10	81½	6	15	10	93½	7	15	10	700	58	6	8
57¾	4	16	3	69¾	5	16	3	81¾	6	16	3	93¾	7	16	3	800	66	13	4
58	4	16	8	70	5	16	8	82	6	16	8	94	7	16	8	900	75	0	0
58¼	4	17	1	70¼	5	17	1	82¼	6	17	1	94¼	7	17	1	1000	83	6	8
58½	4	17	6	70½	5	17	6	82½	6	17	6	94½	7	17	6	2000	166	13	4
58¾	4	17	11	70¾	5	17	11	82¾	6	17	11	94¾	7	17	11	3000	250	0	0
59	4	18	4	71	5	18	4	83	6	18	4	95	7	18	4	4000	333	6	8
59¼	4	18	9	71¼	5	18	9	83¼	6	18	9	95¼	7	18	9	5000	416	13	4
59½	4	19	2	71½	5	19	2	83½	6	19	2	95½	7	19	2	6000	500	0	0
59¾	4	19	7	71¾	5	19	7	83¾	6	19	7	95¾	7	19	7	7000	583	6	8
60	5	0	0	72	6	0	0	84	7	0	0	96	8	0	0	8000	666	13	4
60¼	5	0	5	72¼	6	0	5	84¼	7	0	5	96¼	8	0	5	9000	750	0	0
60½	5	0	10	72½	6	0	10	84½	7	0	10	96½	8	0	10	10000	833	6	8
60¾	5	1	3	72¾	6	1	3	84¾	7	1	3	96¾	8	1	3	20000	1666	13	4
61	5	1	8	73	6	1	8	85	7	1	8	97	8	1	8	30000	2500	0	0
61¼	5	2	1	73¼	6	2	1	85¼	7	2	1	97¼	8	2	1	40000	3333	6	8
61½	5	2	6	73½	6	2	6	85½	7	2	6	97½	8	2	6	50000	4166	13	4
61¾	5	2	11	73¾	6	2	11	85¾	7	2	11	97¾	8	2	11				

TABLE 7.

For the DRAWBACK on Victuallers' Small Beer, at 4d. per Barrel, each Barrel containing 34 gallons;

From ¼ of a Barrel to 50,000 Barrels.

Bar.	l.	s.	d.	Bar	l.	s.	d.	Bar	l.	s.	d.	Bar	l.	s.	d.	Bar	l.	s.	d.
				10	0	3	4	20	0	6	8	30	0	10	0	40	0	13	4
¼	0	0	1	10¼	0	3	5	20¼	0	6	9	30¼	0	10	1	40¼	0	13	5
½	0	0	2	10½	0	3	6	20½	0	6	10	30½	0	10	2	40½	0	13	6
¾	0	0	3	10¾	0	3	7	20¾	0	6	11	30¾	0	10	3	40¾	0	13	7
1	0	0	4	11	0	3	8	21	0	7	0	31	0	10	4	41	0	13	8
1¼	0	0	5	11¼	0	3	9	21¼	0	7	1	31¼	0	10	5	41¼	0	13	9
1½	0	0	6	11½	0	3	10	21½	0	7	2	31½	0	10	6	41½	0	13	10
1¾	0	0	7	11¾	0	3	11	21¾	0	7	3	31¾	0	10	7	41¾	0	13	11
2	0	0	8	12	0	4	0	22	0	7	4	32	0	10	8	42	0	14	0
2¼	0	0	9	12¼	0	4	1	22¼	0	7	5	32¼	0	10	9	42¼	0	14	1
2½	0	0	10	12½	0	4	2	22½	0	7	6	32½	0	10	10	42½	0	14	2
2¾	0	0	11	12¾	0	4	3	22¾	0	7	7	32¾	0	10	11	42¾	0	14	3
3	0	1	0	13	0	4	4	23	0	7	8	33	0	11	0	43	0	14	4
3¼	0	1	1	13¼	0	4	5	23¼	0	7	9	33¼	0	11	1	43¼	0	14	5
3½	0	1	2	13½	0	4	6	23½	0	7	10	33½	0	11	2	43½	0	14	6
3¾	0	1	3	13¾	0	4	7	23¾	0	7	11	33¾	0	11	3	43¾	0	14	7
4	0	1	4	14	0	4	8	24	0	8	0	34	0	11	4	44	0	14	8
4¼	0	1	5	14¼	0	4	9	24¼	0	8	1	34¼	0	11	5	44¼	0	14	9
4½	0	1	6	14½	0	4	10	24½	0	8	2	34½	0	11	6	44½	0	14	10
4¾	0	1	7	14¾	0	4	11	24¾	0	8	3	34¾	0	11	7	44¾	0	14	11
5	0	1	8	15	0	5	0	25	0	8	4	35	0	11	8	45	0	15	0
5¼	0	1	9	15¼	0	5	1	25¼	0	8	5	35¼	0	11	9	45¼	0	15	1
5½	0	1	10	15½	0	5	2	25½	0	8	6	35½	0	11	10	45½	0	15	2
5¾	0	1	11	15¾	0	5	3	25¾	0	8	7	35¾	0	11	11	45¾	0	15	3
6	0	2	0	16	0	5	4	26	0	8	8	36	0	12	0	46	0	15	4
6¼	0	2	1	16¼	0	5	5	26¼	0	8	9	36¼	0	12	1	46¼	0	15	5
6½	0	2	2	16½	0	5	6	26½	0	8	10	36½	0	12	2	46½	0	15	6
6¾	0	2	3	16¾	0	5	7	26¾	0	8	11	36¾	0	12	3	46¾	0	15	7
7	0	2	4	17	0	5	8	27	0	9	0	37	0	12	4	47	0	15	8
7¼	0	2	5	17¼	0	5	9	27¼	0	9	1	37¼	0	12	5	47¼	0	15	9
7½	0	2	6	17½	0	5	10	27½	0	9	2	37½	0	12	6	47½	0	15	10
7¾	0	2	7	17¾	0	5	11	27¾	0	9	3	37¾	0	12	7	47¾	0	15	11
8	0	2	8	18	0	6	0	28	0	9	4	38	0	12	8	48	0	16	0
8¼	0	2	9	18¼	0	6	1	28¼	0	9	5	38¼	0	12	9	48¼	0	16	1
8½	0	2	10	18½	0	6	2	28½	0	9	6	38½	0	12	10	48½	0	16	2
8¾	0	2	11	18¾	0	6	3	28¾	0	9	7	38¾	0	12	11	48¾	0	16	3
9	0	3	0	19	0	6	4	29	0	9	8	39	0	13	0	49	0	16	4
9¼	0	3	1	19¼	0	6	5	29¼	0	9	9	39¼	0	13	1	49¼	0	16	5
9½	0	3	2	19½	0	6	6	29½	0	9	10	39½	0	13	2	49½	0	16	6
9¾	0	3	3	19¾	0	6	7	29¾	0	9	11	39¾	0	13	3	49¾	0	16	7

See

TABLE for the Drawback on Victuallers' Small Beer continued.

Bar.	l.	s.	d.	Bar.	l.	s.	d.	Bar.	l.	s.	d.	Bar.	l.	s.	d.	Bar.	l.	s.	d.
50	0	16	8	62	1	0	8	74	1	4	8	86	1	8	8	98	1	12	8
50¼	0	16	9	62¼	1	0	9	74¼	1	4	9	86¼	1	8	9	98¼	1	12	9
50½	0	16	10	62½	1	0	10	74½	1	4	10	86½	1	8	10	98½	1	12	10
50¾	0	16	11	62¾	1	0	11	74¾	1	4	11	86¾	1	8	11	98¾	1	12	11
51	0	17	0	63	1	1	0	75	1	5	0	87	1	9	0	99	1	13	0
51¼	0	17	1	63¼	1	1	1	75¼	1	5	1	87¼	1	9	1	99¼	1	13	1
51½	0	17	2	63½	1	1	2	75½	1	5	2	87½	1	9	2	99½	1	13	2
51¾	0	17	3	63¾	1	1	3	75¾	1	5	3	87¾	1	9	3	99¾	1	13	3
52	0	17	4	64	1	1	4	76	1	5	4	88	1	9	4	100	1	13	4
52¼	0	17	5	64¼	1	1	5	76¼	1	5	5	88¼	1	9	5	101	1	13	8
52½	0	17	6	64½	1	1	6	76½	1	5	6	88½	1	9	6	102	1	14	0
52¾	0	17	7	64¾	1	1	7	76¾	1	5	7	88¾	1	9	7	103	1	14	4
53	0	17	8	65	1	1	8	77	1	5	8	89	1	9	8	104	1	14	8
53¼	0	17	9	65¼	1	1	9	77¼	1	5	9	89¼	1	9	9	105	1	15	0
53½	0	17	10	65½	1	1	10	77½	1	5	10	89½	1	9	10	106	1	15	4
53¾	0	17	11	65¾	1	1	11	77¾	1	5	11	89¾	1	9	11	107	1	15	8
54	0	18	0	66	1	2	0	78	1	6	0	90	1	10	0	108	1	16	0
54¼	0	18	1	66¼	1	2	1	78¼	1	6	1	90¼	1	10	1	109	1	16	4
54½	0	18	2	66½	1	2	2	78½	1	6	2	90½	1	10	2	110	1	16	8
54¾	0	18	3	66¾	1	2	3	78¾	1	6	3	90¾	1	10	3	120	2	0	0
55	0	18	4	67	1	2	4	79	1	6	4	91	1	10	4	130	2	3	4
55¼	0	18	5	67¼	1	2	5	79¼	1	6	5	91¼	1	10	5	140	2	6	8
55½	0	18	6	67½	1	2	6	79½	1	6	6	91½	1	10	6	150	2	10	0
55¾	0	18	7	67¾	1	2	7	79¾	1	6	7	91¾	1	10	7	160	2	13	4
56	0	18	8	68	1	2	8	80	1	6	8	92	1	10	8	170	2	16	8
56¼	0	18	9	68¼	1	2	9	80¼	1	6	9	92¼	1	10	9	200	3	6	8
56½	0	18	10	68½	1	2	10	80½	1	6	10	92½	1	10	10	300	5	0	0
56¾	0	18	11	68¾	1	2	11	80¾	1	6	11	92¾	1	10	11	400	6	13	4
57	0	19	0	69	1	3	0	81	1	7	0	93	1	11	0	500	8	6	8
57¼	0	19	1	69¼	1	3	1	81¼	1	7	1	93¼	1	11	1	600	10	0	0
57½	0	19	2	69½	1	3	2	81½	1	7	2	93½	1	11	2	700	11	13	4
57¾	0	19	3	69¾	1	3	3	81¾	1	7	3	93¾	1	11	3	800	13	6	8
58	0	19	4	70	1	3	4	82	1	7	4	94	1	11	4	900	15	0	0
58¼	0	19	5	70¼	1	3	5	82¼	1	7	5	94¼	1	11	5	1000	16	13	4
58½	0	19	6	70½	1	3	6	82½	1	7	6	94½	1	11	6	2000	33	6	8
58¾	0	19	7	70¾	1	3	7	82¾	1	7	7	94¾	1	11	7	3000	50	0	0
59	0	19	8	71	1	3	8	83	1	7	8	95	1	11	8	4000	66	13	4
59¼	0	19	9	71¼	1	3	9	83¼	1	7	9	95¼	1	11	9	5000	83	6	8
59½	0	19	10	71½	1	3	10	83½	1	7	10	95½	1	11	10	6000	100	0	0
59¾	0	19	11	71¾	1	3	11	83¾	1	7	11	95¾	1	11	11	7000	116	13	4
60	1	0	0	72	1	4	0	84	1	8	0	96	1	12	0	8000	133	6	8
60¼	1	0	1	72¼	1	4	1	84¼	1	8	1	96¼	1	12	1	9000	150	0	0
60½	1	0	2	72½	1	4	2	84½	1	8	2	96½	1	12	2	10000	166	13	4
60¾	1	0	3	72¾	1	4	3	84¾	1	8	3	96¾	1	12	3	20000	333	6	8
61	1	0	4	73	1	4	4	85	1	8	4	97	1	12	4	30000	500	0	0
61¼	1	0	5	73¼	1	4	5	85¼	1	8	5	97¼	1	12	5	40000	666	13	4
61½	1	0	6	73½	1	4	6	85½	1	8	6	97½	1	12	6	50000	833	6	8
61¾	1	0	7	73¾	1	4	7	85¾	1	8	7	97¾	1	12	7				

TABLE 8.

A TABLE to MONEY COUCH BUSHELS.

Bush.	Old Duty			Add. Duty			15per Cent.			New Add. D.			5 per Cent.			Total Duty			
	s.	d.	pa.	s.	d.	p.	s.	d.	pa.	s.	d.	pa.	s.	d.	pa.	l.	s.	d.	pa.
,1	0	0¼	92	0	0	96	0	0	144	0	0¼	92	0	0	096	0	0	1¼	04
,2	0	0½	84	0	0¼	92	0	0	288	0	0½	84	0	0	192	0	0	2½	08
,3	0	1¼	76	0	0½	88	0	0	432	0	1¼	76	0	0	288	0	0	3¾	12
,4	0	1½	68	0	0¾	84	0	0	576	0	1½	68	0	0	384	0	0	5	16
,5	0	2¼	60	0	1	80	0	0	720	0	2¼	60	0	0	480	0	0	6¼	20
,6	0	2¾	52	0	1¼	76	0	0	864	0	2¾	52	0	0	576	0	0	7½	24
,7	0	3¼	44	0	1½	72	0	0¼	008	0	3¼	44	0	0	672	0	0	8¾	28
,8	0	3¾	36	0	1¾	68	0	0¼	152	0	3¾	36	0	0	768	0	0	10	32
,9	0	4¼	38	0	2	64	0	0¼	296	0	4¼	28	0	0	864	0	0	11¼	36
1	0	4¾	2	0	2¼	6	0	0¼	44	0	4¾	2	0	0	96	0	1	0½	4
2	0	9½	4	0	4¾	2	0	0½	88	0	9½	4	0	0¼	92	0	2	1	8
3	1	2¼	6	0	7	8	0	1	32	1	2¼	6	0	0½	88	0	3	1¾	2
4	1	7	8	0	9½	4	0	1¼	76	1	7	8	0	0¾	84	0	4	2¼	6
5	2	0	0	1	0	0	0	1¾	20	2	0	0	0	1	80	0	5	3	0
6	2	4¾	2	1	2¼	6	0	2	64	2	4¾	2	0	1¼	76	0	6	3½	4
7	2	9½	4	1	4¾	2	0	2½	08	2	9½	4	0	1½	72	0	7	4	8
8	3	2¼	6	1	7	8	0	2¾	52	3	2¼	6	0	1¾	68	0	8	4¾	2
9	3	7	8	1	9½	4	0	3	96	3	7	8	0	2	64	0	9	5¼	6
10	4	0	0	2	0	0	0	3½	40	4	0	0	0	2¼	60	0	10	6	0
11	4	4¾	2	2	2¼	6	0	3¾	84	4	4¾	2	0	2½	56	0	11	6½	4
12	4	9½	4	2	4¾	2	0	4¼	28	4	9½	4	0	2¾	52	0	12	7	8
13	5	2¼	6	2	7	8	0	4½	72	5	2¼	6	0	3	48	0	13	7¾	2
14	5	7	8	2	9½	4	0	5	16	5	7	8	0	3¼	44	0	14	8¼	6
15	6	0	0	3	0	0	0	5¼	60	6	0	0	0	3½	40	0	15	9	0
16	6	4¾	2	3	2¼	6	0	5¾	04	6	4¾	2	0	3¾	36	0	16	9½	4
17	6	9½	4	3	4¾	2	0	6	48	6	9½	4	0	4	32	0	17	10	8
18	7	2¼	6	3	7	8	0	6¼	92	7	2¼	6	0	4¼	28	0	18	10¾	2
19	7	7	8	3	9½	4	0	6¾	36	7	7	8	0	4½	24	0	19	11¼	6
20	8	0	0	4	0	0	0	7	80	8	0	0	0	4¾	20	1	1	0	0
21	8	4¾	2	4	2¼	6	0	7½	24	8	4¾	2	0	5	16	1	2	0½	4
22	8	9½	4	4	4¾	2	0	7¾	68	8	9½	4	0	5¼	12	1	3	1	8
23	9	2¼	6	4	7	8	0	8¼	12	9	2¼	6	0	5½	08	1	4	1¾	2
24	9	7	8	4	9½	4	0	8½	56	9	7	8	0	5¾	04	1	5	2¼	6
25	10	0	0	5	0	0	0	9	0	10	0	0	0	6	0	1	6	3	0
26	10	4¾	2	5	2¼	6	0	9¼	44	10	4¾	2	0	6	96	1	7	3½	4
27	10	9½	4	5	4¾	2	0	9½	88	10	9½	4	0	6¼	92	1	8	4	8
28	11	2¼	6	5	7	8	0	10	32	11	2¼	6	0	6½	88	1	9	4¾	2
29	11	7	8	5	9½	4	0	10¼	76	11	7	8	0	6¾	84	1	10	5¼	6
30	12	0	0	6	0	0	0	10¾	20	12	0	0	0	7	80	1	11	6	0
31	12	4¾	2	6	2¼	6	0	11	64	12	4¾	2	0	7¼	76	1	12	6½	4
32	12	9½	4	6	4¾	2	0	11½	08	12	9½	4	0	7½	72	1	13	7	8
33	13	2¼	6	6	7	8	0	11¾	52	13	2¼	6	0	7¾	68	1	14	7¾	2
34	13	7	8	6	9½	4	1	0	96	13	7	8	0	8	64	1	15	8¼	6
35	14	0	0	7	0	0	1	0½	40	14	0	0	0	8¼	60	1	16	9	0
36	14	4¾	2	7	2¼	6	1	0¾	84	14	4¾	2	0	8½	56	1	17	9½	4
37	14	9½	4	7	4¾	2	1	1¼	28	14	9½	4	0	8¾	52	1	18	10	8
38	15	2¼	6	7	7	8	1	1½	72	15	2¼	6	0	9	48	1	19	10¾	2
39	15	7	8	7	9½	4	1	2	16	15	7	8	0	9¼	44	2	0	11¼	6
40	16	0	0	8	0	0	1	2¼	60	16	0	0	0	9½	40	2	2	0	0
41	16	4¾	2	8	2¼	6	1	2¾	04	16	4¾	2	0	9¾	36	2	3	0½	4
42	16	9½	4	8	4¾	2	1	3	48	16	9½	4	0	10	32	2	4	1	8
43	17	2¼	6	8	7	8	1	3¼	92	17	2¼	6	0	10¼	28	2	5	1¾	2
44	17	7	8	8	9½	4	1	3¾	36	17	7	8	0	10½	24	2	6	2¼	6
45	18	0	0	9	0	0	1	4	80	18	0	0	0	10¾	20	2	7	3	0
46	18	4¾	2	9	2¼	6	1	4½	24	18	4¾	2	0	11	16	2	8	3½	4
47	18	9½	4	9	4¾	2	1	4¾	68	18	9½	4	0	11¼	12	2	9	4	8
48	19	2¼	6	9	7	8	1	5¼	12	19	2¼	6	0	11½	08	2	10	4¾	2
49	19	7	8	9	9½	4	1	5½	56	19	7	8	0	11¾	04	2	11	5¼	6
50	20	0	0	10	0	0	1	6	0	20	0	0	1	0	0	2	12	6	0

TABLE to Money Couch Bushels continued.

Bush.	Old Duty.				Ad. Duty.				15 per Cent.			New Ad. Duty				5 per Cent.			Total Duty.			
	l.	s.	d.	p.	l.	s.	d.	p.	s.	d.	pa.	l.	s.	d.	p.	s.	d.	pa.	l.	s.	d.	p.
51	1	0	4¾	2	0	10	2¼	6	1	6¼	44	1	0	4¾	2	1	0	96	2	13	6½	4
52	1	0	9½	4	0	10	4¾	2	1	6½	88	1	0	9½	4	1	0¼	92	2	14	7	8
53	1	1	2¼	6	0	10	7	8	1	7	32	1	1	2¼	6	1	0½	88	2	15	7¾	2
54	1	1	7	8	0	10	9½	4	1	7¼	76	1	1	7	8	1	0¾	84	2	16	8¼	6
55	1	2	0	0	0	11	0	0	1	7¾	20	1	2	0	0	1	1	80	2	17	9	0
56	1	2	4¾	2	0	11	2¼	6	1	8	64	1	2	4¾	2	1	1¼	76	2	18	9½	4
57	1	2	9½	4	0	11	4¾	2	1	8½	08	1	2	9½	4	1	1½	72	2	19	10	8
58	1	3	2¼	6	0	11	7	8	1	8¾	52	1	3	2¼	6	1	1¾	68	3	0	10¾	2
59	1	3	7	8	0	11	9½	4	1	9	96	1	3	7	8	1	2	64	3	1	11¼	6
60	1	4	0	0	0	12	0	0	1	9½	40	1	4	0	0	1	2¼	60	3	3	0	0
61	1	4	4¾	2	0	12	2¼	6	1	9¾	84	1	4	4¾	2	1	2½	56	3	4	0½	4
62	1	4	9½	4	0	12	4¾	2	1	10¼	28	1	4	9½	4	1	2¾	52	3	5	1	8
63	1	5	2¼	6	0	12	7	8	1	10½	72	1	5	2¼	6	1	3	48	3	6	1¾	2
64	1	5	7	8	0	12	9½	4	1	11	16	1	5	7	8	1	3¼	44	3	7	2¼	6
65	1	6	0	0	0	13	0	0	1	11¼	60	1	6	0	0	1	3½	40	3	8	3	0
66	1	6	4¾	2	0	13	2¼	6	1	11¾	04	1	6	4¾	2	1	3¾	36	3	9	3½	4
67	1	6	9½	4	0	13	4¾	2	2	0	48	1	6	9½	4	1	4	32	3	10	4	8
68	1	7	2¼	6	0	13	7	8	2	0¼	92	1	7	2¼	6	1	4¼	28	3	11	4¾	2
69	1	7	7	8	0	13	9½	4	2	0¾	36	1	7	7	8	1	4½	24	3	12	5¼	6
70	1	8	0	0	0	14	0	0	2	1	80	1	8	0	0	1	4¾	20	3	13	6	0
71	1	8	4¾	2	0	14	2¼	6	2	1½	24	1	8	4¾	2	1	5	16	3	14	6½	4
72	1	8	9½	4	0	14	4¾	2	2	1¾	68	1	8	9½	4	1	5¼	12	3	15	7	8
73	1	9	2¼	6	0	14	7	8	2	2¼	12	1	9	2¼	6	1	5½	08	3	16	7¾	2
74	1	9	7	8	0	14	9½	4	2	2½	56	1	9	7	8	1	5¾	04	3	17	8¼	6
75	1	10	0	0	0	15	0	0	2	3	0	1	10	0	0	1	6	0	3	18	9	0
76	1	10	4¾	2	0	15	2¼	6	2	3¼	44	1	10	4¾	2	1	6	96	3	19	9½	4
77	1	10	9½	4	0	15	4¾	2	2	3½	88	1	10	9½	4	1	6¼	92	4	0	10	8
78	1	11	2¼	6	0	15	7	8	2	4	32	1	11	2¼	6	1	6½	88	4	1	10¾	2
79	1	11	7	8	0	15	9½	4	2	4¼	76	1	11	7	8	1	6¾	84	4	2	11¼	6
80	1	12	0	0	0	16	0	0	2	4¾	20	1	12	0	0	1	7	80	4	4	0	0
81	1	12	4¾	2	0	16	2¼	6	2	5	64	1	12	4¾	2	1	7¼	76	4	5	0½	4
82	1	12	9½	4	0	16	4¾	2	2	5½	08	1	12	9½	4	1	7½	72	4	6	1	8
83	1	13	2¼	6	0	16	7	8	2	5¾	52	1	13	2¼	6	1	7¾	68	4	7	1¾	2
84	1	13	7	8	0	16	9½	4	2	6	96	1	13	7	8	1	8	64	4	8	2¼	6
85	1	14	0	0	0	17	0	0	2	6½	40	1	14	0	0	1	8¼	60	4	9	3	0
86	1	14	4¾	2	0	17	2¼	6	2	6¾	84	1	14	4¾	2	1	8½	56	4	10	3½	4
87	1	14	9½	4	0	17	4¾	2	2	7¼	28	1	14	9½	4	1	8¾	52	4	11	4	8
88	1	15	2¼	6	0	17	7	8	2	7½	72	1	15	2¼	6	1	9	48	4	12	4¾	2
89	1	15	7	8	0	17	9½	4	2	8	16	1	15	7	8	1	9¼	44	4	13	5¼	6
90	1	16	0	0	0	18	0	0	2	8½	60	1	16	0	0	1	9½	40	4	14	6	0
91	1	16	4¾	2	0	18	2¼	6	2	8¾	04	1	16	4¾	2	1	9¾	36	4	15	6½	4
92	1	16	9½	4	0	18	4¾	2	2	9	48	1	16	9½	4	1	10	32	4	16	7	8
93	1	17	2¼	6	0	18	7	8	2	9¼	92	1	17	2¼	6	1	10¼	28	4	17	7¾	2
94	1	17	7	8	0	18	9½	4	2	9¾	36	1	17	7	8	1	10½	24	4	18	8¼	6
95	1	18	0	0	0	19	0	0	2	10	80	1	18	0	0	1	10¾	20	4	19	9	0
96	1	18	4¾	2	0	19	2¼	6	2	10½	24	1	18	4¾	2	1	11	16	5	0	9½	4
97	1	18	9½	4	0	19	4¾	2	2	10¾	68	1	18	9½	4	1	11¼	12	5	1	10	8
98	1	19	2¼	6	0	19	7	8	2	11¼	12	1	19	2¼	6	1	11½	08	5	2	10¾	2
99	1	19	7	8	0	19	9½	4	2	11½	56	1	19	7	8	1	11¾	04	5	3	11¼	6
100	2	0	0	0	1	0	0	0	3	0	0	2	0	0	0	2	0	0	5	5	0	0
200	4	0	0	0	2	0	0	0	6	0	0	4	0	0	0	4	0	0	10	10	0	0
300	6	0	0	0	3	0	0	0	9	0	0	6	0	0	0	6	0	0	15	15	0	0
400	8	0	0	0	4	0	0	0	12	0	0	8	0	0	0	8	0	0	21	0	0	0
500	10	0	0	0	5	0	0	0	15	0	0	10	0	0	0	10	0	0	26	5	0	0
600	12	0	0	0	6	0	0	0	18	0	0	12	0	0	0	12	0	0	31	10	0	0
700	14	0	0	0	7	0	0	0	21	0	0	14	0	0	0	14	0	0	36	15	0	0
800	16	0	0	0	8	0	0	0	24	0	0	16	0	0	0	16	0	0	42	0	0	0
900	18	0	0	0	9	0	0	0	27	0	0	18	0	0	0	18	0	0	47	5	0	0
1000	20	0	0	0	10	0	0	0	30	0	0	20	0	0	0	20	0	0	52	10	0	0

TABLE 9.

A TABLE to MONEY FLOOR BUSHELS.

Bush.	Old Duty			Add. Duty			15 per Cent.			New Add. D.			5 per Cent.			Total Duty			
	s.	d.	pa.	s.	d.	pa.	s.	d.	pa.	s.	d.	pa.	s.	d.	pa.	l.	s.	d.	pa.
,1	0	0¼	2	0	0	6	0	0	09	0	0¼	2	0	0	06	0	0	0¾	15
,2	0	0½	4	0	0¼	2	0	0	18	0	0½	4	0	0	12	0	0	1½	30
,3	0	0¾	6	0	0¼	8	0	0	27	0	0¾	6	0	0	18	0	0	2¼	45
,4	0	1	8	0	0½	4	0	0	36	0	1	8	0	0	24	0	0	3	60
,5	0	1½	0	0	0¾	0	0	0	45	0	1½	0	0	0	30	0	0	3¾	75
,6	0	1¾	2	0	0¾	6	0	0	54	0	1¾	2	0	0	36	0	0	4½	90
,7	0	2	4	0	1	2	0	0	63	0	2	4	0	0	42	0	0	5½	05
,8	0	2¼	6	0	1	8	0	0	72	0	2¼	6	0	0	48	0	0	6¼	20
,9	0	2½	8	0	1¼	4	0	0	81	0	2½	8	0	0	54	0	0	7	35
1	0	3	0	0	1½	0	0	0	9	0	3	0	0	0	6	0	0	7¾	5
2	0	6	0	0	3	0	0	0¼	8	0	6	0	0	0¼	2	0	1	3¾	0
3	0	9	0	0	4½	0	0	0½	7	0	9	0	0	0¼	8	0	1	11½	5
4	1	0	0	0	6	0	0	0¾	6	1	0	0	0	0½	4	0	2	7½	0
5	1	3	0	0	7½	0	0	1	5	1	3	0	0	0¾	0	0	3	3¼	5
6	1	6	0	0	9	0	0	1¼	4	1	6	0	0	0¾	6	0	3	11¼	0
7	1	9	0	0	10½	0	0	1¾	3	1	9	0	0	1	2	0	4	7	5
8	2	0	0	1	0	0	0	1¾	2	2	0	0	0	1	8	0	5	3	0
9	2	3	0	1	1½	0	0	2	1	2	3	0	0	1¼	4	0	5	10¼	5
10	2	6	0	1	3	0	0	2¼	0	2	6	0	0	1½	0	0	6	6¾	0
11	2	9	0	1	4½	0	0	2¼	9	2	9	0	0	1½	6	0	7	2¼	5
12	3	0	0	1	6	0	0	2½	8	3	0	0	0	1¾	2	0	7	10½	0
13	3	3	0	1	7½	0	0	2¾	7	3	3	0	0	1¾	8	0	8	6¼	5
14	3	6	0	1	9	0	0	3	6	3	6	0	0	2	4	0	9	2¼	0
15	3	9	0	1	10½	0	0	3¼	5	3	9	0	0	2¼	0	0	9	10	5
16	4	0	0	2	0	0	0	3½	4	4	0	0	0	2¼	6	0	10	6	0
17	4	3	0	2	1½	0	0	3¾	3	4	3	0	0	2½	2	0	11	1¾	5
18	4	6	0	2	3	0	0	4	2	4	6	0	0	2½	8	0	11	9¾	0
19	4	9	0	2	4½	0	0	4¼	1	4	9	0	0	2¾	4	0	12	5½	5
20	5	0	0	2	6	0	0	4½	0	5	0	0	0	3	0	0	13	1½	0
21	5	3	0	2	7½	0	0	4½	9	5	3	0	0	3	6	0	13	9¼	5
22	5	6	0	2	9	0	0	4¾	8	5	6	0	0	3¼	2	0	14	5¼	0
23	5	9	0	2	10½	0	0	5	7	5	9	0	0	3¼	8	0	15	1	5
24	6	0	0	3	0	0	0	5¼	6	6	0	0	0	3½	4	0	15	9	0
25	6	3	0	3	1½	0	0	5½	5	6	3	0	0	3¾	0	0	16	4¾	5
26	6	6	0	3	3	0	0	5¾	4	6	6	0	0	3¾	6	0	17	0¾	0
27	6	9	0	3	4½	0	0	6	3	6	9	0	0	4	2	0	17	8½	5
28	7	0	0	3	6	0	0	6¼	2	7	0	0	0	4	8	0	18	4½	0
29	7	3	0	3	7½	0	0	6½	1	7	3	0	0	4¼	4	0	19	0¼	5
30	7	6	0	3	9	0	0	6¾	0	7	6	0	0	4½	0	0	19	8¼	0
31	7	9	0	3	10½	0	0	6¾	9	7	9	0	0	4½	6	1	0	4	5
32	8	0	0	4	0	0	0	7	8	8	0	0	0	4¾	2	1	1	0	0
33	8	3	0	4	1½	0	0	7¼	7	8	3	0	0	4¾	8	1	1	7¾	5
34	8	6	0	4	3	0	0	7½	6	8	6	0	0	5	4	1	2	3¾	0
35	8	9	0	4	4½	0	0	7¾	5	8	9	0	0	5¼	0	1	2	11½	5
36	9	0	0	4	6	0	0	8	4	9	0	0	0	5¼	6	1	3	7½	0
37	9	3	0	4	7½	0	0	8¼	3	9	3	0	0	5½	2	1	4	3¼	5
38	9	6	0	4	9	0	0	8½	2	9	6	0	0	5½	8	1	4	11¼	0
39	9	9	0	4	10½	0	0	8¾	1	9	9	0	0	5¾	4	1	5	7	5
40	10	0	0	5	0	0	0	9	0	10	0	0	0	6	0	1	6	3	0
41	10	3	0	5	1½	0	0	9	9	10	3	0	0	6	6	1	6	10¾	5
42	10	6	0	5	3	0	0	9¼	8	10	6	0	0	6¼	2	1	7	6¾	0
43	10	9	0	5	4½	0	0	9½	7	10	9	0	0	6¼	8	1	8	2½	5
44	11	0	0	5	6	0	0	9¾	6	11	0	0	0	6½	4	1	8	10½	0
45	11	3	0	5	7½	0	0	10	5	11	3	0	0	6¾	0	1	9	6¼	5
46	11	6	0	5	9	0	0	10¼	4	11	6	0	0	6¾	6	1	10	2¼	0
47	11	9	0	5	10½	0	0	10½	3	11	9	0	0	7	2	1	10	10	5
48	12	0	0	6	0	0	0	10¾	2	12	0	0	0	7	8	1	11	6	0
49	12	3	0	6	1½	0	0	11	1	12	3	0	0	7¼	4	1	12	1¾	5
50	12	6	0	6	3	0	0	11¼	0	12	6	0	0	7½	0	1	12	9¾	0

TABLE to Money Floor Bushels continued.

Bush.	Old Duty.				Ad. Duty.			15l. p. Cent.			New Ad. Duty.				5 per Cent.			Total Duty			
	l.	s.	d.	p.	l.	s.	d.	s.	d.	p.	l.	s.	d.	p.	s.	d.	p.	l.	s.	d.	p.
51	0	12	9	0	0	6	4½	0	11¼	9	0	12	9	0	0	7½	6	1	13	5½	5
52	0	13	0	0	0	6	6	0	11½	8	0	13	0	0	0	7¾	2	1	14	1½	0
53	0	13	3	0	0	6	7½	0	11¾	7	0	13	3	0	0	7¾	8	1	14	9¼	5
54	0	13	6	0	0	6	9	1	0	6	0	13	6	0	0	8	4	1	15	5¼	0
55	0	13	9	0	0	6	10½	1	0¼	5	0	13	9	0	0	8¼	0	1	16	1	5
56	0	14	0	0	0	7	0	1	0½	4	0	14	0	0	0	8¼	6	1	16	9	0
57	0	14	3	0	0	7	1½	1	0¾	3	0	14	3	0	0	8½	2	1	17	4¾	5
58	0	14	6	0	0	7	3	1	1	2	0	14	6	0	0	8½	8	1	18	0¼	0
59	0	14	9	0	0	7	4½	1	1¼	1	0	14	9	0	0	8¾	4	1	18	8½	5
60	0	15	0	0	0	7	6	1	1½	0	0	15	0	0	0	9	0	1	19	4½	0
61	0	15	3	0	0	7	7½	1	1½	9	0	15	3	0	0	9	6	2	0	0¼	5
62	0	15	6	0	0	7	9	1	1¾	8	0	15	6	0	0	9¼	2	2	0	8¼	0
63	0	15	9	0	0	7	10½	1	2	7	0	15	9	0	0	9¼	8	2	1	4	5
64	0	16	0	0	0	8	0	1	2¼	6	0	16	0	0	0	9½	4	2	2	0	0
65	0	16	3	0	0	8	1½	1	2½	5	0	16	3	0	0	9¾	0	2	2	7¾	5
66	0	16	6	0	0	8	3	1	2¾	4	0	16	6	0	0	9¾	6	2	3	3¾	0
67	0	16	9	0	0	8	4½	1	3	3	0	16	9	0	0	10	2	2	3	11½	5
68	0	17	0	0	0	8	6	1	3¼	2	0	17	0	0	0	10	8	2	4	7½	0
69	0	17	3	0	0	8	7½	1	3½	1	0	17	3	0	0	10¼	4	2	5	3¼	5
70	0	17	6	0	0	8	9	1	3¾	0	0	17	6	0	0	10½	0	2	5	11¼	0
71	0	17	9	0	0	8	10½	1	3¾	9	0	17	9	0	0	10½	6	2	6	7	5
72	0	18	0	0	0	9	0	1	4	8	0	18	0	0	0	10¾	2	2	7	3	0
73	0	18	3	0	0	9	1½	1	4¼	7	0	18	3	0	0	10¾	8	2	7	10¾	5
74	0	18	6	0	0	9	3	1	4½	6	0	18	6	0	0	11	4	2	8	6¾	0
75	0	18	9	0	0	9	4½	1	4¾	5	0	18	9	0	0	11¼	0	2	9	2½	5
76	0	19	0	0	0	9	6	1	5	4	0	19	0	0	0	11¼	6	2	9	10½	0
77	0	19	3	0	0	9	7½	1	5¼	3	0	19	3	0	0	11½	2	2	10	6¼	5
78	0	19	6	0	0	9	9	1	5½	2	0	19	6	0	0	11½	8	2	11	2¼	0
79	0	19	9	0	0	9	10½	1	5¾	1	0	19	9	0	0	11¾	4	2	11	10	5
80	1	0	0	0	0	10	0	1	6	0	1	0	0	0	1	0	0	2	12	6	0
81	1	0	3	0	0	10	1½	1	6	9	1	0	3	0	1	0	6	2	13	1¾	5
82	1	0	6	0	0	10	3	1	6¼	8	1	0	6	0	1	0¼	2	2	13	9¾	0
83	1	0	9	0	0	10	4½	1	6½	7	1	0	9	0	1	0¼	8	2	14	5½	5
84	1	1	0	0	0	10	6	1	6¾	6	1	1	0	0	1	0½	4	2	15	1½	0
85	1	1	3	0	0	10	7½	1	7	5	1	1	3	0	1	0¾	0	2	15	9¼	5
86	1	1	6	0	0	10	9	1	7¼	4	1	1	6	0	1	0¾	6	2	16	5¼	0
87	1	1	9	0	0	10	10½	1	7½	3	1	1	9	0	1	1	2	2	17	1	5
88	1	2	0	0	0	11	0	1	7¾	2	1	2	0	0	1	1	8	2	17	9	0
89	1	2	3	0	0	11	1½	1	8	1	1	2	3	0	1	1¼	4	2	18	4¾	5
90	1	2	6	0	0	11	3	1	8¼	0	1	2	6	0	1	1½	0	2	19	0¾	0
91	1	2	9	0	0	11	4½	1	8¼	9	1	2	9	0	1	1½	6	2	19	8½	5
92	1	3	0	0	0	11	6	1	8½	8	1	3	0	0	1	1¾	2	3	0	4½	0
93	1	3	3	0	0	11	7½	1	8¾	7	1	3	3	0	1	1¾	8	3	1	0¼	5
94	1	3	6	0	0	11	9	1	9	6	1	3	6	0	1	2	4	3	1	8¼	0
95	1	3	9	0	0	11	10½	1	9¼	5	1	3	9	0	1	2¼	0	3	2	4	5
96	1	4	0	0	0	12	0	1	9½	4	1	4	0	0	1	2¼	6	3	3	0	0
97	1	4	3	0	0	12	1½	1	9¾	3	1	4	3	0	1	2½	2	3	3	7¾	5
98	1	4	6	0	0	12	3	1	10	2	1	4	6	0	1	2½	8	3	4	3¾	0
99	1	4	9	0	0	12	4½	1	10¼	1	1	4	9	0	1	2¾	4	3	4	11½	5
100	1	5	0	0	0	12	6	1	10½	0	1	5	0	0	1	3	0	3	5	7½	0
200	2	10	0	0	1	5	0	3	9	0	2	10	0	0	2	6	0	6	11	3	0
300	3	15	0	0	1	17	6	5	7½	0	3	15	0	0	3	9	0	9	16	10½	0
400	5	0	0	0	2	10	0	7	6	0	5	0	0	0	5	0	0	13	2	6	0
500	6	5	0	0	3	2	6	9	4½	0	6	5	0	0	6	3	0	16	8	1½	0
600	7	10	0	0	3	15	0	11	3	0	7	10	0	0	7	6	0	19	13	9	0
700	8	15	0	0	4	7	6	13	1½	0	8	15	0	0	8	9	0	22	19	4½	0
800	10	0	0	0	5	0	0	15	0	0	10	0	0	0	10	0	0	26	5	0	0
900	11	5	0	0	5	12	6	16	10½	0	11	5	0	0	11	3	0	29	10	7½	0
1000	12	10	0	0	6	5	0	18	9	0	12	10	0	0	12	6	0	32	16	3	0

TABLE 9.

A TABLE to MONEY FLOOR BUSHELS.

Bush.	Old Duty			Add. Duty			15 per Cent.			New Add. D.			5 per Cent.			Total Duty			
	s.	d.	pa.	s.	d.	pa.	s.	d.	pa.	s.	d.	pa.	s.	d.	pa.	l.	s.	d.	pa.
,1	0	0¼	2	0	0	6	0	0	09	0	0¼	2	0	0	06	0	0	0¾	15
,2	0	0½	4	0	0¼	2	0	0	18	0	0½	4	0	0	12	0	0	1½	30
,3	0	0¾	6	0	0¼	8	0	0	27	0	0¾	6	0	0	18	0	0	2¼	45
,4	0	1	8	0	0½	4	0	0	36	0	1	8	0	0	24	0	0	3	60
,5	0	1½	0	0	0¾	0	0	0	45	0	1½	0	0	0	30	0	0	3¾	75
,6	0	1¾	2	0	0¾	6	0	0	54	0	1¾	2	0	0	36	0	0	4½	90
,7	0	2	4	0	1	2	0	0	63	0	2	4	0	0	42	0	0	5½	05
,8	0	2¼	6	0	1	8	0	0	72	0	2¼	6	0	0	48	0	0	6¼	20
,9	0	2½	8	0	1¼	4	0	0	81	0	2½	8	0	0	54	0	0	7	35
1	0	3	0	0	1½	0	0	0	9	0	3	0	0	0	6	0	0	7¾	5
2	0	6	0	0	3	0	0	0¼	8	0	6	0	0	0¼	2	0	1	3¾	0
3	0	9	0	0	4½	0	0	0½	7	0	9	0	0	0¼	8	0	1	11½	5
4	1	0	0	0	6	0	0	0¾	6	1	0	0	0	0½	4	0	2	7½	0
5	1	3	0	0	7½	0	0	1	5	1	3	0	0	0¾	0	0	3	3¼	5
6	1	6	0	0	9	0	0	1¼	4	1	6	0	0	0¾	6	0	3	11¼	0
7	1	9	0	0	10½	0	0	1½	3	1	9	0	0	1	2	0	4	7	5
8	2	0	0	1	0	0	0	1¾	2	2	0	0	0	1	8	0	5	3	0
9	2	3	0	1	1½	0	0	2	1	2	3	0	0	1¼	4	0	5	10¾	5
10	2	6	0	1	3	0	0	2¼	0	2	6	0	0	1½	0	0	6	6¾	0
11	2	9	0	1	4½	0	0	2¼	9	2	9	0	0	1½	6	0	7	2½	5
12	3	0	0	1	6	0	0	2½	8	3	0	0	0	1¾	2	0	7	10½	0
13	3	3	0	1	7½	0	0	2¾	7	3	3	0	0	1¾	8	0	8	6¼	5
14	3	6	0	1	9	0	0	3	6	3	6	0	0	2	4	0	9	2¼	0
15	3	9	0	1	10½	0	0	3¼	5	3	9	0	0	2¼	0	0	9	10	5
16	4	0	0	2	0	0	0	3½	4	4	0	0	0	2¼	6	0	10	6	0
17	4	3	0	2	1½	0	0	3¾	3	4	3	0	0	2½	2	0	11	1¾	5
18	4	6	0	2	3	0	0	4	2	4	6	0	0	2½	8	0	11	9¾	0
19	4	9	0	2	4½	0	0	4¼	1	4	9	0	0	2¾	4	0	12	5½	5
20	5	0	0	2	6	0	0	4½	0	5	0	0	0	3	0	0	13	1½	0
21	5	3	0	2	7½	0	0	4½	9	5	3	0	0	3	6	0	13	9¼	5
22	5	6	0	2	9	0	0	4¾	8	5	6	0	0	3¼	2	0	14	5¼	0
23	5	9	0	2	10½	0	0	5	7	5	9	0	0	3¼	8	0	15	1	5
24	6	0	0	3	0	0	0	5¼	6	6	0	0	0	3½	4	0	15	9	0
25	6	3	0	3	1½	0	0	5½	5	6	3	0	0	3¾	0	0	16	4¾	5
26	6	6	0	3	3	0	0	5¾	4	6	6	0	0	3¾	6	0	17	0¾	0
27	6	9	0	3	4½	0	0	6	3	6	9	0	0	4	2	0	17	8½	5
28	7	0	0	3	6	0	0	6¼	2	7	0	0	0	4	8	0	18	4½	0
29	7	3	0	3	7½	0	0	6½	1	7	3	0	0	4¼	4	0	19	0¼	5
30	7	6	0	3	9	0	0	6¾	0	7	6	0	0	4½	0	0	19	8¼	0
31	7	9	0	3	10½	0	0	6¾	9	7	9	0	0	4½	6	1	0	4	5
32	8	0	0	4	0	0	0	7	8	8	0	0	0	4¾	2	1	1	0	0
33	8	3	0	4	1½	0	0	7¼	7	8	3	0	0	4¾	8	1	1	7¾	5
34	8	6	0	4	3	0	0	7½	6	8	6	0	0	5	4	1	2	3¾	0
35	8	9	0	4	4½	0	0	7¾	5	8	9	0	0	5¼	0	1	2	11½	5
36	9	0	0	4	6	0	0	8	4	9	0	0	0	5¼	6	1	3	7½	0
37	9	3	0	4	7½	0	0	8¼	3	9	3	0	0	5½	2	1	4	3¼	5
38	9	6	0	4	9	0	0	8½	2	9	6	0	0	5½	8	1	4	11¼	0
39	9	9	0	4	10½	0	0	8¾	1	9	9	0	0	5¾	4	1	5	7	5
40	10	0	0	5	0	0	0	9	0	10	0	0	0	6	0	1	6	3	0
41	10	3	0	5	1½	0	0	9	9	10	3	0	0	6	6	1	6	10¾	5
42	10	6	0	5	3	0	0	9¼	8	10	6	0	0	6¼	2	1	7	6¾	0
43	10	9	0	5	4½	0	0	9½	7	10	9	0	0	6¼	8	1	8	2½	5
44	11	0	0	5	6	0	0	9¾	6	11	0	0	0	6½	4	1	8	10½	0
45	11	3	0	5	7½	0	0	10	5	11	3	0	0	6¾	0	1	9	6¼	5
46	11	6	0	5	9	0	0	10¼	4	11	6	0	0	6¾	6	1	10	2¼	0
47	11	9	0	5	10½	0	0	10½	3	11	9	0	0	7	2	1	10	10	5
48	12	0	0	6	0	0	0	10¾	2	12	0	0	0	7	8	1	11	6	0
49	12	3	0	6	1½	0	0	11	1	12	3	0	0	7¼	4	1	12	1¾	5
50	12	6	0	6	3	0	0	11¼	0	12	6	0	0	7½	0	1	12	9¾	0

TABLE to Money Floor Bushels continued.

Bush.	Old Duty.				Ad. Duty.			15l. p.Cent.			New Ad. Duty.				5 per Cent.			Total Duty			
	l.	s.	d.	p.	l.	s.	d.	s.	d.	p.	l.	s.	d.	p.	s.	d.	p.	l.	s.	d.	p.
51	0	12	9	0	0	6	4½	0	11¼	9	0	12	9	0	0	7½	6	1	13	5½	5
52	0	13	0	0	0	6	6	0	11½	8	0	13	0	0	0	7¾	2	1	14	1½	0
53	0	13	3	0	0	6	7½	0	11¾	7	0	13	3	0	0	7¾	8	1	14	9½	5
54	0	13	6	0	0	6	9	1	0	6	0	13	6	0	0	8	4	1	15	5¼	0
55	0	13	9	0	0	6	10½	1	0¼	5	0	13	9	0	0	8¼	0	1	16	1	5
56	0	14	0	0	0	7	0	1	0½	4	0	14	0	0	0	8¼	6	1	16	9	0
57	0	14	3	0	0	7	1½	1	0¾	3	0	14	3	0	0	8½	2	1	17	4¾	5
58	0	14	6	0	0	7	3	1	1	2	0	14	6	0	0	8½	8	1	18	0¼	0
59	0	14	9	0	0	7	4½	1	1¼	1	0	14	9	0	0	8¾	4	1	18	8½	5
60	0	15	0	0	0	7	6	1	1½	0	0	15	0	0	0	9	0	1	19	4½	0
61	0	15	3	0	0	7	7½	1	1½	9	0	15	3	0	0	9	6	2	0	0¼	5
62	0	15	6	0	0	7	9	1	1¾	8	0	15	6	0	0	9¼	2	2	0	8¼	0
63	0	15	9	0	0	7	10½	1	2	7	0	15	9	0	0	9¼	8	2	1	4	5
64	0	16	0	0	0	8	0	1	2¼	6	0	16	0	0	0	9½	4	2	2	0	0
65	0	16	3	0	0	8	1½	1	2½	5	0	16	3	0	0	9¾	0	2	2	7¾	5
66	0	16	6	0	0	8	3	1	2¾	4	0	16	6	0	0	9¾	6	2	3	3¾	0
67	0	16	9	0	0	8	4½	1	3	3	0	16	9	0	0	10	2	2	3	11½	5
68	0	17	0	0	0	8	6	1	3¼	2	0	17	0	0	0	10	8	2	4	7½	0
69	0	17	3	0	0	8	7½	1	3½	1	0	17	3	0	0	10¼	4	2	5	3¼	5
70	0	17	6	0	0	8	9	1	3¾	0	0	17	6	0	0	10½	0	2	5	11¼	0
71	0	17	9	0	0	8	10½	1	3¾	9	0	17	9	0	0	10½	6	2	6	7	5
72	0	18	0	0	0	9	0	1	4	8	0	18	0	0	0	10¾	2	2	7	3	0
73	0	18	3	0	0	9	1½	1	4¼	7	0	18	3	0	0	10¾	8	2	7	10¾	5
74	0	18	6	0	0	9	3	1	4½	6	0	18	6	0	0	11	4	2	8	6½	0
75	0	18	9	0	0	9	4½	1	4¾	5	0	18	9	0	0	11¼	0	2	9	2½	5
76	0	19	0	0	0	9	6	1	5	4	0	19	0	0	0	11¼	6	2	9	10½	0
77	0	19	3	0	0	9	7½	1	5¼	3	0	19	3	0	0	11½	2	2	10	6¼	5
78	0	19	6	0	0	9	9	1	5½	2	0	19	6	0	0	11½	8	2	11	2¼	0
79	0	19	9	0	0	9	10½	1	5¾	1	0	19	9	0	0	11¾	4	2	11	10	5
80	1	0	0	0	0	10	0	1	6	0	1	0	0	0	1	0	0	2	12	6	0
81	1	0	3	0	0	10	1½	1	6	9	1	0	3	0	1	0	6	2	13	1¾	5
82	1	0	6	0	0	10	3	1	6¼	8	1	0	6	0	1	0¼	2	2	13	9¾	0
83	1	0	9	0	0	10	4½	1	6½	7	1	0	9	0	1	0¼	8	2	14	5½	5
84	1	1	0	0	0	10	6	1	6¾	6	1	1	0	0	1	0½	4	2	15	1½	0
85	1	1	3	0	0	10	7½	1	7	5	1	1	3	0	1	0¾	0	2	15	9¼	5
86	1	1	6	0	0	10	9	1	7¼	4	1	1	6	0	1	0¾	6	2	16	5¼	0
87	1	1	9	0	0	10	10½	1	7½	3	1	1	9	0	1	1	2	2	17	1	5
88	1	2	0	0	0	11	0	1	7¾	2	1	2	0	0	1	1	8	2	17	9	0
89	1	2	3	0	0	11	1½	1	8	1	1	2	3	0	1	1¼	4	2	18	4¾	5
90	1	2	6	0	0	11	3	1	8¼	0	1	2	6	0	1	1½	0	2	19	0¼	0
91	1	2	9	0	0	11	4½	1	8¼	9	1	2	9	0	1	1½	6	2	19	8½	5
92	1	3	0	0	0	11	6	1	8½	8	1	3	0	0	1	1¾	2	3	0	4½	0
93	1	3	3	0	0	11	7½	1	8¾	7	1	3	3	0	1	1¾	8	3	1	0¼	5
94	1	3	6	0	0	11	9	1	9	6	1	3	6	0	1	2	4	3	1	8¼	0
95	1	3	9	0	0	11	10½	1	9¼	5	1	3	9	0	1	2¼	0	3	2	4	5
96	1	4	0	0	0	12	0	1	9½	4	1	4	0	0	1	2¼	6	3	3	0	0
97	1	4	3	0	0	12	1½	1	9¾	3	1	4	3	0	1	2½	2	3	3	7¾	5
98	1	4	6	0	0	12	3	1	10	2	1	4	6	0	1	2½	8	3	4	3¾	0
99	1	4	9	0	0	12	4½	1	10¼	1	1	4	9	0	1	2¾	4	3	4	11½	5
100	1	5	0	0	0	12	6	1	10½	0	1	5	0	0	1	3	0	3	5	7½	0
200	2	10	0	0	1	5	0	3	9	0	2	10	0	0	2	6	0	6	11	3	0
300	3	15	0	0	1	17	6	5	7½	0	3	15	0	0	3	9	0	9	16	10½	0
400	5	0	0	0	2	10	0	7	6	0	5	0	0	0	5	0	0	13	2	6	0
500	6	5	0	0	3	2	6	9	4½	0	6	5	0	0	6	3	0	16	8	1½	0
600	7	10	0	0	3	15	0	11	3	0	7	10	0	0	7	6	0	19	13	9	0
700	8	15	0	0	4	7	6	13	1½	0	8	15	0	0	8	9	0	22	19	4½	0
800	10	0	0	0	5	0	0	15	0	0	10	0	0	0	10	0	0	26	5	0	0
900	11	5	0	0	5	12	6	16	10½	0	11	5	0	0	11	3	0	29	10	7½	0
1000	12	10	0	0	6	5	0	18	9	0	12	10	0	0	12	6	0	32	16	3	0

TABLE 10. A TABLE of 5l. and 15l. per Cent.

1 Farthing.

Value.	Duty at 5l. p. Ct.		Duty at 15 per Cent.		
d.	d.	f. p.	s.	p.	f. pa.
¼	0	0,05	0	0	0,15
½	0	0,1	0	0	0,3
¾	0	0,15	0	0	0,45
1	0	0,2	0	0	0,6
1¼	0	0,25	0	0	0,75
1½	0	0,3	0	0	0,9
1¾	0	0,35	0	0	1,05
2	0	0,4	0	0	1,2
2¼	0	0,45	0	0	1,35
2½	0	0,5	0	0	1,5
2¾	0	0,55	0	0	1,65
3	0	0,6	0	0	1,8
3¼	0	0,65	0	0	1,95
3½	0	0,7	0	0	2,1
3¾	0	0,75	0	0	2,25
4	0	0,8	0	0	2,4
4¼	0	0,85	0	0	2,55
4½	0	0,9	0	0	2,7
4¾	0	0,95	0	0	2,85
5	0	1,0	0	0	3,0
5¼	0	1,05	0	0	3,15
5½	0	1,1	0	0	3,3
5¾	0	1,15	0	0	3,45
6	0	1,2	0	0	3,6
6¼	0	1,25	0	0	3,75
6½	0	1,3	0	0	3,9
6¾	0	1,35	0	1	0,05
7	0	1,4	0	1	0,2
7¼	0	1,45	0	1	0,35
7½	0	1,5	0	1	0,5
7¾	0	1,55	0	1	0,65
8	0	1,6	0	1	0,8
8¼	0	1,65	0	1	0,95
8½	0	1,7	0	1	1,1
8¾	0	1,75	0	1	1,25
9	0	1,8	0	1	1,4
9¼	0	1,85	0	1	1,55
9½	0	1,9	0	1	1,7
9¾	0	1,95	0	1	1,85
10	0	2,0	0	1	2,0
10¼	0	2,05	0	1	2,15
10½	0	2,1	0	1	3,3
10¾	0	2,15	0	1	2,45
11	0	2,2	0	1	2,6
11¼	0	2,25	0	1	2,75
11½	0	2,3	0	1	2,9
11¾	0	2,35	0	1	3,05

1 Shilling.

Value.		Duty at 5 per Cent.			Duty at 15 per Cent.		
s.	d.	s.	d.	f. ps.	s.	d.	f. ps.
1	0	0	0	2,4	0	1	3,2
1	0¼	0	0	2,45	0	1	3,35
1	0½	0	0	2,5	0	1	3,5
1	0¾	0	0	2,55	0	1	3,65
1	1	0	0	2,6	0	1	3,8
1	1¼	0	0	2,65	0	1	3,95
1	1½	0	0	2,7	0	2	0,1
1	1¾	0	0	2,75	0	2	0,25
1	2	0	0	2,8	0	2	0,4
1	2¼	0	0	2,85	0	2	0,55
1	2½	0	0	2,9	0	2	0,7
1	2¾	0	0	2,95	0	2	0,85
1	3	0	0	3,0	0	2	1,0
1	3¼	0	0	3,05	0	2	1,15
1	3½	0	0	3,1	0	2	1,3
1	3¾	0	0	3,15	0	2	1,45
1	4	0	0	3,2	0	2	1,6
1	4¼	0	0	3,25	0	2	1,75
1	4½	0	0	3,3	0	2	1,9
1	4¾	0	0	3,35	0	2	2,05
1	5	0	0	3,4	0	2	2,2
1	5¼	0	0	3,45	0	2	2,35
1	5½	0	0	3,5	0	2	2,5
1	5¾	0	0	3,55	0	2	2,65
1	6	0	0	3,6	0	2	2,8
1	6¼	0	0	3,65	0	2	2,95
1	6½	0	0	3,7	0	2	3,1
1	6¾	0	0	3,75	0	2	3,25
1	7	0	0	3,8	0	2	3,4
1	7¼	0	0	3,85	0	2	3,55
1	7½	0	0	3,9	0	2	3,7
1	7¾	0	0	3,95	0	2	3,85
1	8	0	1	0,0	0	3	0,0
1	8¼	0	1	0,05	0	3	0,15
1	8½	0	1	0,1	0	3	0,3
1	8¾	0	1	0,15	0	3	0,45
1	9	0	1	0,2	0	3	0,6
1	9¼	0	1	0,25	0	3	0,75
1	9½	0	1	0,3	0	3	0,9
1	9¾	0	1	0,35	0	3	1,05
1	10	0	1	0,4	0	3	1,2
1	10¼	0	1	0,45	0	3	1,35
1	10½	0	1	0,5	0	3	1,5
1	10¾	0	1	0,55	0	3	1,65
1	11	0	1	0,6	0	3	1,8
1	11¼	0	1	0,65	0	3	1,95
1	11½	0	1	0,7	0	3	2,1
1	11¾	0	1	0,75	0	3	2,25

TABLE of 5l. and 15l. per Cent. continued.

2 Shillings.

Value. s.	d. f.	Duty at 5 per Cent. s.	d.	f. p.	Duty at 15 per Cent. s.	d.	f. p.
2	0	0	1	0,8	0	3	2,4
2	0¼	0	1	0,85	0	3	2,55
2	0½	0	1	0,9	0	3	2,7
2	0¾	0	1	0,95	0	3	2,85
2	1	0	1	1,0	0	3	3,0
2	1¼	0	1	1,05	0	3	3,15
2	1½	0	1	1,1	0	3	3,3
2	1¾	0	1	1,15	0	3	3,45
2	2	0	1	1,2	0	3	3,6
2	2¼	0	1	1,25	0	3	3,75
2	2½	0	1	1,3	0	3	3,9
2	2¾	0	1	1,35	0	4	0,05
2	3	0	1	1,4	0	4	0,2
2	3¼	0	1	1,45	0	4	0,35
2	3½	0	1	1,5	0	4	0,5
2	3¾	0	1	1,55	0	4	0,65
2	4	0	1	1,6	0	4	0,8
2	4¼	0	1	1,65	0	4	0,95
2	4½	0	1	1,7	0	4	1,1
2	4¾	0	1	1,75	0	4	1,25
2	5	0	1	1,8	0	4	1,4
2	5¼	0	1	1,85	0	4	1,55
2	5½	0	1	1,9	0	4	1,7
2	5¾	0	1	1,95	0	4	1,85
2	6	0	1	2,0	0	4	2,0
2	6¼	0	1	2,05	0	4	2,15
2	6½	0	1	2,1	0	4	2,3
2	6¾	0	1	2,15	0	4	2,45
2	7	0	1	2,2	0	4	2,6
2	7¼	0	1	2,25	0	4	2,75
2	7½	0	1	2,3	0	4	2,9
2	7¾	0	1	2,35	0	4	3,05
2	8	0	1	2,4	0	4	3,2
2	8¼	0	1	2,45	0	4	3,35
2	8½	0	1	2,5	0	4	3,5
2	8¾	0	1	2,55	0	4	3,65
2	9	0	1	2,6	0	4	3,8
2	9¼	0	1	2,65	0	4	3,95
2	9½	0	1	2,7	0	5	0,1
2	9¾	0	1	2,75	0	5	0,25
2	10	0	1	2,8	0	5	0,4
2	10¼	0	1	2,85	0	5	0,55
2	10½	0	1	2,9	0	5	0,7
2	10¾	0	1	2,95	0	5	0,85
2	11	0	1	3,0	0	5	1,0
2	11¼	0	1	3,05	0	5	1,15
2	11½	0	1	3,1	0	5	1,3
2	11¾	0	1	3,15	0	5	1,45

3 Shillings.

Value. s.	d. f.	Duty at 5 per Cent. s.	d.	f. p.	Duty at 15 per Cent. s.	d.	f. p.
3	0	0	1	3,2	0	5	1,6
3	0¼	0	1	3,25	0	5	1,75
3	0½	0	1	3,3	0	5	1,9
3	0¾	0	1	3,35	0	5	2,05
3	1	0	1	3,4	0	5	2,2
3	1¼	0	1	3,45	0	5	2,35
3	1½	0	1	3,5	0	5	2,5
3	1¾	0	1	3,55	0	5	2,65
3	2	0	1	3,6	0	5	2,8
3	2¼	0	1	3,65	0	5	2,95
3	2½	0	1	3,7	0	5	3,1
3	2¾	0	1	3,75	0	5	3,25
3	3	0	1	3,8	0	5	3,4
3	3¼	0	1	3,85	0	5	3,55
3	3½	0	1	3,9	0	5	3,7
3	3¾	0	1	3,95	0	5	3,85
3	4	0	2	0,0	0	6	0,0
3	4¼	0	2	0,05	0	6	0,15
3	4½	0	2	0,1	0	6	0,3
3	4¾	0	2	0,15	0	6	0,45
3	5	0	2	0,2	0	6	0,6
3	5¼	0	2	0,25	0	6	0,75
3	5½	0	2	0,3	0	6	0,9
3	5¾	0	2	0,35	0	6	1,05
3	6	0	2	0,4	0	6	1,2
3	6¼	0	2	0,45	0	6	1,35
3	6½	0	2	0,5	0	6	1,5
3	6¾	0	2	0,55	0	6	1,65
3	7	0	2	0,6	0	6	1,8
3	7¼	0	2	0,65	0	6	1,95
3	7½	0	2	0,7	0	6	2,1
3	7¾	0	2	0,75	0	6	2,25
3	8	0	2	0,8	0	6	2,4
3	8¼	0	2	0,85	0	6	2,55
3	8½	0	2	0,9	0	6	2,7
3	8¾	0	2	0,95	0	6	2,85
3	9	0	2	1,0	0	6	3,0
3	9¼	0	2	1,05	0	6	3,15
3	9½	0	2	1,1	0	6	3,3
3	9¾	0	2	1,15	0	6	3,45
3	10	0	2	1,2	0	6	3,6
3	10¼	0	2	1,25	0	6	3,75
3	10½	0	2	1,3	0	6	3,9
3	10¾	0	2	1,35	0	7	0,05
3	11	0	2	1,4	0	7	0,2
3	11¼	0	2	1,45	0	7	0,35
3	11½	0	2	1,5	0	7	0,5
3	11¾	0	2	1,55	0	7	0,65

TABLE of 5l. and 15l. per Cent. continued.

4 Shillings.

Value. s.	d.	Duty at 5 per Cent. d.	f. p.	Duty at 15 per Cent. s.	d.	f. p.
4	0	2	1,6	0	7	0,8
4	0¼	2	1,65	0	7	0,95
4	0½	2	1,7	0	7	1,1
4	0¾	2	1,75	0	7	1,25
4	1	2	1,8	0	7	1,4
4	1¼	2	1,85	0	7	1,55
4	1½	2	1,9	0	7	1,7
4	1¾	2	1,95	0	7	1,85
4	2	2	2,0	0	7	2,0
4	2¼	2	2,05	0	7	2,15
4	2½	2	2,1	0	7	2,3
4	2¾	2	2,15	0	7	2,45
4	3	2	2,2	0	7	2,6
4	3¼	2	2,25	0	7	2,75
4	3½	2	2,3	0	7	2,9
4	3¾	2	2,35	0	7	3,05
4	4	2	2,4	0	7	3,2
4	4¼	2	2,45	0	7	3,35
4	4½	2	2,5	0	7	3,5
4	4¾	2	2,55	0	7	3,65
4	5	2	2,6	0	7	3,8
4	5¼	2	2,65	0	7	3,95
4	5½	2	2,7	0	8	0,1
4	5¾	2	2,75	0	8	0,25
4	6	2	2,8	0	8	0,4
4	6¼	2	2,85	0	8	0,55
4	6½	2	2,9	0	8	0,7
4	6¾	2	2,95	0	8	0,85
4	7	2	3,0	0	8	1,0
4	7¼	2	3,05	0	8	1,15
4	7½	2	3,1	0	8	1,3
4	7¾	2	3,15	0	8	1,45
4	8	2	3,2	0	8	1,6
4	8¼	2	3,25	0	8	1,75
4	8½	2	3,3	0	8	1,9
4	8¾	2	3,35	0	8	2,05
4	9	2	3,4	0	8	2,2
4	9¼	2	3,45	0	8	2,35
4	9½	2	3,5	0	8	2,5
4	9¾	2	3,55	0	8	2,65
4	10	2	3,6	0	8	2,8
4	10¼	2	3,65	0	8	2,95
4	10½	2	3,7	0	8	3,1
4	10¾	2	3,75	0	8	3,25
4	11	2	3,8	0	8	3,4
4	11¼	2	3,85	0	8	3,55
4	11½	2	3,9	0	8	3,7
4	11¾	2	3,95	0	8	3,85

5 Shillings.

Value. s.	d.	Duty at 5 per Cent. d.	f. p.	Duty at 15 per Cent. s.	d.	f. p.
5	0	3	0,0	0	9	0,0
5	0¼	3	0,05	0	9	0,15
5	0½	3	0,1	0	9	0,3
5	0¾	3	0,15	0	9	0,45
5	1	3	0,2	0	9	0,6
5	1¼	3	0,25	0	9	0,75
5	1½	3	0,3	0	9	0,9
5	1¾	3	0,35	0	9	1,05
5	2	3	0,4	0	9	1,2
5	2¼	3	0,45	0	9	1,35
5	2½	3	0,5	0	9	1,5
5	2¾	3	0,55	0	9	1,65
5	3	3	0,6	0	9	1,8
5	3¼	3	0,65	0	9	1,95
5	3½	3	0,7	0	9	2,1
5	3¾	3	0,75	0	9	2,25
5	4	3	0,8	0	9	2,4
5	4¼	3	0,85	0	9	2,55
5	4½	3	0,9	0	9	2,7
5	4¾	3	0,95	0	9	2,85
5	5	3	1,0	0	9	3,0
5	5¼	3	1,05	0	9	3,15
5	5½	3	1,1	0	9	3,3
5	5¾	3	1,15	0	9	3,45
5	6	3	1,2	0	9	3,6
5	6¼	3	1,25	0	9	3,75
5	6½	3	1,3	0	9	3,9
5	6¾	3	1,35	0	10	0,05
5	7	3	1,4	0	10	0,2
5	7¼	3	1,45	0	10	0,35
5	7½	3	1,5	0	10	0,5
5	7¾	3	1,55	0	10	0,65
5	8	3	1,6	0	10	0,8
5	8¼	3	1,65	0	10	0,95
5	8½	3	1,7	0	10	1,1
5	8¾	3	1,75	0	10	1,25
5	9	3	1,8	0	10	1,4
5	9¼	3	1,85	0	10	1,55
5	9½	3	1,9	0	10	1,7
5	9¾	3	1,95	0	10	1,85
5	10	3	2,0	0	10	2,0
5	10¼	3	2,05	0	10	2,15
5	10½	3	2,1	0	10	2,3
5	10¾	3	2,15	0	10	2,45
5	11	3	2,2	0	10	2,6
5	11¼	3	2,25	0	10	2,75
5	11½	3	2,3	0	10	2,9
5	11¾	3	2,35	0	10	3,05

TABLE of 5l. and 15l. per Cent. continued.

6 Shillings.

Value.		Duty at 5 per Cent.		Duty at 15 per Cent.		
s.	d.	d.	f. p.	s.	d.	f. p.
6	0	3	2,4	0	10	3,2
6	0¼	3	2,45	0	10	3,35
6	0½	3	2,5	0	10	3,5
6	0¾	3	2,55	0	10	3,65
6	1	3	2,6	0	10	3,8
6	1¼	3	2,65	0	10	3,95
6	1½	3	2,7	0	11	0,1
6	1¾	3	2,75	0	11	0,25
6	2	3	2,8	0	11	0,4
6	2¼	3	2,85	0	11	0,55
6	2½	3	2,9	0	11	0,7
6	2¾	3	2,95	0	11	0,85
6	3	3	3,0	0	11	1,0
6	3¼	3	3,05	0	11	1,15
6	3½	3	3,1	0	11	1,3
6	3¾	3	3,15	0	11	1,45
6	4	3	3,2	0	11	1,6
6	4¼	3	3,25	0	11	1,75
6	4½	3	3,3	0	11	1,9
6	4¾	3	3,35	0	11	2,05
6	5	3	3,4	0	11	2,2
6	5¼	3	3,45	0	11	2,35
6	5½	3	3,5	0	11	2,5
6	5¾	3	3,55	0	11	2,65
6	6	3	3,6	0	11	2,8
6	6¼	3	3,65	0	11	2,95
6	6½	3	3,7	0	11	3,1
6	6¾	3	3,75	0	11	3,25
6	7	3	3,8	0	11	3,4
6	7¼	3	3,85	0	11	3,55
6	7½	3	3,9	0	11	3,7
6	7¾	3	3,95	0	11	3,85
6	8	4	0,0	1	0	0,0
6	8¼	4	0,05	1	0	0,15
6	8½	4	0,1	1	0	0,3
6	8¾	4	0,15	1	0	0,45
6	9	4	0,2	1	0	0,6
6	9¼	4	0,25	1	0	0,75
6	9½	4	0,3	1	0	0,9
6	9¾	4	0,35	1	0	1,05
6	10	4	0,4	1	0	1,2
6	10¼	4	0,45	1	0	1,35
6	10½	4	0,5	1	0	1,5
6	10¾	4	0,55	1	0	1,65
6	11	4	0,6	1	0	1,8
6	11¼	4	0,65	1	0	1,95
6	11½	4	0,7	1	0	2,1
6	11¾	4	0,75	1	0	2,25

7 Shillings.

Value.		Duty at 5 per Cent.		Duty at 15 per Cent.		
s.	d.	d.	f. p.	s.	d.	f. p.
7	0	4	0,8	1	0	2,4
7	0¼	4	0,85	1	0	2,55
7	0½	4	0,9	1	0	2,7
7	0¾	4	0,95	1	0	2,85
7	1	4	1,0	1	0	3,0
7	1¼	4	1,05	1	0	3,15
7	1½	4	1,1	1	0	3,3
7	1¾	4	1,15	1	0	3,45
7	2	4	1,2	1	0	3,6
7	2¼	4	1,25	1	0	3,75
7	2½	4	1,3	1	0	3,9
7	2¾	4	1,35	1	1	0,05
7	3	4	1,4	1	1	0,2
7	3¼	4	1,45	1	1	0,35
7	3½	4	1,5	1	1	0,5
7	3¾	4	1,55	1	1	0,65
7	4	4	1,6	1	1	0,8
7	4¼	4	1,65	1	1	0,95
7	4½	4	1,7	1	1	1,1
7	4¾	4	1,75	1	1	1,25
7	5	4	1,8	1	1	1,4
7	5¼	4	1,85	1	1	1,55
7	5½	4	1,9	1	1	1,7
7	5¾	4	1,95	1	1	1,85
7	6	4	2,0	1	1	2,0
7	6¼	4	2,05	1	1	2,15
7	6½	4	2,1	1	1	2,3
7	6¾	4	2,15	1	1	2,45
7	7	4	2,2	1	1	2,6
7	7¼	4	2,25	1	1	2,75
7	7½	4	2,3	1	1	2,9
7	7¾	4	2,35	1	1	3,05
7	8	4	2,4	1	1	3,2
7	8¼	4	2,45	1	1	3,35
7	8½	4	2,5	1	1	3,5
7	8¾	4	2,55	1	1	3,65
7	9	4	2,6	1	1	3,8
7	9¼	4	2,65	1	1	3,95
7	9½	4	2,7	1	2	0,1
7	9¾	4	2,75	1	2	0,25
7	10	4	2,8	1	2	0,44
7	10¼	4	2,85	1	2	0,59
7	10½	4	2,9	1	2	0,74
7	10¾	4	2,95	1	2	0,89
7	11	4	3,0	1	2	1,04
7	11¼	4	3,05	1	2	1,19
7	11½	4	3,1	1	2	1,34
7	11¾	4	3,15	1	1	1,45

TABLE of 5l. and 15l. per Cent. continued.

8 Shillings.

Value.		Duty at 5 per Cent.		Duty at 15 per Cent.		
s.	d.	d.	f. p.	s.	d.	f. p.
8	0	4	3,2	1	2	1,6
8	0¼	4	3,25	1	2	1,75
8	0½	4	3,3	1	2	1,9
8	0¾	4	3,35	1	2	2,05
8	1	4	3,4	1	2	2,2
8	1¼	4	3,45	1	2	2,35
8	1½	4	3,5	1	2	2,5
8	1¾	4	3,55	1	2	2,65
8	2	4	3,6	1	2	2,8
8	2¼	4	3,65	1	2	2,95
8	2½	4	3,7	1	2	3,1
8	2¾	4	3,75	1	2	3,25
8	3	4	3,8	1	2	3,4
8	3¼	4	3,85	1	2	3,55
8	3½	4	3,9	1	2	3,7
8	3¾	4	3,95	1	2	3,85
8	4	5	0,0	1	3	0,0
8	4¼	5	0,05	1	3	0,15
8	4½	5	0,1	1	3	0,3
8	4¾	5	0,15	1	3	0,45
8	5	5	0,2	1	3	0,6
8	5¼	5	0,25	1	3	0,75
8	5½	5	0,3	1	3	0,9
8	5¾	5	0,35	1	3	1,05
8	6	5	0,4	1	3	1,2
8	6¼	5	0,45	1	3	1,35
8	6½	5	0,5	1	3	1,5
8	6¾	5	0,55	1	3	1,65
8	7	5	0,6	1	3	1,8
8	7¼	5	0,65	1	3	1,95
8	7½	5	0,7	1	3	2,1
8	7¾	5	0,75	1	3	2,25
8	8	5	0,8	1	3	2,4
8	8¼	5	0,85	1	3	2,55
8	8½	5	0,9	1	3	2,7
8	8¾	5	0,95	1	3	2,85
8	9	5	1,0	1	3	3,0
8	9¼	5	1,05	1	3	3,15
8	9½	5	1,1	1	3	3,3
8	9¾	5	1,15	1	3	3,45
8	10	5	1,2	1	3	3,6
8	10¼	5	1,25	1	3	3,75
8	10½	5	1,3	1	3	3,9
8	10¾	5	1,35	1	4	0,05
8	11	5	1,4	1	4	0,2
8	11¼	5	1,45	1	4	0,35
8	11½	5	1,5	1	4	0,5
8	11¾	5	1,55	1	4	0,65

9 Shillings.

Value.		Duty at 5 per Cent.		Duty at 15 per Cent.		
s.	d.	d.	f. p.	s.	d.	f. p.
9	0	5	1,6	1	4	0,8
9	0¼	5	1,65	1	4	0,95
9	0½	5	1,7	1	4	1,1
9	0¾	5	1,75	1	4	1,25
9	1	5	1,8	1	4	1,4
9	1¼	5	1,85	1	4	1,55
9	1½	5	1,9	1	4	1,7
9	1¾	5	1,95	1	4	1,85
9	2	5	2,0	1	4	2,0
9	2¼	5	2,05	1	4	2,15
9	2½	5	2,1	1	4	2,3
9	2¾	5	2,15	1	4	2,45
9	3	5	2,2	1	4	2,6
9	3¼	5	2,25	1	4	2,75
9	3½	5	2,3	1	4	2,9
9	3¾	5	2,35	1	4	3,05
9	4	5	2,4	1	4	3,2
9	4¼	5	2,45	1	4	3,35
9	4½	5	2,5	1	4	3,5
9	4¾	5	2,55	1	4	3,65
9	5	5	2,6	1	4	3,8
9	5¼	5	2,65	1	4	3,95
9	5½	5	2,7	1	5	0,1
9	5¾	5	2,75	1	5	0,25
9	6	5	2,8	1	5	0,4
9	6¼	5	2,85	1	5	0,55
9	6½	5	2,9	1	5	0,7
9	6¾	5	2,95	1	5	0,85
9	7	5	3,0	1	5	1,0
9	7¼	5	3,05	1	5	1,15
9	7½	5	3,1	1	5	1,3
9	7¾	5	3,15	1	5	1,45
9	8	5	3,2	1	5	1,6
9	8¼	5	3,25	1	5	1,75
9	8½	5	3,3	1	5	1,9
9	8¾	5	3,35	1	5	2,05
9	9	5	3,4	1	5	2,2
9	9¼	5	3,45	1	5	2,35
9	9½	5	3,5	1	5	2,5
9	9¾	5	3,55	1	5	2,65
9	10	5	3,6	1	5	2,8
9	10¼	5	3,65	1	5	2,95
9	10½	5	3,7	1	5	3,1
9	10¾	5	3,75	1	5	3,25
9	11	5	3,8	1	5	3,4
9	11¼	5	3,85	1	5	3,55
9	11½	5	3,9	1	5	3,7
9	11¾	5	3,95	1	5	3,85

A TABLE of 5l. and 15l. per Cent. continued.

10 Shillings.

Value. s.	d.	Duty at 5 per Cent. s.	d.	f. p.	Duty at 15 per Cent. s.	d.	f. p.
10	0	0	6	0,0	1	6	0,0
10	0¼	0	6	0,05	1	6	0,15
10	0½	0	6	0,1	1	6	0,3
10	0¾	0	6	0,15	1	6	0,45
10	1	0	6	0,2	1	6	0,6
10	1¼	0	6	0,25	1	6	0,75
10	1½	0	6	0,3	1	6	0,9
10	1¾	0	6	0,35	1	6	1,05
10	2	0	6	0,4	1	6	1,2
10	2¼	0	6	0,45	1	6	1,35
10	2½	0	6	0,5	1	6	1,5
10	2¾	0	6	0,55	1	6	1,65
10	3	0	6	0,6	1	6	1,8
10	3¼	0	6	0,65	1	6	1,95
10	3½	0	6	0,7	1	6	2,1
10	3¾	0	6	0,75	1	6	2,25
10	4	0	6	0,8	1	6	2,4
10	4¼	0	6	0,85	1	6	2,55
10	4½	0	6	0,9	1	6	2,7
10	4¾	0	6	0,95	1	6	2,85
10	5	0	6	1,0	1	6	3,0
10	5¼	0	6	1,05	1	6	3,15
10	5½	0	6	1,1	1	6	3,3
10	5¾	0	6	1,15	1	6	3,45
10	6	0	6	1,2	1	6	3,6
10	6¼	0	6	1,25	1	6	3,75
10	6½	0	6	1,3	1	6	3,9
10	6¾	0	6	1,35	1	7	0,05
10	7	0	6	1,4	1	7	0,2
10	7¼	0	6	1,45	1	7	0,35
10	7½	0	6	1,5	1	7	0,5
10	7¾	0	6	1,55	1	7	0,65
10	8	0	6	1,6	1	7	0,8
10	8¼	0	6	1,65	1	7	0,95
10	8½	0	6	1,7	1	7	1,1
10	8¾	0	6	1,75	1	7	1,25
10	9	0	6	1,8	1	7	1,4
10	9¼	0	6	1,85	1	7	1,55
10	9½	0	6	1,9	1	7	1,7
10	9¾	0	6	1,95	1	7	1,85
10	10	0	6	2,0	1	7	2,0
10	10¼	0	6	2,05	1	7	2,15
10	10½	0	6	2,1	1	7	2,3
10	10¾	0	6	2,15	1	7	2,45
10	11	0	6	2,2	1	7	2,6
10	11¼	0	6	2,25	1	7	2,75
10	11½	0	6	2,3	1	7	2,9
10	11¾	0	6	2,35	1	7	3,05

11 Shillings.

Value. s.	d.	Duty at 5 per Cent. s.	d.	f. p.	Duty at 15 per Cent. s.	d.	f. p.
11	0	0	6	2,4	1	7	3,2
11	0¼	0	6	2,45	1	7	3,35
11	0½	0	6	2,5	1	7	3,5
11	0¾	0	6	2,55	1	7	3,65
11	1	0	6	2,6	1	7	3,8
11	1¼	0	6	2,65	1	7	3,95
11	1½	0	6	2,7	1	8	0,1
11	1¾	0	6	2,75	1	8	0,25
11	2	0	6	2,8	1	8	0,4
11	2¼	0	6	2,85	1	8	0,55
11	2½	0	6	2,9	1	8	0,7
11	2¾	0	6	2,95	1	8	0,85
11	3	0	6	3,0	1	8	1,0
11	3¼	0	6	3,05	1	8	1,15
11	3½	0	6	3,1	1	8	1,3
11	3¾	0	6	3,15	1	8	1,45
11	4	0	6	3,2	1	8	1,6
11	4¼	0	6	3,25	1	8	1,75
11	4½	0	6	3,3	1	8	1,9
11	4¾	0	6	3,35	1	8	2,05
11	5	0	6	3,4	1	8	2,2
11	5¼	0	6	3,45	1	8	2,35
11	5½	0	6	3,5	1	8	2,5
11	5¾	0	6	3,55	1	8	2,65
11	6	0	6	3,6	1	8	2,8
11	6¼	0	6	3,65	1	8	2,95
11	6½	0	6	3,7	1	8	3,1
11	6¾	0	6	3,75	1	8	3,25
11	7	0	6	3,8	1	8	3,4
11	7¼	0	6	3,85	1	8	3,55
11	7½	0	6	3,9	1	8	3,7
11	7¾	0	6	3,95	1	8	3,85
11	8	0	7	0,0	1	9	0,0
11	8¼	0	7	0,05	1	9	0,15
11	8½	0	7	0,1	1	9	0,3
11	8¾	0	7	0,15	1	9	0,45
11	9	0	7	0,2	1	9	0,6
11	9¼	0	7	0,25	1	9	0,75
11	9½	0	7	0,3	1	9	0,9
11	9¾	0	7	0,35	1	9	1,05
11	10	0	7	0,4	1	9	1,2
11	10¼	0	7	0,45	1	9	1,35
11	10½	0	7	0,5	1	9	1,5
11	10¾	0	7	0,55	1	9	1,65
11	11	0	7	0,6	1	9	1,8
11	11¼	0	7	0,65	1	9	1,95
11	11½	0	7	0,7	1	9	2,1
11	11¾	0	7	0,75	1	9	2,25

TABLE of 5l. and 15l. per Cent. continued.

12 Shillings.

Value s.	Value d.	Duty at 5 per Cent. d.	f. p.	Duty at 15 per Cent. s.	d.	f. p.
12	0	7	0,8	1	9	2,4
12	0¼	7	0,85	1	9	2,55
12	0½	7	0,9	1	9	2,7
12	0¾	7	0,95	1	9	2,85
12	1	7	1,0	1	9	3,0
12	1¼	7	1,05	1	9	3,15
12	1½	7	1,1	1	9	3,3
12	1¾	7	1,15	1	9	3,45
12	2	7	1,2	1	9	3,6
12	2¼	7	1,25	1	9	3,75
12	2½	7	1,3	1	9	3,9
12	2¾	7	1,35	1	10	0,05
12	3	7	1,4	1	10	0,2
12	3¼	7	1,45	1	10	0,35
12	3½	7	1,5	1	10	0,5
12	3¾	7	1,55	1	10	0,65
12	4	7	1,6	1	10	0,8
12	4¼	7	1,65	1	10	0,95
12	4½	7	1,7	1	10	1,1
12	4¾	7	1,75	1	10	1,25
12	5	7	1,8	1	10	1,4
12	5¼	7	1,85	1	10	1,55
12	5½	7	1,9	1	10	1,7
12	5¾	7	1,95	1	10	1,85
12	6	7	2,0	1	10	2,0
12	6¼	7	2,05	1	10	2,15
12	6½	7	2,1	1	10	2,3
12	6¾	7	2,15	1	10	2,45
12	7	7	2,2	1	10	2,6
12	7¼	7	2,25	1	10	2,75
12	7½	7	2,3	1	10	2,9
12	7¾	7	2,35	1	10	3,05
12	8	7	2,4	1	10	3,2
12	8¼	7	2,45	1	10	3,35
12	8½	7	2,5	1	10	3,5
12	8¾	7	2,55	1	10	3,65
12	9	7	2,6	1	10	3,8
12	9¼	7	2,65	1	10	3,95
12	9½	7	2,7	1	11	0,1
12	9¾	7	2,75	1	11	0,25
12	10	7	2,8	1	11	0,4
12	10¼	7	2,85	1	11	0,55
12	10½	7	2,9	1	11	0,7
12	10¾	7	2,95	1	11	0,85
12	11	7	3,0	1	11	1,0
12	11¼	7	3,05	1	11	1,15
12	11½	7	3,1	1	11	1,3
12	11¾	7	3,15	1	11	1,45

13 Shillings.

Value s.	Value d.	Duty at 5 per Cent. d.	f. p.	Duty at 15 per Cent. s.	d.	f. p.
13	0	7	3,2	1	11	1,6
13	0¼	7	3,25	1	11	1,75
13	0½	7	3,3	1	11	1,9
13	0¾	7	3,35	1	11	2,05
13	1	7	3,4	1	11	2,2
13	1¼	7	3,45	1	11	2,35
13	1½	7	3,5	1	11	2,5
13	1¾	7	3,55	1	11	2,65
13	2	7	3,6	1	11	2,8
13	2¼	7	3,65	1	11	2,95
13	2½	7	3,7	1	11	3,1
13	2¾	7	3,75	1	11	3,25
13	3	7	3,8	1	11	3,4
13	3¼	7	3,85	1	11	3,55
13	3½	7	3,9	1	11	3,7
13	3¾	7	3,95	1	11	3,85
13	4	8	0,0	2	0	0,0
13	4¼	8	0,05	2	0	0,15
13	4½	8	0,1	2	0	0,3
13	4¾	8	0,15	2	0	0,45
13	5	8	0,2	2	0	0,6
13	5¼	8	0,25	2	0	0,75
13	5½	8	0,3	2	0	0,9
13	5¾	8	0,35	2	0	1,05
13	6	8	0,4	2	0	1,2
13	6¼	8	0,45	2	0	1,35
13	6½	8	0,5	2	0	1,5
13	6¾	8	0,55	2	0	1,65
13	7	8	0,6	2	0	1,8
13	7¼	8	0,65	2	0	1,95
13	7½	8	0,7	2	0	2,1
13	7¾	8	0,75	2	0	2,25
13	8	8	0,8	2	0	2,4
13	8¼	8	0,85	2	0	2,55
13	8½	8	0,9	2	0	2,7
13	8¾	8	0,95	2	0	2,85
13	9	8	1,0	2	0	3,0
13	9¼	8	1,05	2	0	3,15
13	9½	8	1,1	2	0	3,3
13	9¾	8	1,15	2	0	3,45
13	10	8	1,2	2	0	3,6
13	10¼	8	1,25	2	0	3,75
13	10½	8	1,3	2	0	3,9
13	10¾	8	1,35	2	1	0,05
13	11	8	1,4	2	1	0,2
13	11¼	8	1,45	2	1	0,35
13	11½	8	1,5	2	1	0,5
13	11¾	8	1,55	2	1	0,65

TABLE of 5l. and 15l. per Cent. continued.

14 Shillings.

Value.		Duty at 5 per Cent.		Duty at 15 per Cent.		
s.	d.	d.	f. p.	s.	d.	f. p.
14	0	8	1,6	2	1	0,8
14	0¼	8	1,65	2	1	0,95
14	0½	8	1,7	2	1	1,1
14	0¾	8	1,75	2	1	1,25
14	1	8	1,8	2	1	1,4
14	1¼	8	1,85	2	1	1,55
14	1½	8	1,9	2	1	1,7
14	1¾	8	1,95	2	1	1,85
14	2	8	2,0	2	1	2,0
14	2¼	8	2,05	2	1	2,15
14	2½	8	2,1	2	1	2,3
14	2¾	8	2,15	2	1	2,45
14	3	8	2,2	2	1	2,6
14	3¼	8	2,25	2	1	2,75
14	3½	8	2,3	2	1	2,9
14	3¾	8	2,35	2	1	3,05
14	4	8	2,4	2	1	3,2
14	4¼	8	2,45	2	1	3,35
14	4½	8	2,5	2	1	3,5
14	4¾	8	2,55	2	1	3,65
14	5	8	2,6	2	1	3,8
14	5¼	8	2,65	2	1	3,95
14	5½	8	2,7	2	2	0,1
14	5¾	8	2,75	2	2	0,25
14	6	8	2,8	2	2	0,4
14	6¼	8	2,85	2	2	0,55
14	6½	8	2,9	2	2	0,7
14	6¾	8	2,95	2	2	0,85
14	7	8	3,0	2	2	1,0
14	7¼	8	3,05	2	2	1,15
14	7½	8	3,1	2	2	1,3
14	7¾	8	3,15	2	2	1,45
14	8	8	3,2	2	2	1,6
14	8¼	8	3,25	2	2	1,75
14	8½	8	3,3	2	2	1,9
14	8¾	8	3,35	2	2	2,05
14	9	8	3,4	2	2	2,2
14	9¼	8	3,45	2	2	2,35
14	9½	8	3,5	2	2	2,5
14	9¾	8	3,55	2	2	2,65
14	10	8	3,6	2	2	2,8
14	10¼	8	3,65	2	2	2,95
14	10½	8	3,7	2	2	3,1
14	10¾	8	3,75	2	2	3,25
14	11	8	3,8	2	2	3,4
14	11¼	8	3,85	2	2	3,55
14	11½	8	3,9	2	2	3,7
14	11¾	8	3,95	2	2	3,85

15 Shillings.

Value.		Duty at 5 per Cent.		Duty at 15 per Cent.		
s.	d.	d.	f. p.	s.	d.	f. p.
15	0	9	0,0	2	3	0,01
15	0¼	9	0,05	2	3	0,15
15	0½	9	0,1	2	3	0,31
15	0¾	9	0,15	2	3	0,45
15	1	9	0,2	2	3	0,61
15	1¼	9	0,25	2	3	0,75
15	1½	9	0,3	2	3	0,91
15	1¾	9	0,35	2	3	1,05
15	2	9	0,4	2	3	1,21
15	2¼	9	0,45	2	3	1,35
15	2½	9	0,5	2	3	1,51
15	2¾	9	0,55	2	3	1,65
15	3	9	0,6	2	3	1,81
15	3¼	9	0,65	2	3	1,95
15	3½	9	0,7	2	3	2,11
15	3¾	9	0,75	2	3	2,25
15	4	9	0,8	2	3	2,41
15	4¼	9	0,85	2	3	2,55
15	4½	9	0,9	2	3	2,71
15	4¾	9	0,95	2	3	2,85
15	5	9	1,0	2	3	3,01
15	5¼	9	1,05	2	3	3,15
15	5½	9	1,1	2	3	3,31
15	5¾	9	1,15	2	3	3,45
15	6	9	1,2	2	3	3,61
15	6¼	9	1,25	2	3	3,75
15	6½	9	1,3	2	3	3,91
15	6¾	9	1,35	2	4	0,05
15	7	9	1,4	2	4	0,21
15	7¼	9	1,45	2	4	0,35
15	7½	9	1,5	2	4	0,51
15	7¾	9	1,55	2	4	0,65
15	8	9	1,6	2	4	0,81
15	8¼	9	1,65	2	4	0,95
15	8½	9	1,7	2	4	1,11
15	8¾	9	1,75	2	4	1,25
15	9	9	1,8	2	4	1,41
15	9¼	9	1,85	2	4	1,55
15	9½	9	1,9	2	4	1,71
15	9¾	9	1,95	2	4	1,85
15	10	9	2,0	2	4	2,01
15	10¼	9	2,05	2	4	2,15
15	10½	9	2,1	2	4	2,31
15	10¾	9	2,15	2	4	2,45
15	11	9	2,2	2	4	2,61
15	11¼	9	2,25	2	4	2,75
15	11½	9	2,3	2	4	2,91
15	11¾	9	2,35	2	4	3,05

TABLE of 5l. and 15l. per Cent. continued.

16 Shillings.								17 Shillings.						
Value.		Duty at 5 per Cent.		Duty at 15 per Cent.				Value.		Duty at 5 per Cent.		Duty at 15 per Cent.		
s.	d.	d.	f. p.	s.	d.	f. p.		s.	d.	d.	f. p.	s.	d.	f. p.
16	0	9	2,4	2	4	3,2		17	0	10	0,8	2	6	2,4
16	0¼	9	2,45	2	4	3,35		17	0¼	10	0,85	2	6	2,55
16	0½	9	2,5	2	4	3,5		17	0½	10	0,9	2	6	2,7
16	0¾	9	2,55	2	4	3,65		17	0¾	10	0,95	2	6	2,85
16	1	9	2,6	2	4	3,8		17	1	10	1,0	2	6	3,0
16	1¼	9	2,65	2	4	3,95		17	1¼	10	1,05	2	6	3,15
16	1½	9	2,7	2	5	0,1		17	1½	10	1,1	2	6	3,3
16	1¾	9	2,75	2	5	0,25		17	1¾	10	1,15	2	6	3,45
16	2	9	2,8	2	5	0,4		17	2	10	1,2	2	6	3,6
16	2¼	9	2,85	2	5	0,55		17	2¼	10	1,25	2	6	3,75
16	2½	9	2,9	2	5	0,7		17	2½	10	1,3	2	6	3,9
16	2¾	9	2,95	2	5	0,85		17	2¾	10	1,35	2	7	0,05
16	3	9	3,0	2	5	1,0		17	3	10	1,4	2	7	0,2
16	3¼	9	3,05	2	5	1,15		17	3¼	10	1,45	2	7	0,35
16	3½	9	3,1	2	5	1,3		17	3½	10	1,5	2	7	0,5
16	3¾	9	3,15	2	5	1,45		17	3¾	10	1,55	2	7	0,65
16	4	9	3,2	2	5	1,6		17	4	10	1,6	2	7	0,8
16	4¼	9	3,25	2	5	1,75		17	4¼	10	1,65	2	7	0,95
16	4½	9	3,3	2	5	1,9		17	4½	10	1,7	2	7	1,1
16	4¾	9	3,35	2	5	2,05		17	4¾	10	1,75	2	7	1,25
16	5	9	3,4	2	5	2,2		17	5	10	1,8	2	7	1,4
16	5¼	9	3,45	2	5	2,35		17	5¼	10	1,85	2	7	1,55
16	5½	9	3,5	2	5	2,5		17	5½	10	1,9	2	7	1,7
16	5¾	9	3,55	2	5	2,65		17	5¾	10	1,95	2	7	1,85
16	6	9	3,6	2	5	2,8		17	6	10	2,0	2	7	2,0
16	6¼	9	3,65	2	5	2,95		17	6¼	10	2,05	2	7	2,15
16	6½	9	3,7	2	5	3,1		17	6½	10	2,1	2	7	2,3
16	6¾	9	3,75	2	5	3,25		17	6¾	10	2,15	2	7	2,45
16	7	9	3,8	2	5	3,4		17	7	10	2,2	2	7	2,6
16	7¼	9	3,85	2	5	3,55		17	7¼	10	2,25	2	7	2,75
16	7½	9	3,9	2	5	3,7		17	7½	10	2,3	2	7	2,9
16	7¾	9	3,95	2	5	3,85		17	7¾	10	2,35	2	7	3,05
16	8	10	0,0	2	6	0,0		17	8	10	2,4	2	7	3,2
16	8¼	10	0,05	2	6	0,15		17	8¼	10	2,45	2	7	3,35
16	8½	10	0,1	2	6	0,3		17	8½	10	2,5	2	7	3,5
16	8¾	10	0,15	2	6	0,45		17	8¾	10	2,55	2	7	3,65
16	9	10	0,2	2	6	0,6		17	9	10	2,6	2	7	3,8
16	9¼	10	0,25	2	6	0,75		17	9¼	10	2,65	2	7	3,95
16	9½	10	0,3	2	6	0,9		17	9½	10	2,7	2	8	0,1
16	9¾	10	0,35	2	6	1,05		17	9¾	10	2,75	2	8	0,25
16	10	10	0,4	2	6	1,2		17	10	10	2,8	2	8	0,4
16	10¼	10	0,45	2	6	1,35		17	10¼	10	2,85	2	8	0,55
16	10½	10	0,5	2	6	1,5		17	10½	10	2,9	2	8	0,7
16	10¾	10	0,55	2	6	1,65		17	10¾	10	2,95	2	8	0,85
16	11	10	0,6	2	6	1,8		17	11	10	3,0	2	8	1,0
16	11¼	10	0,65	2	6	1,95		17	11¼	10	3,05	2	8	1,15
16	11½	10	0,7	2	6	2,1		17	11½	10	3,1	2	8	1,3
16	11¾	10	0,75	2	6	2,25		17	11¾	10	3,15	2	8	1,45

TABLE of 5l. and 15l. per Cent. continued.

18 Shillings.

Value.		Duty at 5 per Cent.		Duty at 15 per Cent.		
s.	d.	d.	f. p.	s.	d.	f. p.
18	0	10	3,2	2	8	1,6
18	0¼	10	3,25	2	8	1,75
18	0½	10	3,3	2	8	1,9
18	0¾	10	3,35	2	8	2,05
18	1	10	3,4	2	8	2,2
18	1¼	10	3,45	2	8	2,35
18	1½	10	3,5	2	8	2,5
18	1¾	10	3,55	2	8	2,65
18	2	10	3,6	2	8	2,8
18	2¼	10	3,65	2	8	2,95
18	2½	10	3,7	2	8	3,1
18	2¾	10	3,75	2	8	3,25
18	3	10	3,8	2	8	3,4
18	3¼	10	3,85	2	8	3,55
18	3½	10	3,9	2	8	3,7
18	3¾	10	3,95	2	8	3,85
18	4	11	0,0	2	9	0,0
18	4¼	11	0,05	2	9	0,15
18	4½	11	0,1	2	9	0,3
18	4¾	11	0,15	2	9	0,45
18	5	11	0,2	2	9	0,6
18	5¼	11	0,25	2	9	0,75
18	5½	11	0,3	2	9	0,9
18	5¾	11	0,35	2	9	1,05
18	6	11	0,4	2	9	1,2
18	6¼	11	0,45	2	9	1,35
18	6½	11	0,5	2	9	1,5
18	6¾	11	0,55	2	9	1,65
18	7	11	0,6	2	9	1,8
18	7¼	11	0,65	2	9	1,95
18	7½	11	0,7	2	9	2,1
18	7¾	11	0,75	2	9	2,25
18	8	11	0,8	2	9	2,4
18	8¼	11	0,85	2	9	2,55
18	8½	11	0,9	2	9	2,7
18	8¾	11	0,95	2	9	2,85
18	9	11	1,0	2	9	3,0
18	9¼	11	1,05	2	9	3,15
18	9½	11	1,1	2	9	3,3
18	9¾	11	1,15	2	9	3,45
18	10	11	1,2	2	9	3,6
18	10¼	11	1,25	2	9	3,75
18	10½	11	1,3	2	9	3,9
18	10¾	11	1,35	2	10	0,05
18	11	11	1,4	2	10	0,2
18	11¼	11	1,45	2	10	0,35
18	11½	11	1,5	2	10	0,5
18	11¾	11	1,55	2	10	0,65

19 Shillings.

Value.		Duty at 5 per Cent.		Duty at 15 per Cent.		
s.	d.	d.	f. p.	s.	d.	f. p.
19	0	11	1,6	2	10	0,8
19	0¼	11	1,65	2	10	0,95
19	0½	11	1,7	2	10	1,1
19	0¾	11	1,75	2	10	1,25
19	1	11	1,8	2	10	1,4
19	1¼	11	1,85	2	10	1,55
19	1½	11	1,9	2	10	1,7
19	1¾	11	1,95	2	10	1,85
19	2	11	2,0	2	10	2,0
19	2¼	11	2,05	2	10	2,15
19	2½	11	2,1	2	10	2,3
19	2¾	11	2,15	2	10	2,45
19	3	11	2,2	2	10	2,6
19	3¼	11	2,25	2	10	2,75
19	3½	11	2,3	2	10	2,9
19	3¾	11	2,35	2	10	3,05
19	4	11	2,4	2	10	3,2
19	4¼	11	2,45	2	10	3,35
19	4½	11	2,5	2	10	3,5
19	4¾	11	2,55	2	10	3,65
12	5	11	2,6	2	10	3,8
19	5¼	11	2,65	2	10	3,95
19	5½	11	2,7	2	11	0,1
19	5¾	11	2,75	2	11	0,25
19	6	11	2,8	2	11	0,4
19	6¼	11	2,85	2	11	0,55
19	6½	11	2,9	2	11	0,7
19	6¾	11	2,95	2	11	0,85
19	7	11	3,0	2	11	1,0
19	7¼	11	3,05	2	11	1,15
19	7½	11	3,1	2	11	1,3
19	7¾	11	3,15	2	11	1,45
19	8	11	3,2	2	11	1,6
19	8¼	11	3,25	2	11	1,75
19	8½	11	3,3	2	11	1,9
19	8¾	11	3,35	2	11	2,05
19	9	11	3,4	2	11	2,2
19	9¼	11	3,45	2	11	2,35
19	9½	11	3,5	2	11	2,5
19	9¾	11	3,55	2	11	2,65
19	10	11	3,6	2	11	2,8
19	10¼	11	3,65	2	11	2,95
19	10½	11	3,7	2	11	3,1
19	10¾	11	3,75	2	11	3,25
19	11	11	3,8	2	11	3,4
19	11¼	11	3,85	2	11	3,55
19	11½	11	3,9	2	11	3,7
19	11¾	11	3,95	2	11	3,85
20	0	12	0,0	3	0	0,0

TABLE 11.

Old duty Cash Table to money drawbacks upon soap, at one-third of a penny per pound.

lb.	s.	d.	q. p.	lb.	s.	d.	q. p.	lb.	l.	s.	d.	q. p.
1	0	0	1,1	46	1	3	1,1	91	0	2	6	1,1
2	0	0	2,2	47	1	3	2,2	92	0	2	6	2,2
3	0	1	0,0	48	1	4	0,0	93	0	2	7	0,0
4	0	1	1,1	49	1	4	1,1	94	0	2	7	1,1
5	0	1	2,2	50	1	4	2,2	95	0	2	7	2,2
6	0	2	0,0	51	1	5	0,0	96	0	2	8	0,0
7	0	2	1,1	52	1	5	1,1	97	0	2	8	1,1
8	0	2	2,2	53	1	5	2,2	98	0	2	8	2,2
9	0	3	0,0	54	1	6	0,0	99	0	2	9	0,0
10	0	3	1,1	55	1	6	1,1	100	0	2	9	1,1
11	0	3	2,2	56	1	6	2,2	101	0	2	9	2,2
12	0	4	0,0	57	1	7	0,0	102	0	2	10	0,0
13	0	4	1,1	58	1	7	1,1	103	0	2	10	1,1
14	0	4	2,2	59	1	7	2,2	104	0	2	10	2,2
15	0	5	0,0	60	1	8	0,0	105	0	2	11	0,0
16	0	5	1,1	61	1	8	1,1	106	0	2	11	1,1
17	0	5	2,2	62	1	8	2,2	107	0	2	11	2,2
18	0	6	0,0	63	1	9	0,0	108	0	3	0	0,0
19	0	6	1,1	64	1	9	1,1	109	0	3	0	1,1
20	0	6	2,2	65	1	9	2,2	110	0	3	0	2,2
21	0	7	0,0	66	1	10	0,0	111	0	3	1	0,0
22	0	7	1,1	67	1	10	1,1	112	0	3	1	1,1
23	0	7	2,2	68	1	10	2,2	113	0	3	1	2,2
24	0	8	0,0	69	1	11	0,0	114	0	3	2	0,0
25	0	8	1,1	70	1	11	1,1	115	0	3	2	1,1
26	0	8	2,2	71	1	11	2,2	116	0	3	2	2,2
27	0	9	0,0	72	2	0	0,0	117	0	3	3	0,0
28	0	9	1,1	73	2	0	1,1	118	0	3	3	1,1
29	0	9	2,2	74	2	0	2,2	119	0	3	3	2,2
30	0	10	0,0	75	2	1	0,0	120	0	3	4	0,0
31	0	10	1,1	76	2	1	1,1	200	0	5	6	2,2
32	0	10	2,2	77	2	1	2,2	300	0	8	4	0,0
33	0	11	0,0	78	2	2	0,0	400	0	11	1	1,1
34	0	11	1,1	79	2	2	1,1	500	0	13	10	2,2
35	0	11	2,2	80	2	2	2,2	600	0	16	8	0,0
36	1	0	0,0	81	2	3	0,0	700	0	19	5	1,1
37	1	0	1,1	82	2	3	1,1	800	1	2	2	2,2
38	1	0	2,2	83	2	3	2,2	900	1	5	0	0,0
39	1	1	0,0	84	2	4	0,0	1000	1	7	9	1,1
40	1	1	1,1	85	2	4	1,1	2000	2	15	6	2,2
41	1	1	2,2	86	2	4	2,2	3000	4	3	4	0,0
42	1	2	0,0	87	2	5	0,0	4000	5	11	1	1,1
43	1	2	1,1	88	2	5	1,1	5000	6	18	10	2,2
44	1	2	2,2	89	2	5	2,2	6000	8	6	8	0,0
45	1	3	0,0	90	2	6	0,0					

TABLE 12.

Cash Table for the additional duty on soap, at a half-penny per pound.

lb.	s.	d.	lb.	s.	d.	lb.	s.	d.
1	0	0½	46	1	11	91	3	9½
2	0	1	47	1	11½	92	3	10
3	0	1½	48	2	0	93	3	10½
4	0	2	49	2	0½	64	3	11
5	0	2½	50	2	1	95	3	11½
6	0	3	51	2	1½	96	4	0
7	0	3½	52	2	2	97	4	0½
8	0	4	53	2	2½	98	4	1
9	0	4½	54	2	3	98	4	1½
10	0	5	55	2	3½	100	4	2
11	0	5½	56	2	4	101	4	2½
12	0	6	57	2	4½	102	4	3
13	0	6½	58	2	5	103	4	3½
14	0	7	59	2	5½	104	4	4
15	0	7½	60	2	6	105	4	4½
16	0	8	61	2	6½	106	4	5
17	0	8½	62	2	7	107	4	5½
18	0	9	63	2	7½	108	4	6
19	0	9½	64	2	8	109	4	6½
20	0	10	65	2	8½	110	4	7
21	0	10½	66	2	9	111	4	7½
22	0	11	67	2	9½	112	4	8
23	0	11½	68	2	10	113	4	8½
24	1	0	69	2	10½	114	4	9
25	1	0½	70	2	11	113	4	9½
26	1	1	71	2	11½	116	4	10
27	1	1½	72	3	0	1:7	4	10½
28	1	2	73	3	0½	118	4	11
29	1	2½	74	3	1	119	4	11½
30	1	3	75	3	1½	120	5	0
31	1	3½	76	3	2	200	8	4
32	1	4	77	3	2½	300	12	6
33	1	4½	78	3	3	400	16	8
34	1	5	79	3	3½	500	20	10
35	1	5½	80	3	4	600	25	0
36	1	6	81	3	4½	700	29	2
37	1	6½	82	3	5	800	33	4
38	1	7	83	3	5½	900	37	6
39	1	7½	84	3	6	1000	41	8
40	1	8	85	3	6½	2000	83	4
41	1	8½	86	3	7			
42	1	9	87	3	7½			
43	1	9½	88	3	8			
44	1	10	89	3	8½			
45	1	10½	90	3	9			

TABLE 13.

Cash Table for the additional duty on soap, at ¼d. per pound.

lb.	s.	d.	q.	lb.	s.	d.	q.	lb.	l.	s.	d.	q.
1	0	0	3	46	2	10	2	91	0	5	8	1
2	0	1	2	47	2	11	1	92	0	5	9	0
3	0	2	1	48	3	0	0	93	0	5	9	3
4	0	3	0	49	3	0	3	94	0	5	10	2
5	0	3	3	50	3	1	2	95	0	5	11	1
6	0	4	2	51	3	2	1	96	0	6	0	0
7	0	5	1	52	3	3	0	97	0	6	0	3
8	0	6	0	53	3	3	3	98	0	6	1	2
9	0	6	3	54	3	4	2	99	0	6	2	1
10	0	7	2	55	3	5	1	100	0	6	3	0
11	0	8	1	56	3	6	0	101	0	6	3	3
12	0	9	0	57	3	6	3	102	0	6	4	2
13	0	9	3	58	3	7	2	103	0	6	5	1
14	0	10	2	59	3	8	1	104	0	6	6	0
15	0	11	1	60	3	9	0	105	0	6	6	3
16	1	0	0	61	3	9	3	106	0	6	7	2
17	1	0	3	62	3	10	2	107	0	6	8	1
18	1	1	2	63	3	11	1	108	0	6	9	0
19	1	2	1	64	4	0	0	109	0	6	9	3
20	1	3	0	65	4	0	3	110	0	6	10	2
21	1	3	3	66	4	1	2	111	0	6	11	1
22	1	4	2	67	4	2	1	112	0	7	0	0
23	1	5	1	68	4	3	0	113	0	7	0	3
24	1	6	0	69	4	3	3	114	0	7	1	2
25	1	6	3	70	4	4	2	115	0	7	2	1
26	1	7	2	71	4	5	1	116	0	7	3	0
27	1	8	1	72	4	6	0	117	0	7	3	3
28	1	9	0	73	4	6	3	118	0	7	4	2
29	1	9	3	74	4	7	2	119	0	7	5	1
30	1	10	2	75	4	8	1	120	0	7	6	0
31	1	11	1	76	4	9	0	200	0	12	6	
32	2	0	0	77	4	9	3	300	0	18	9	
33	2	0	3	78	4	10	2	400	1	5	0	
34	2	1	2	79	4	11	1	500	1	11	3	
35	2	2	1	80	5	0	0	600	1	17	6	
36	2	3	0	81	5	0	3	700	2	3	9	
37	2	3	3	82	5	1	2	800	2	10	0	
38	2	4	2	83	5	2	1	900	2	16	3	
39	2	5	1	84	5	3	0	1000	3	2	6	
40	2	6	0	85	5	3	3	2000	6	5	0	
41	2	6	3	86	5	4	2	3000	9	7	6	
42	2	7	2	87	5	5	1	4000	12	10	0	
43	2	8	1	88	5	6	0	5000	15	12	6	
44	2	9	0	89	5	6	3	6000	18	15	0	
45	2	9	3	90	5	7	2	7000	21	17	6	

TABLE 14.

Cash Table for the duty on tallow, candles, & hops, at 1d. per pound.

lb.	s.	d.	lb.	s.	d.	lb.	l.	s.	d.
1	0	1	46	3	10	91	0	7	7
2	0	2	47	3	11	92	0	7	8
3	0	3	48	4	0	93	0	7	9
4	0	4	49	4	1	94	0	7	10
5	0	5	50	4	2	95	0	7	11
6	0	6	51	4	3	96	0	8	0
7	0	7	52	4	4	97	0	8	1
8	0	8	53	4	5	98	0	8	2
9	0	9	54	4	6	99	0	8	3
10	0	10	55	4	7	100	0	8	4
11	0	11	56	4	8	101	0	8	5
12	1	0	57	4	9	102	0	8	6
13	1	1	58	4	10	103	0	8	7
14	1	2	59	4	11	104	0	8	8
15	1	3	60	5	0	105	0	8	9
16	1	4	61	5	1	106	0	8	10
17	1	5	62	5	2	107	0	8	11
18	1	6	63	5	3	108	0	9	0
19	1	7	64	5	4	109	0	9	1
20	1	8	65	5	5	110	0	9	2
21	1	9	66	5	6	111	0	9	3
22	1	10	67	5	7	112	0	9	4
23	1	11	68	5	8	113	0	9	5
24	2	0	69	5	9	114	0	9	6
25	2	1	70	5	10	115	0	9	7
26	2	2	71	5	11	116	0	9	8
27	2	3	72	6	0	117	0	9	9
28	2	4	73	5	1	118	0	9	10
29	2	5	74	6	2	119	0	9	11
30	2	6	75	6	3	120	0	10	0
31	2	7	76	6	4	200	0	16	8
32	2	8	77	6	5	300	1	5	0
33	2	9	78	6	6	400	1	13	4
34	2	10	79	6	7	500	2	1	8
35	2	11	80	6	8	600	2	10	0
36	3	0	81	6	9	700	2	18	4
37	3	1	82	6	10	800	3	6	8
38	3	2	83	6	11	900	3	15	0
39	3	3	84	7	0	1000	4	3	4
40	3	4	85	7	1	2000	8	6	8
41	3	5	86	7	2	3000	12	10	0
42	3	6	87	7	3	4000	16	13	4
43	3	7	88	7	4	5000	20	16	8
44	3	8	89	7	5	6000	25	0	0
45	3	9	90	7	6	7000	29	3	4

TABLE 15.

Cash Table for the duty on tawed sheep and lamb-skins, at 1¼d. per pound.

lb.	s.	d.	lb.	s.	d.	lb.	l.	s.	d.
1	0	1¼	46	4	9½	91	0	9	5¾
2	0	2½	47	4	10¾	92	0	9	7
3	0	3¾	48	5	0	93	0	9	8¼
4	0	5	49	5	1¼	94	0	9	9½
5	0	6¼	50	5	2½	95	0	9	10¾
6	0	7½	51	5	3¾	96	0	10	0
7	0	8¾	52	5	5	97	0	10	1¼
8	0	10	53	5	6¼	98	0	10	2½
9	0	11¼	54	5	7½	99	0	10	3¾
10	1	0½	55	5	8¾	100	0	10	5
11	1	1¾	56	5	10	101	0	10	6¼
12	1	3	57	5	11¼	102	0	10	7½
13	1	4¼	58	6	0½	103	0	10	8¾
14	1	5½	59	6	1¾	104	0	10	10
15	1	6¾	60	6	3	105	0	10	11¼
16	1	8	61	6	4¼	106	0	11	0½
17	1	9¼	62	6	5½	107	0	11	1¾
18	1	10½	63	6	6¾	108	0	11	3
19	1	11¾	64	6	8	109	0	11	4¼
20	2	1	65	6	9¼	110	0	11	5½
21	2	2¼	66	6	10½	111	0	11	6¾
22	2	3½	67	6	11¾	112	0	11	8
23	2	4¾	68	7	1	113	0	11	9¼
24	2	6	69	7	2¼	114	0	11	10½
25	2	7¼	70	7	3½	115	0	11	11¾
26	2	8½	71	7	4¾	116	0	12	1
27	2	9¾	72	7	6	117	0	12	2¼
28	2	11	73	7	7¼	118	0	12	3½
29	3	0¼	74	7	8½	119	0	12	4¾
30	3	1½	75	7	9¾	120	0	12	6
31	3	2¾	76	7	11	200	1	0	10
32	3	4	77	8	0¼	300	1	11	3
33	3	5¼	78	8	1½	400	2	1	8
34	3	6½	79	8	2¾	500	2	12	1
35	3	7¾	80	8	4	600	3	2	6
36	3	9	81	8	5¼	700	3	12	11
37	3	10¼	82	8	6½	800	4	3	4
38	3	11½	83	8	7¾	900	4	13	9
39	4	0¾	84	8	9	1000	5	4	2
40	4	2	85	8	10¼	2000	10	8	4
41	4	3¼	86	8	11½	3000	15	12	6
42	4	4½	87	9	0¾	4000	20	16	8
43	4	5¾	88	9	2	5000	26	0	10
44	4	7	89	9	3¼	6000	31	5	0
45	4	8¼	90	9	4½	7000	36	9	2

TABLE 16.

Cash Table for the duty on sheep, lamb, &c. tanned; calf and kid tawed; and soap, at 1½d. per lb.

lb.	s.	d.	lb.	s.	d.	lb.	l.	s.	d.
1	0	1½	46	5	9	91	0	11	4½
2	0	3	47	5	10½	92	0	11	6
3	0	4½	48	6	0	93	0	11	7½
4	0	6	49	6	1½	94	0	11	9
5	0	7½	50	6	3	95	0	11	10½
6	0	9	51	6	4½	96	0	12	0
7	0	10½	52	6	6	97	0	12	1½
8	1	0	53	6	7½	98	0	12	3
9	1	1½	54	6	9	99	0	12	4½
10	1	3	55	6	10½	100	0	12	6
11	1	4½	56	7	0	101	0	12	7½
12	1	6	57	7	1½	102	0	12	9
13	1	7½	58	7	3	103	0	12	10½
14	1	9	59	7	4½	104	0	13	0
15	1	10½	60	7	6	105	0	13	1½
16	2	0	61	7	7½	106	0	13	3
17	2	1½	62	7	9	107	0	13	4½
18	2	3	63	7	10½	108	0	13	6
19	2	4½	64	8	0	109	0	13	7½
20	2	6	65	8	1½	110	0	13	9
21	2	7½	66	8	3	111	0	13	10½
22	2	9	67	8	4½	112	0	14	0
23	2	10½	68	8	6	113	0	14	1½
24	3	0	69	8	7½	114	0	14	3
25	3	1½	70	8	9	115	0	14	4½
26	3	3	71	8	10½	116	0	14	6
27	3	4½	72	9	0	117	0	14	7½
28	3	6	73	9	1½	118	0	14	9
29	3	7½	74	9	3	119	0	14	10½
30	3	9	75	9	4½	120	0	15	0
31	3	10½	76	9	6	200	1	5	0
32	4	0	77	9	7½	300	1	17	6
33	4	1½	78	9	9	400	2	10	0
34	4	3	79	9	10½	500	3	2	6
35	4	4½	80	10	0	600	3	15	0
36	4	6	81	10	1½	700	4	7	6
37	4	7½	82	10	3	800	5	0	0
38	4	9	83	10	4½	900	5	12	6
39	4	10½	84	10	6	1000	6	5	0
40	5	0	85	10	7½	2000	12	10	0
41	5	1½	86	10	9	3000	18	15	0
42	5	3	87	10	10½	4000	25	0	0
43	5	4½	88	11	0	5000	31	5	0
44	5	6	89	11	1½	6000	37	10	0
45	5	7½	90	11	3	7000	43	5	0

TABLE 17.

Cash Table for the duty on roans tanned skins, pieces in oil, and starch, at 2d. per pound.

lb.	s.	d.	lb.	s.	d.	lb.	l.	s.	d.
1	0	2	46	7	8	91	0	15	2
2	0	4	47	7	10	92	0	15	4
3	0	6	48	8	0	93	0	15	6
4	0	8	49	8	2	94	0	15	8
5	0	10	50	8	4	95	0	15	10
6	1	0	51	8	6	96	0	16	0
7	1	2	52	8	8	97	0	16	2
8	1	4	53	8	10	98	0	16	4
9	1	6	54	9	0	99	0	16	6
10	1	8	55	9	2	100	0	16	8
11	1	10	56	9	4	101	0	16	10
12	2	0	57	9	6	102	0	17	0
13	2	2	58	9	8	103	0	17	2
14	2	4	59	9	10	104	0	17	4
15	2	6	60	10	0	105	0	17	6
16	2	8	61	10	2	106	0	17	8
17	2	10	62	10	4	107	0	17	10
18	3	0	63	10	6	108	0	18	0
19	3	2	64	10	8	109	0	18	2
20	3	4	65	10	10	110	0	18	4
21	3	6	66	11	0	111	0	18	6
22	3	8	67	11	2	112	0	18	8
23	3	10	68	11	4	113	0	18	10
24	4	0	69	11	6	114	0	19	0
25	4	2	70	11	8	115	0	19	2
26	4	4	71	11	10	116	0	19	4
27	4	6	72	12	0	117	0	19	6
28	4	8	73	12	2	118	0	19	8
29	4	10	74	12	4	119	0	19	10
30	5	0	75	12	6	120	1	0	0
31	5	2	76	12	8	200	1	13	4
32	5	4	77	12	10	300	2	10	0
33	5	6	78	13	0	400	3	6	8
34	5	8	79	13	2	500	4	3	4
35	5	10	80	13	4	600	5	0	0
36	6	0	81	13	6	700	5	16	8
37	6	2	82	13	8	800	6	13	4
38	6	4	83	13	10	900	7	10	0
39	6	6	84	14	0	1000	8	6	8
40	6	8	85	14	2	2000	16	13	4
41	6	10	86	14	4	3000	25	0	0
42	7	0	87	14	6	4000	33	6	8
43	7	2	88	14	8	5000	41	13	4
44	7	4	89	14	10	6000	50	0	0
45	7	6	90	15	0	7000	58	6	8

TABLE 18.

Cash Table for the duty on sheep and lamb in oil, &c. at 3d. per pound.

lb.	s.	d.	lb.	l.	s.	d.	lb.	l.	s.	d.
1	0	3	46	0	11	6	91	1	2	9
2	0	6	47	0	11	9	92	1	3	0
3	0	9	48	0	12	0	93	1	3	3
4	1	0	49	0	12	3	94	1	3	6
5	1	3	50	0	12	6	95	1	3	9
6	1	6	51	0	12	9	96	1	4	0
7	1	9	52	0	13	0	97	1	4	3
8	2	0	53	0	13	3	98	1	4	6
9	2	3	54	0	13	6	99	1	4	9
10	2	6	55	0	13	9	100	1	5	0
11	2	9	56	0	13	0	101	1	5	3
12	3	0	57	0	14	3	102	1	5	6
13	3	3	58	0	14	6	103	1	5	9
14	3	6	59	0	14	9	104	1	6	0
15	3	9	60	0	15	0	105	1	6	3
16	4	0	61	0	15	3	106	1	6	6
17	4	3	62	0	15	6	107	1	6	9
18	4	6	63	0	15	9	108	1	7	0
19	4	9	64	0	16	0	109	1	7	3
20	5	0	65	0	16	3	110	1	7	6
21	5	3	66	0	16	6	111	1	7	9
22	5	6	67	0	16	9	112	1	8	0
23	5	9	68	0	17	0	113	1	8	3
24	6	0	69	0	17	3	114	1	8	6
25	6	3	70	0	17	6	115	1	8	9
26	6	6	71	0	17	9	116	1	9	0
27	6	9	72	0	18	0	117	1	9	3
28	7	0	73	0	18	3	118	1	9	6
29	7	3	74	0	18	6	119	1	9	9
30	7	6	75	0	18	9	120	1	10	0
31	7	9	76	0	19	0	200	2	10	0
32	8	0	77	0	19	3	300	3	15	0
33	8	3	78	0	19	6	400	5	0	0
34	8	6	79	0	19	9	500	6	5	0
35	8	9	80	1	0	0	600	7	10	0
36	9	0	81	1	0	3	700	8	15	0
37	9	3	82	1	0	6	800	10	0	0
38	9	6	83	1	0	9	900	11	5	0
39	9	9	84	1	1	0	1000	12	10	0
40	10	0	85	1	1	3	2000	25	0	0
41	10	3	86	1	1	6	3000	37	10	0
42	10	6	87	1	1	9	4000	50	0	0
43	10	9	88	1	2	0	5000	62	10	0
44	11	0	89	1	2	3	6000	75	0	0
45	11	3	90	1	2	6	7000	87	10	0

TABLE 19.

Cash Table for the duty on goats tanned with shumack, at 4d. per pound.

lb.	l.	s.	d.	lb.	l.	s.	d.
1	0	0	4	53	0	18	8
2	0	0	8	54	0	17	0
3	0	1	0	55	0	18	4
4	0	1	4	56	0	18	8
5	0	1	8	57	0	19	0
6	0	2	0	58	0	19	4
7	0	2	4	59	0	19	8
8	0	2	8	60	1	0	0
9	0	3	0	61	1	0	4
10	0	3	4	62	1	0	8
11	0	3	8	63	1	1	0
12	0	4	0	64	1	1	4
13	0	4	4	65	1	1	8
14	0	4	8	66	1	2	0
15	0	5	0	67	1	2	4
16	0	5	4	68	1	2	8
17	0	5	8	69	1	3	0
18	0	6	0	70	1	3	4
19	0	6	4	71	1	3	8
20	0	6	8	72	1	4	0
21	0	7	0	73	1	4	4
22	0	7	4	74	1	4	8
23	0	7	8	75	1	5	0
24	0	8	0	76	1	5	4
25	0	8	4	77	1	5	8
26	0	8	8	78	1	6	0
27	0	9	0	79	1	6	4
28	0	9	4	80	1	6	8
29	0	9	8	81	1	7	0
30	0	10	0	82	1	7	4
31	0	10	4	83	1	7	8
32	0	10	8	84	1	8	0
33	0	11	0	85	1	8	4
34	0	11	4	86	1	8	8
35	0	11	8	87	1	9	0
36	0	12	0	88	1	9	4
37	0	12	4	89	1	9	8
38	0	12	8	90	1	10	0
39	0	13	0	91	1	10	4
40	0	13	4	92	1	10	8
41	0	13	8	93	1	11	0
42	0	14	0	94	1	11	4
43	0	14	4	95	1	11	8
44	0	14	8	96	1	12	0
45	0	15	0	97	1	12	4
46	0	15	4	98	1	12	8
47	0	15	8	99	1	13	0
48	0	16	0	100	1	13	4
49	0	16	4	200	3	6	0
50	0	16	8	300	5	0	0
51	0	17	0	400	6	13	4
52	0	17	4	500	8	6	8

TABLE 20.

Cash Table for duty on buck and doe tawed; hides, deer, go at, &c. in oil, at 6d. per lb.

lb.	l.	s.	d.	lb.	l.	s.	d.
1	0	0	6	53	1	6	6
2	0	1	0	54	1	7	0
3	0	1	6	55	1	7	6
4	0	2	0	56	1	8	0
5	0	2	6	57	1	8	6
6	0	3	0	58	1	9	0
7	0	3	6	59	1	9	6
8	0	4	0	60	1	10	0
9	0	4	6	61	1	10	6
10	0	5	0	62	1	11	0
11	0	5	6	63	1	11	0
12	0	6	0	64	1	12	0
13	0	6	6	65	1	12	6
14	0	7	0	66	1	13	0
15	0	7	6	67	1	13	6
16	0	8	0	68	1	14	0
17	0	8	6	69	1	14	6
18	0	9	0	70	1	15	0
19	0	9	6	71	1	15	6
20	0	10	0	72	1	16	0
21	0	10	6	73	1	16	6
22	0	11	0	74	1	17	0
23	0	11	6	75	1	17	6
24	0	12	0	76	1	18	0
25	0	12	6	77	1	18	6
26	0	13	0	78	1	19	0
27	0	13	6	79	1	19	6
28	0	14	0	80	2	0	0
29	0	14	6	81	2	0	6
30	0	15	0	82	2	1	0
31	0	15	6	83	2	1	6
32	0	16	0	84	2	2	0
33	0	16	6	85	2	2	6
34	0	17	0	86	2	3	0
35	0	17	6	87	2	3	6
36	0	18	0	88	2	4	0
37	0	18	6	89	2	4	6
38	0	19	0	90	2	5	0
39	0	19	6	91	2	5	6
40	1	0	0	92	2	6	0
41	1	0	6	93	2	6	6
42	1	1	0	94	2	7	0
43	1	1	6	95	2	7	6
44	1	2	0	96	2	8	0
45	1	2	6	97	2	8	6
46	1	3	0	98	2	9	0
47	1	3	6	99	2	9	6
48	1	4	0	100	2	10	0
49	1	4	6	200	5	0	0
50	1	5	0	300	7	10	0
51	1	5	6	400	10	0	0
52	1	6	0	500	12	10	0

TABLE 21.

Cash Table for the duty on wax candles, gilt wire, and calf-skins in oil, at 8d. per pound.

lb.	l.	s.	d.	lb.	l.	s.	d.
1	0	0	8	53	1	15	4
2	0	1	4	54	1	16	0
3	0	2	0	55	1	16	8
4	0	2	8	56	1	17	4
5	0	3	4	57	1	18	0
6	0	4	0	58	1	18	8
7	0	4	8	59	1	19	4
8	0	5	4	60	2	0	0
9	0	6	0	61	2	0	8
10	0	6	8	62	2	1	4
11	0	7	4	63	2	2	0
12	0	8	0	64	2	2	8
13	0	8	8	65	2	3	4
14	0	9	4	66	2	4	0
15	0	10	0	67	2	4	8
16	0	10	8	68	2	5	4
17	0	11	4	69	2	6	0
18	0	12	0	70	2	6	8
19	0	12	8	71	2	7	4
20	0	13	4	72	2	8	0
21	0	14	0	73	2	8	8
22	0	14	8	74	2	9	4
23	0	15	4	75	2	10	0
24	0	16	0	76	2	10	8
25	0	16	8	77	2	11	4
26	0	17	4	78	2	12	0
27	0	18	0	79	2	12	8
28	0	18	8	80	2	13	4
29	0	19	4	81	2	14	0
30	1	0	0	82	2	14	8
31	1	0	8	83	2	15	4
32	1	1	4	84	2	16	0
33	1	2	0	85	2	16	8
34	1	2	8	86	2	17	4
35	1	3	4	87	2	18	0
36	1	4	0	88	2	18	8
37	1	4	8	89	2	19	4
38	1	5	4	90	3	0	0
39	1	6	0	91	3	0	8
40	1	6	8	92	3	1	4
41	1	7	4	93	3	2	0
42	1	8	0	94	3	2	8
43	1	8	8	95	3	3	4
44	1	9	4	96	3	4	0
45	1	10	0	97	3	4	8
46	1	10	8	98	3	5	4
47	1	11	4	99	3	6	0
48	1	12	0	100	3	6	8
49	1	12	8	200	6	13	4
50	1	13	4	300	10	0	0
51	1	14	0	400	13	6	8
52	1	14	8	500	16	13	4

TABLE 22.

Cash Table for the duty on common bottle glass, at 3s. 6d. per hundred weight.

Cwt.	l.	s.	d.	Cwt.	l.	s.	d
1	0	3	6	53	9	5	6
2	0	7	0	54	9	9	0
3	0	10	6	55	9	12	6
4	0	14	0	56	9	16	0
5	0	17	6	57	9	19	6
6	1	1	0	58	10	3	0
7	1	4	6	59	10	6	6
8	1	8	0	60	10	10	0
9	1	11	6	61	10	13	6
10	1	15	0	62	10	17	0
11	1	18	6	63	11	0	6
12	2	2	0	64	11	4	0
13	2	5	6	65	11	7	6
14	2	9	0	66	11	11	0
15	2	12	6	67	11	14	6
16	2	16	0	68	11	18	0
17	2	19	6	69	12	1	6
18	3	3	0	70	12	5	0
19	3	6	6	71	12	8	6
20	3	10	0	72	12	12	0
21	3	13	6	73	12	15	6
22	3	17	0	74	12	19	0
23	4	0	6	75	13	2	6
24	4	4	0	76	13	6	0
25	4	7	6	77	13	9	6
26	4	11	0	78	13	13	0
27	4	14	6	79	13	16	6
28	4	18	0	80	14	0	0
29	5	1	6	81	14	3	6
30	5	5	0	82	14	7	0
31	5	8	6	83	14	10	6
32	5	12	0	84	14	14	0
33	5	15	6	85	14	17	6
34	5	19	0	86	15	1	0
35	6	2	6	87	15	4	6
36	6	6	0	88	15	8	0
37	6	9	6	89	15	11	6
38	6	13	0	90	15	15	0
39	6	16	6	91	15	18	6
40	7	0	0	92	16	2	0
41	7	3	6	93	16	5	6
42	7	7	0	94	16	9	0
43	7	10	6	95	16	12	6
44	7	14	0	96	16	16	0
45	7	17	6	97	16	19	6
46	8	1	0	98	17	3	0
47	8	4	6	99	17	6	6
48	8	8	0	100	17	10	0
49	8	11	6	200	35	0	0
50	8	15	0	300	52	10	0
51	8	18	6	400	70	0	0
52	9	2	0	500	87	10	0

TABLE 23.

Cask Table for the duty on malt Cyder, at 4s. per hogshead.

Hhd.	l.	s.	d.	Hhd.	l.	s.	d.
1	0	4	0	53	10	12	0
2	0	8	0	54	10	16	0
3	0	12	0	55	11	0	0
4	0	16	0	56	11	4	0
5	1	0	0	57	11	8	0
6	1	4	0	58	11	12	0
7	1	8	0	59	11	16	0
8	1	12	0	60	12	0	0
9	1	16	0	61	12	4	0
10	2	0	0	62	12	8	0
11	2	4	0	63	12	12	0
12	2	8	0	64	12	16	0
13	2	12	0	65	13	0	0
14	2	16	0	66	13	4	0
15	3	0	0	67	13	8	0
16	3	4	0	68	13	12	0
17	3	8	0	69	13	16	0
18	3	12	0	70	14	0	0
19	3	16	0	71	14	4	0
20	4	0	0	72	14	8	0
21	4	4	0	73	14	12	0
22	4	8	0	74	14	16	0
23	4	12	0	75	15	0	0
24	4	16	0	76	15	4	0
25	5	0	0	77	15	8	0
26	5	4	0	78	15	12	0
27	5	8	0	79	15	16	0
28	5	12	0	80	16	0	0
29	5	16	0	81	16	4	0
30	6	0	0	82	16	8	0
31	6	4	0	83	16	12	0
32	6	8	0	84	16	16	0
33	6	12	0	85	17	0	0
34	6	16	0	86	17	4	0
35	7	0	0	87	17	8	0
36	7	4	0	88	17	12	0
37	7	8	0	89	17	16	0
38	7	12	0	90	18	0	0
39	7	16	0	91	18	4	0
40	8	0	0	92	18	8	0
41	8	4	0	93	18	12	0
42	8	8	0	94	18	16	0
43	8	12	0	95	19	0	0
44	8	16	0	96	19	4	0
45	9	0	0	97	19	8	0
46	9	4	0	98	19	12	0
47	9	8	0	99	19	16	0
48	9	12	0	100	20	0	0
49	9	16	0	200	40	0	0
50	10	0	0	300	60	0	0
51	10	4	0	400	80	0	0
52	10	8	0	500	100	0	0

TABLE 24.

Cash Table for the duty on cyder, in the year 1766, at 6s. per hogshead.

Hhd.	l.	s.	d.	Hhd.	l.	s.	d.
1	0	6	0	53	15	18	0
2	0	12	0	54	16	4	0
3	0	18	0	55	16	10	0
4	1	4	0	56	16	16	0
5	1	10	0	57	17	2	0
6	1	16	0	58	17	8	0
7	2	2	0	59	17	14	0
8	2	8	0	60	18	0	0
2	2	14	0	61	18	6	0
10	3	0	0	62	18	12	0
11	3	6	0	63	18	18	0
12	3	12	0	64	19	4	0
13	3	18	0	65	19	10	0
14	4	4	0	66	19	16	0
15	4	10	0	67	20	2	0
16	4	16	0	68	20	8	0
17	5	2	0	69	20	14	0
18	5	8	0	70	21	0	0
19	5	14	0	71	21	6	0
20	6	0	0	72	21	12	0
21	6	6	0	73	21	18	0
22	6	12	0	74	22	4	0
23	6	18	0	75	22	10	0
24	7	4	0	76	22	16	0
25	7	10	0	77	23	2	0
26	7	16	0	78	23	8	0
27	8	2	0	79	23	14	0
28	8	8	0	80	24	0	0
29	8	14	0	81	24	6	0
30	9	0	0	82	24	12	0
31	9	6	0	83	24	18	0
32	9	12	0	84	25	4	0
33	9	18	0	85	25	10	0
34	10	4	0	86	25	10	0
35	10	10	0	87	26	2	0
36	10	16	0	88	26	8	0
37	11	2	0	89	26	14	0
38	11	8	0	90	27	0	0
39	11	14	0	91	27	6	0
40	12	0	0	92	27	12	0
41	12	6	0	93	27	18	0
42	12	12	0	94	28	4	0
43	12	18	0	95	28	10	0
44	13	4	0	96	28	16	0
45	13	10	0	97	29	2	0
46	13	16	0	98	29	8	0
47	14	2	0	99	29	14	0
48	14	8	0	100	30	0	0
49	14	14	0	200	60	0	0
50	15	0	0	300	90	0	0
51	15	6	0	400	120	0	0
52	15	12	0	500	150	0	0

TABLE 25.

Cash Table for the excise duty on Cyder, at 6s. 8d. per hogshead.

Hhd.	l.	s.	d.	Hhd.	l.	s.	d.
1	0	6	8	53	17	13	4
2	0	13	4	54	18	0	0
3	1	0	0	55	18	6	8
4	1	6	8	56	18	13	4
5	1	13	4	57	19	0	0
6	2	0	0	58	19	6	8
7	2	6	8	59	19	13	4
8	2	13	4	60	20	0	0
9	3	0	0	61	20	6	8
10	3	6	8	62	20	13	4
11	3	13	4	63	21	0	0
12	4	0	0	64	21	6	8
13	4	6	8	65	21	13	4
14	4	13	4	66	22	0	0
15	5	0	0	67	22	6	8
16	5	6	8	68	22	13	4
17	5	13	4	69	23	0	0
18	6	0	0	70	23	6	8
19	6	6	8	71	23	13	4
20	6	13	4	72	24	0	0
21	7	0	0	73	24	6	8
22	7	6	8	74	24	13	4
23	7	13	4	75	25	0	0
24	8	0	0	76	25	6	8
25	8	6	8	77	25	13	4
26	8	13	4	78	26	0	0
27	9	0	0	79	26	6	8
28	9	6	8	80	26	13	4
29	9	13	4	81	27	0	0
30	10	0	0	82	27	6	8
31	10	6	8	83	27	13	4
32	10	13	4	84	28	0	0
33	11	0	0	85	28	6	8
34	11	6	8	86	28	13	4
35	11	13	4	87	29	0	0
36	12	0	0	88	29	6	8
37	12	6	8	89	29	13	4
38	12	13	4	90	30	0	0
39	13	0	0	91	30	6	8
40	13	6	8	92	30	13	4
41	13	13	4	93	31	0	0
42	14	0	0	94	31	6	8
43	14	6	8	95	31	13	4
44	14	13	4	96	32	0	0
45	15	0	0	97	32	6	8
46	15	6	8	98	32	13	4
47	15	13	4	99	33	0	0
48	16	0	0	100	33	6	8
49	16	6	8	200	66	13	4
50	16	13	4	300	100	0	0
51	17	0	0	400	133	6	8
52	17	6	8	500	166	13	4

TABLE 26.

Cash Table for the duty on broad glass, at 7s. per hundred weight.

Cwt.	l.	s.	d.	Cwt.	l.	s.	d.
1	0	7	0	53	18	11	0
2	0	14	0	54	18	18	0
3	1	1	0	55	19	5	0
4	1	8	0	56	19	12	0
5	1	15	0	57	19	19	0
6	2	2	0	58	20	6	0
7	2	9	0	59	20	13	0
8	2	16	0	60	21	0	0
9	3	3	0	61	21	7	0
10	3	10	0	62	21	14	0
11	3	17	0	63	22	1	0
12	4	4	0	64	22	8	0
13	4	11	0	65	22	15	0
14	4	18	0	66	23	2	0
15	5	5	0	67	23	9	0
16	5	12	0	68	23	16	0
17	5	19	0	69	24	3	0
18	6	6	0	70	24	10	0
19	6	13	0	71	24	17	0
20	7	0	0	72	25	4	0
21	7	7	0	73	25	11	0
22	7	14	0	74	25	18	0
23	8	1	0	75	26	5	0
24	8	8	0	76	26	12	0
25	8	15	0	77	26	19	0
26	9	2	0	78	27	6	0
27	9	9	0	79	27	13	0
28	9	16	0	80	28	0	0
29	10	3	0	81	28	7	0
30	10	10	0	82	28	14	0
31	10	17	0	83	29	1	0
32	11	4	0	84	29	8	0
33	11	11	0	85	29	15	0
34	11	18	0	86	30	2	0
35	12	5	0	87	30	9	0
36	12	12	0	88	30	16	0
37	12	19	0	89	31	3	0
38	13	6	0	90	31	10	0
39	13	13	0	91	31	17	0
40	14	0	0	92	32	4	0
41	14	7	0	93	32	11	0
42	14	14	0	94	32	18	0
43	15	1	0	95	33	5	0
44	15	8	0	96	33	12	0
45	15	15	0	97	33	19	0
46	16	2	0	98	34	6	0
47	16	9	0	99	34	13	0
48	16	16	0	100	35	0	0
49	17	3	0	200	70	0	0
50	17	10	0	300	105	0	0
51	17	17	0	400	140	0	0
52	18	4	0	500	175	0	0

TABLE 27.

Cash Table for the duty on vinegar, at 8s. 9d. per barrel.

Bar.	l.	s.	d.	Bar.	l.	s.	d.
1	0	8	9	53	23	3	9
2	0	17	6	54	23	12	6
3	1	6	3	55	24	1	3
4	1	15	0	56	24	10	0
5	2	3	9	57	24	18	9
6	2	12	6	58	25	7	6
7	3	1	3	59	25	16	3
8	3	10	0	60	26	5	0
9	3	18	9	61	26	13	9
10	4	7	6	62	27	2	6
11	4	16	3	63	27	11	3
12	5	5	0	64	28	0	0
13	5	13	9	65	28	8	9
14	6	2	6	66	28	17	6
15	6	11	3	67	29	6	3
16	7	0	0	68	29	15	0
17	7	8	9	69	30	3	9
18	7	17	6	70	30	12	6
19	8	6	3	71	31	1	3
20	8	15	0	72	31	10	0
21	9	3	9	73	31	18	9
22	9	12	6	74	32	7	6
23	10	1	3	75	32	16	3
24	10	10	0	76	33	5	0
25	10	18	9	77	33	13	9
26	11	7	6	78	34	2	6
27	11	16	3	79	34	11	3
28	12	5	0	80	35	0	0
29	12	13	9	81	35	8	9
30	13	2	6	82	35	17	6
31	13	11	3	83	36	6	3
32	14	0	0	84	36	15	0
33	14	8	9	85	37	3	9
34	14	17	6	86	37	12	6
35	15	6	3	87	38	1	3
36	15	15	0	88	38	10	0
37	16	3	9	89	38	18	9
38	16	12	6	90	39	7	6
39	17	1	3	91	39	16	3
40	17	10	0	92	40	5	0
41	17	18	9	93	40	13	9
42	18	7	6	94	41	2	6
43	18	16	3	95	41	11	3
44	19	5	0	96	42	0	0
45	19	13	9	97	42	8	9
46	20	2	6	98	42	17	6
47	20	11	3	99	43	6	3
48	21	0	0	100	43	15	0
49	21	8	9	200	87	10	0
50	21	17	6	300	131	5	0
51	22	6	3	400	175	0	0
52	22	15	0	500	218	15	0

TABLE 28.

Cash Table for the duty on crown glass, at 14s. per hundred weight.

Cwt.	l.	s.	d.	Cwt.	l.	s.	d.
1	0	14	0	53	37	2	0
2	1	8	0	54	37	16	0
3	2	2	0	55	38	10	0
4	2	16	0	56	39	4	0
5	3	10	0	57	39	18	0
6	4	4	0	58	40	12	0
7	4	18	0	59	41	6	0
8	5	12	0	60	42	0	0
9	6	6	0	61	42	14	0
10	7	0	0	62	43	8	0
11	7	14	0	63	44	2	0
12	8	8	0	64	44	16	0
13	9	2	0	65	45	10	0
14	9	16	0	66	46	4	0
15	10	10	0	67	46	18	0
16	11	4	0	68	47	12	0
17	11	18	0	69	48	6	0
18	12	12	0	70	49	0	0
19	13	6	0	71	49	14	0
20	14	0	0	72	50	8	0
21	14	14	0	73	51	2	0
22	15	8	0	74	51	16	0
23	16	2	0	75	52	10	0
24	16	16	0	76	53	4	0
25	17	10	0	77	53	18	0
26	18	4	0	78	54	12	0
27	18	18	0	79	55	6	0
28	19	12	0	80	56	0	0
29	20	6	0	81	56	14	0
30	21	0	0	82	57	8	0
31	21	14	0	83	58	2	0
32	22	8	0	84	58	16	0
33	23	2	0	85	59	10	0
34	23	16	0	86	60	4	0
35	24	10	0	87	60	18	0
36	25	4	0	88	61	12	0
37	25	18	0	89	62	6	0
38	26	12	0	90	63	0	0
39	27	6	0	91	63	14	0
40	28	0	0	92	64	8	0
41	28	14	0	93	65	2	0
42	29	8	0	94	65	16	0
43	30	2	0	95	66	10	0
44	30	16	0	96	67	4	0
45	31	10	0	97	67	18	0
46	32	4	0	98	68	12	0
47	32	18	0	99	69	6	0
48	33	12	0	100	70	0	0
49	34	6	0	200	140	0	0
50	35	0	0	300	210	0	0
51	35	14	0	400	280	0	0
52	36	8	0	500	350	0	0

TABLE 29.

Cash Table for the duty on sweets, at 18s. per barrel, of $31\frac{1}{2}$ gallons to the barrel.

Bar.	l.	s.	d.	Bar.	l.	s.	d.
1	0	18	0	53	47	14	0
2	1	16	0	54	48	12	0
3	2	14	0	55	49	10	0
4	3	12	0	56	50	8	0
5	4	10	0	57	51	6	0
6	5	8	0	58	52	4	0
7	6	6	0	59	53	2	0
8	7	4	0	60	54	0	0
9	8	2	0	61	54	18	0
10	9	0	0	62	55	16	0
11	9	18	0	63	56	14	0
12	10	16	0	64	57	12	0
13	11	14	0	65	58	10	0
14	12	12	0	66	59	8	0
15	13	10	0	67	60	6	0
16	14	8	0	68	61	4	0
17	15	6	0	69	62	2	0
18	16	4	0	70	63	0	0
19	17	2	0	71	63	18	0
20	18	0	0	72	64	16	0
21	18	18	0	73	65	14	0
22	19	16	0	74	66	12	0
23	20	14	0	75	67	10	0
24	21	12	0	76	68	8	0
25	22	10	0	77	69	6	0
26	23	8	0	78	70	4	0
27	24	6	0	79	71	2	0
28	25	4	0	80	72	0	0
29	26	2	0	81	72	18	0
30	27	0	0	82	73	16	0
31	27	18	0	83	74	14	0
32	28	16	0	84	75	12	0
33	29	14	0	85	76	10	0
34	30	12	0	86	77	8	0
35	31	10	0	87	78	6	0
36	32	8	0	88	79	4	0
37	33	6	0	89	80	2	0
38	34	4	0	90	81	0	0
39	35	2	0	91	81	18	0
40	36	0	0	92	82	16	0
41	36	18	0	93	83	14	0
42	37	16	0	94	84	12	0
43	38	14	0	95	85	10	0
44	39	12	0	96	86	8	0
45	40	10	0	97	87	6	0
46	41	8	0	98	88	4	0
47	42	6	0	99	89	2	0
48	43	4	0	100	90	0	0
49	44	2	0	200	180	0	0
50	45	0	0	300	270	0	0
51	45	18	0	400	360	0	0
52	46	16	0	500	450	0	0

TABLE 30.

Cash Table for the duty on plate, or flint glass, at 18s. 8d. per hundred weight.

Cwt.	l.	s.	d.	Cwt.	l.	s.	d.
1	0	18	8	53	49	9	4
2	1	17	4	54	50	8	0
3	2	16	0	55	51	6	8
4	3	14	8	56	52	5	4
5	4	13	4	57	53	4	0
6	5	12	0	58	54	2	8
7	6	10	8	59	55	1	4
8	7	9	4	60	56	0	0
9	8	8	0	61	56	18	8
10	9	6	8	62	57	17	4
11	10	5	4	63	58	16	0
12	11	4	0	64	59	14	8
13	12	2	8	65	60	13	4
14	13	1	4	66	61	12	0
15	14	0	0	67	62	10	8
16	14	18	8	68	63	9	4
17	15	17	4	69	64	8	0
18	16	16	0	70	65	6	8
19	17	14	8	71	66	5	4
20	18	13	4	72	67	4	0
21	19	12	0	73	68	2	8
22	20	10	8	74	69	1	4
23	21	9	4	75	70	0	0
24	22	8	0	76	70	18	8
25	23	6	8	77	71	17	4
26	24	5	4	78	72	16	0
27	25	4	0	79	73	14	8
28	26	2	8	80	74	13	4
29	27	1	4	81	75	12	0
30	28	0	0	82	76	10	8
31	28	18	8	83	77	9	4
32	29	17	4	84	78	8	0
33	30	16	0	85	79	6	8
34	31	14	8	86	80	5	4
35	32	13	4	87	81	4	0
36	33	12	0	88	82	2	8
37	34	10	8	89	83	1	4
38	35	9	4	90	84	0	0
39	36	8	0	91	84	18	8
40	37	6	8	92	85	17	4
41	38	5	4	93	86	16	0
42	39	4	0	94	87	14	8
43	40	2	8	95	88	13	4
44	41	1	4	96	89	12	0
45	42	0	0	97	90	10	8
46	42	18	8	98	91	9	4
47	43	17	4	99	92	8	0
48	44	16	0	100	93	6	8
49	45	14	8	200	186	13	4
50	46	13	4	300	280	0	0
51	47	12	0	400	373	6	8
52	48	10	8	500	466	13	4

TABLE 31. A TABLE of the RATES of HIDES, &c.

How dressed	SPECIES, and how charged.	Rates. l.	s.	d.	q.
Tanned.	Sheep and lamb, buts and backs, hides, calves and kips, hogs, dogs, and seals, per pound	0	0	1	2
	Roans, per pound	0	0	2	0
	Goats tanned with shomack, per ditto	0	0	4	0
	Skins and pieces, ad valorem, per cent.	30	0	0	0
Tawed.	Sheep and lamb, per pound	0	0	1	1
	Calves, kipps, and seals, per ditto	0	0	1	2
	Buck and doe, per ditto	0	0	6	0
	Calves and slink without hair, dog and kid, per dozen	0	1	0	0
	Beaver and goat, per ditto	0	2	0	0
	Calves slink with hair, per ditto	0	3	0	0
	Horse hides, per hide	0	1	0	0
	All other hides, per ditto	0	3	0	0
	Skins and pieces, &c. ad valorem, per cent.	30	0	0	0
Dressed in oil.	Sheep and lamb, per pound	0	0	3	0
	Hides, deer, goat and beaver, per ditto	0	0	6	0
	Calves skins, per ditto	0	0	8	0
	Skins and pieces, &c. ad valorem, { per ditto	0	0	2	0
	{ per cent.	15	0	0	0
	Vellum, per dozen	0	3	0	0
	Parchment, per ditto	0	1	6	0

To save the officer and trader the trouble of computing the amount of the duty of those goods which pay duty ad valorum, after the rates of 15l. and 30l. per cent. I have inserted the following tables.

TABLE 32.

Ad Valorum TABLE of equal parts, at 15l. per cent. for skins, &c. dressed in oil.

Value. s.	d.	Duty. s.	d.	q.	Value. s.	d.	Duty. s.	d.	q.
0	5	0	0	3	10	5	1	6	3
0	10	0	1	2	10	10	1	7	2
1	3	0	2	1	11	3	1	8	1
1	8	0	3	0	11	8	1	9	0
2	1	0	3	3	12	1	1	9	3
2	6	0	4	2	12	6	1	10	2
2	11	0	5	1	12	11	1	11	1
3	4	0	6	0	13	4	2	0	0
3	9	0	6	3	13	9	2	0	3
4	2	0	7	2	14	2	2	1	2
4	7	0	8	1	14	7	2	2	1
5	0	0	9	0	15	0	2	3	0
5	5	0	9	3	15	5	2	3	3
5	10	0	10	2	15	10	2	4	2
6	3	0	11	1	16	3	2	5	1
6	8	1	0	0	16	8	2	6	0
7	1	1	0	3	17	1	2	6	3
7	6	1	1	2	17	6	2	7	2
7	11	1	2	1	17	11	2	8	1
8	4	1	3	0	18	4	2	9	0
8	9	1	3	3	18	9	2	9	3
9	2	1	4	2	19	2	1	10	2
9	7	1	5	1	19	7	2	11	1
10	0	1	6	0	20	0	3	0	0

TABLE 33.

Ad Valorum TABLE of equal parts, at 30l. per cent. for tanners and tawers.

Value. s.	d.	q.	Duty. s.	d.	q.	Value. s.	d.	q.	Duty. s.	d.	q.
0	2	2	0	0	3	5	2	2	1	6	3
0	5	0	0	1	2	5	5	0	1	7	2
0	7	2	0	2	1	5	7	2	1	8	1
0	10	0	0	3	0	5	10	0	1	9	0
1	0	2	0	3	3	6	0	2	1	9	3
1	3	0	0	4	2	6	3	0	1	10	2
1	5	2	0	5	1	6	5	2	1	11	1
1	8	0	0	6	0	6	8	0	2	0	0
1	10	2	0	6	3	6	10	2	2	0	3
2	1	0	0	7	2	7	1	0	2	1	2
2	3	2	0	8	1	7	3	2	2	2	1
2	6	0	0	9	0	7	6	0	2	3	0
2	8	2	0	9	3	7	8	2	2	3	3
2	11	0	0	10	2	7	11	0	2	4	2
3	1	2	0	11	1	8	1	2	2	5	1
3	4	0	1	0	0	8	4	0	2	6	0
3	6	2	1	0	3	8	6	2	2	6	3
3	9	0	1	1	2	8	9	0	2	7	2
3	11	2	1	2	1	8	11	2	2	8	1
4	2	0	1	3	0	9	2	0	2	9	0
4	4	2	1	3	3	9	4	2	2	9	3
4	7	0	1	4	2	9	7	0	2	10	2
4	9	2	1	5	1	9	9	2	2	11	1
5	0	0	1	6	0	10	0	0	3	0	0

TABLE 34.

TABLE of the value, dimensions, and duty on PAPER.

Denomination.	Value per ream and upwards. l.	s.	d.	Not exceeding the dimensions of Inches.		Inches	Duty. s.	d.
WRITING.								
Imperial, per ream	2	11	0	22	by	$30\frac{1}{4}$	9	0
Super Royal	1	18	0	$19\frac{1}{4}$		$27\frac{1}{2}$	6	9
Royal	1	9	0	$19\frac{1}{4}$		24	5	0
Medium	1	2	6	$17\frac{1}{2}$		$22\frac{1}{2}$	4	0
Demy	0	16	0	$15\frac{1}{2}$		20	2	9
Thick Post	0	13	0	$15\frac{1}{4}$		$19\frac{1}{2}$	2	3
Thin Post	0	10	0	$15\frac{1}{4}$		$19\frac{1}{2}$	1	9
Small Post	0	7	6	$13\frac{1}{2}$		$16\frac{1}{2}$	1	3
Fool's Cap	0	9	0	$13\frac{1}{2}$		$16\frac{1}{2}$	1	6
Pot	0	6	0	$12\frac{1}{2}$		$15\frac{1}{2}$	1	0
WRITING, or Copper-plate Printing.								
Double Atlas	15	0	0	55		$31\frac{1}{2}$	30	0
Demy	0	12	0	$15\frac{1}{2}$		20	1	9
Copy, or Bastard	0	7	6	16		$20\frac{1}{4}$	1	0
Fool's Cap	0	6	0	$13\frac{1}{2}$		$16\frac{3}{4}$	0	10
Littris Fool's Cap	0	6	0	$13\frac{1}{2}$		$17\frac{1}{4}$	0	10
Pot	0	4	0	$12\frac{1}{2}$		$15\frac{1}{2}$	0	8
Grand Eagle, or Double Elephant	4	0	0	$26\frac{3}{4}$		40	11	0
Columbier	2	10	0	$23\frac{1}{2}$		$34\frac{1}{2}$	7	0
Atlas	3	0	0	$26\frac{1}{4}$		34	10	0
Atlas	2	0	0	$26\frac{1}{4}$		34	6	6
Small Atlas	1	10	0	25		31	5	0
Imperial	1	10	0	22		$30\frac{1}{4}$	4	9
Super Royal	1	5	0	$19\frac{1}{2}$		$27\frac{1}{2}$	3	6
Long Royal	1	0	0	$27\frac{1}{2}$		18	3	0
Royal	0	18	0	$19\frac{1}{4}$		24	2	6
Demy	0	13	0	17		22	1	9
Short Demy, or Crowns	0	9	0	14 or 15	or	$20\frac{1}{4}$ or 20	1	3
Large Fan	0	14	0	$23\frac{1}{2}$		$20\frac{1}{2}$	2	0
Small Fan	0	11	0	$22\frac{1}{4}$		$13\frac{1}{4}$	1	6
Elephant	0	15	0	23		28	2	3
Paper for Bank, or Bankers Bills, or Notes, allowing two Bills or Notes in each Sheet, and so in Proportion for any greater or lesser Number of Bills or Notes in each Sheet							2	0
PRINTING.								
Double Demy, per bundle	1	18	0	26		$38\frac{1}{2}$	5	6
Royal	1	4	0	$19\frac{1}{2}$ or 20	or	$24\frac{1}{4}$ or 26	3	6
Royal, inferior	0	14	0	$19\frac{1}{2}$		$24\frac{1}{4}$	2	0
Medium	1	0	0	18		23	2	9
Demy, single	0	17	0	$17\frac{1}{2}$ or $19\frac{1}{4}$	or	22 or $21\frac{1}{2}$	2	6
Demy, inferior	0	10	0	$17\frac{1}{2}$		22	1	6
Double Crown	0	17	0	20		30	2	4
Double Crown, inferior	0	12	0	20		30	1	9
Single Crown	0	13	0	15		20	2	0
Single Crown, inferior	0	8	0	15		20	1	3
Demy Tissue	0	8	0	$17\frac{1}{2}$		22	1	3
Crown Tissue	0	5	0	15		20	0	10
Double Pot	0	9	0	17		$25\frac{1}{2}$	1	6

TABLE of the duty on PAPER, &c. continued.

Denomination.	Not exceeding the dimensions of: Inches.	Inches.	Duty. s.	d.
ORDINARY and COLOURED.				
Cartridge, per ream	21 by	26	1	9
Cartridge, square	24½	25½	2	0
Cartridge	19¼	24	1	6
Elephant, common	23	28	1	3
Sugar Blue	21½	33	2	0
Sugar Blue, smaller size	18¾	27	1	6
Sugar Blue, demy size	17½	22	1	3
Sugar Blue, crown size	15	20	1	3
Purple Royal	19½	24¼	1	0
Blue Elephant	23	28	1	6
Blue Royal, per bundle	19½	24¼	2	0
Blue Demy and Blossom	17	22	1	3
Blue Crown, single	15	20	0	9
Whited BROWNS, and BROWNS.				
Royal Hand Thick, per ream	24	19½	0	10
Royal Hand, per bundle	24	19½	1	0
Lumber Hand	23	18	1	0
Double Two Pound	24	16	0	9
Single Two Pound	16	11	0	4
Middle Hand Double	33	21	1	6
Middle Hand	22	16	0	9
Small Hand Double	32	20	1	0
Small Hand	19½	16	0	6
Couples, Pound and Half Pound	12 and 9	10 and 7½	0	4
Imperial Cap, per ream	29	22	1	0
Havon Cap	24	20	0	9
Bag Cap	23½	19	0	8
Kentish Cap	21	18	0	6
Four Pounds	20	16	0	6
Small Cap	20	15	0	4
Double Four Pounds	33	20	1	0
Single Two Pounds, per bundle	16	11	0	6
Couples, Pound and Half Pound	12 and 9	10 and 7½	0	4
Pasteboard, Millboard, and Scaleboard, and Glazed Paper, for Clothiers and Hot Pressers, per Cwt. —			4	6

Note. All the several and respective rates and duties in the preceding Table, on paper, pasteboard, millboard, and scaleboard, are subject to an additional duty of 10 per Cent. on the produce thereof.

All other paper that shall happen to be made of a new kind or fabrick, of any colour or kind, which is not in the foregoing Table, is to pay a Duty of 18l. per Cent. ad valorem.

A bundle of paper contains two reams.

TABLE 35.

Ad valorum TABLE, of equal Parts, at 18l. per Cent. for the duty on Paper.

Value l.	Value s.	Value d.	Duty l.	Duty s.	Duty d.	Duty q.	100 pts.
0	0	0¼	0	0	0	0	18
0	0	0½	0	0	0	0	36
0	0	0¾	0	0	0	0	54
0	0	1	0	0	0	0	72
0	0	1½	0	0	0	0	08
0	0	2	0	0	0	1	44
0	0	2½	0	0	0	1	80
0	0	3	0	0	0	2	16
0	0	3½	0	0	0	2	52
0	0	4	0	0	0	2	88
0	0	4½	0	0	0	3	24
0	0	5	0	0	0	3	60
0	0	5½	0	0	1	3	96
0	0	6	0	0	1	0	32
0	0	6½	0	0	1	0	68
0	0	7	0	0	1	1	04
0	0	7½	0	0	1	1	40
0	0	8	0	0	1	1	76
0	0	8½	0	0	1	2	12
0	0	9	0	0	1	2	48
0	0	9½	0	0	1	2	84
0	0	10	0	0	1	3	20
0	0	10½	0	0	1	3	56
0	0	11	0	0	1	3	92
0	0	11½	0	0	2	0	28
0	1	0	0	0	2	0	64
0	2	0	0	0	4	1	28
0	3	0	0	0	6	1	92
0	4	0	0	0	8	2	56
0	5	0	0	0	10	3	20
0	6	0	0	1	0	3	84
0	7	0	0	1	3	0	48
0	8	0	0	1	5	1	12
0	9	0	0	1	7	1	76
0	10	0	0	1	9	2	40
0	11	0	0	1	11	3	04
0	12	0	0	2	1	3	68
0	13	0	0	2	4	0	32
0	14	0	0	2	6	0	96
0	15	0	0	2	8	1	60
0	16	0	0	2	10	2	24
0	17	0	0	3	0	2	88
0	18	0	0	3	2	3	52
0	19	0	0	3	5	0	16
							10 pts.
1	0	0	0	3	7	0	8
2	0	0	0	7	2	1	6
3	0	0	0	10	9	2	4
4	0	0	0	14	4	3	2
5	0	0	0	18	0	0	0
6	0	0	1	1	7	0	8

Val. £.	Duty l.	Duty s.	Duty d.	Duty q.	10 pts.
7	1	5	2	1	6
8	1	8	9	2	4
9	1	12	4	3	2
10	1	16	0	0	0
11	1	19	7	0	8
12	2	3	2	1	6
13	2	6	9	2	4
14	2	10	4	3	2
15	2	14	0	0	0
16	2	17	7	0	8
17	3	1	2	1	6
18	3	4	9	2	4
19	3	8	4	3	2
20	3	12	0	0	0
21	3	15	7	0	8
22	3	19	2	1	6
23	4	2	9	2	4
24	4	6	4	3	2
25	4	10	0	0	0
26	4	13	7	0	8
27	4	17	2	1	6
28	5	0	9	2	4
29	5	4	4	3	2
30	5	8	0	0	0
31	5	11	7	0	8
32	5	15	2	1	6
33	5	18	9	2	4
34	6	2	4	3	2
35	6	6	0	0	0
36	6	9	7	0	8
37	6	13	2	1	6
38	6	16	9	2	4
39	7	0	4	3	2
40	7	4	0	0	0
41	7	7	7	0	8
42	7	11	2	1	6
43	7	14	9	2	4
44	7	18	4	3	2
45	8	2	0	0	0
46	8	5	7	0	8
47	8	9	2	1	6
48	8	12	9	2	4
49	8	16	4	3	2
50	9	0	0	0	0
51	9	3	7	0	8
52	9	7	2	1	6
53	9	10	9	2	4
54	9	14	4	3	2
55	9	18	0	0	0

Val. £.	Duty l.	Duty s.	Duty d.	Duty q.	10 pts.
56	10	1	7	0	8
57	10	5	2	1	6
58	10	8	9	2	4
59	10	12	4	3	2
60	10	16	0	0	0
61	10	19	7	0	8
62	11	3	2	1	6
63	11	6	9	2	4
64	11	10	4	3	2
65	11	14	0	0	0
66	11	17	7	0	8
67	12	1	2	1	6
68	12	4	9	2	4
69	12	8	4	3	2
70	12	12	0	0	0
71	12	15	7	0	8
72	12	19	2	1	6
73	13	2	9	2	4
74	13	6	4	3	2
75	13	10	0	0	0
76	13	13	7	0	8
77	13	17	2	1	6
78	14	0	9	2	4
79	14	4	4	3	2
80	14	8	0	0	0
81	14	11	7	0	8
82	14	15	2	1	6
83	14	18	9	2	4
84	15	2	4	3	2
85	15	6	0	0	0
86	15	9	7	0	8
87	15	13	2	1	6
88	15	16	9	2	4
89	16	0	4	3	2
90	16	4	0	0	0
91	16	7	7	0	8
92	16	11	2	1	6
93	16	14	9	2	4
94	16	18	4	3	2
95	17	2	0	0	0
96	17	5	7	0	8
97	17	9	2	1	6
98	17	12	9	2	4
99	17	16	4	3	2
100	18	0	0	0	0
200	36	0	0	0	0
300	54	0	0	0	0
400	72	0	0	0	0

TABLE 36. For reducing hundreds, quarters, and pounds, into pounds grofs; and pounds grofs into pounds net; the allowance for the bag or tare, which is one pound in ten pounds, being deducted.

Hund.	Quar.	Pounds	Grofs weight in lbs.	Net wt. in lbs. & 10ths.	Hund.	Quar.	Pounds	Grofs weight in lbs.	Net wt. in lbs. & 10ths.	Hund.	Quar.	Pounds	Grofs weight in lbs.	Net wt. in lbs. & 10ths.
			lb.	lb. 10th.					lb. 10th.				lb.	lb. 10th
0	0	0	1	0,9	0	1	20	48	43,2	0	3	11	95	85,5
0	0	0	2	1,8	0	1	21	49	44,1	0	3	12	96	86,4
0	0	0	3	2,7	0	1	22	50	45,0	0	3	13	97	87,3
0	0	0	4	3,6	0	1	23	51	45,9	0	3	14	98	88,2
0	0	0	5	4,5	0	1	24	52	46,8	0	3	15	99	89,1
0	0	0	6	5,4	0	4	25	53	47,7	0	3	16	100	90,0
0	0	0	7	6,3	0	1	20	54	48,6	0	3	17	101	90,9
0	0	0	8	7,2	0	1	27	55	49,5	0	3	18	102	91,8
0	0	0	9	8,1	0	2	0	56	50,4	0	3	19	103	92,7
0	0	0	10	9,0	0	2	1	57	51,3	0	3	20	104	93,6
0	0	0	11	9,9	0	2	2	58	52,2	0	3	21	105	94,5
0	0	0	12	10,8	0	2	3	59	53,1	0	3	22	106	95,4
0	0	0	13	11,7	0	2	4	60	54,0	0	3	23	107	96,3
0	0	0	14	12,6	0	2	5	61	54,9	0	3	24	108	97,2
0	0	0	15	13,5	0	2	6	62	55,8	0	3	25	109	98,1
0	0	0	16	14,4	0	2	7	63	56,7	0	3	26	110	99,0
0	0	0	17	15,3	0	2	8	64	57,6	0	3	27	111	99,9
0	0	0	18	16,2	0	2	9	65	58,5	1	0	0	112	100,8
0	0	0	19	17,1	0	2	10	66	59,4	2	0	0	224	201,6
0	0	0	20	18,0	0	2	11	67	60,3	3	0	0	236	302,4
0	0	0	21	18,9	0	2	12	68	61,2	4	0	0	448	403,2
0	0	0	22	19,8	0	2	13	69	62,1	5	0	0	560	504,0
0	0	0	23	20,7	0	2	14	70	63,0	6	0	0	672	604,8
0	0	0	24	21,6	0	2	15	71	63,9	7	0	0	784	705,6
0	0	0	25	22,5	0	2	16	72	64,8	8	0	0	896	806,4
0	0	0	26	23,4	0	2	17	73	65,7	9	0	0	1008	907,2
0	0	0	27	24,3	0	2	18	74	66,6	10	0	0	1120	1008,0
0	1	0	28	25,2	0	2	19	75	67,5	11	0	0	1232	1108,8
0	1	1	29	26,1	0	2	20	76	68,4	12	0	0	1344	1209,6
0	1	2	30	27,0	0	2	21	77	69,3	13	0	0	1456	1310,4
0	1	3	31	27,9	0	2	22	78	70,2	14	0	0	1568	1411,2
0	1	4	32	28,8	0	2	23	79	71,1	15	0	0	1680	1512,0
0	1	5	33	29,7	0	2	24	80	72,0	16	0	0	1792	1612,8
0	1	6	34	30,6	0	2	25	81	72,9	17	0	0	1904	1713,6
0	1	7	35	31,5	0	2	26	82	73,8	18	0	0	2016	1814,4
0	1	8	36	32,4	0	2	27	83	74,7	19	0	0	2128	1915,2
0	1	9	37	33,3	0	3	0	84	75,6	20	0	0	2240	2016,0
0	1	10	38	34,2	0	3	1	85	76,5	30	0	0	3360	3024,0
0	1	11	39	35,1	0	3	2	86	77,4	40	0	0	4480	4032,0
0	1	12	40	36,0	0	3	3	87	78,3	50	0	0	5600	5040,0
0	1	13	41	36,9	0	3	4	88	79,2	60	0	0	6720	6048,0
0	1	14	42	37,8	0	3	5	89	80,1	70	0	0	7840	7056,0
0	1	15	43	38,7	0	3	6	90	81,0	80	0	0	8960	8064,0
0	1	16	44	39,6	0	3	7	91	81,9	90	0	0	10080	9072,0
0	1	17	45	40,5	0	3	8	92	82,8	100	0	0	11200	10080,0
0	1	18	46	41,4	0	3	9	93	83,7	200	0	0	22400	20160,0
0	1	19	47	42,3	0	3	10	94	84,6	300	0	0	33600	30240,0

T A B L E 37.

A New TABLE, to find the number of days, from the 5th of July to any given day of any month, by inspection.

July	Days	Aug.	Days	Sept.	Days	Oct.	Days	Nov.	Days	Dec.	Days	Jan.	Days	Feb.	Days	Mar.	Days	Apr.	Days	May	Days	June	Days
1	361	1	27	1	58	1	88	1	119	1	149	1	180	1	211	1	239	1	270	1	300	1	331
2	362	2	28	2	59	2	89	2	120	2	150	2	181	2	212	2	240	2	271	2	301	2	332
3	363	3	29	3	60	3	90	3	121	3	151	3	182	3	213	3	241	3	272	3	302	3	333
4	364	4	30	4	61	4	91	4	122	4	152	4	183	4	214	4	242	4	273	4	303	4	334
5	365	5	31	5	62	5	92	5	123	5	153	5	184	5	215	5	243	5	274	5	304	5	335
6	1	6	32	6	63	6	93	6	124	6	154	6	185	6	216	6	244	6	275	6	305	6	336
7	2	7	33	7	64	7	94	7	125	7	155	7	186	7	217	7	245	7	276	7	306	7	337
8	3	8	34	8	65	8	95	8	126	8	156	8	187	8	218	8	246	8	277	8	307	8	338
9	4	9	35	9	66	9	96	9	127	9	157	9	188	9	219	9	247	9	278	9	308	9	339
10	5	10	36	10	67	10	97	10	128	10	158	10	189	10	220	10	248	10	279	10	309	10	340
11	6	11	37	11	68	11	98	11	129	11	159	11	190	11	221	11	249	11	280	11	310	11	341
12	7	12	38	12	69	12	99	12	130	12	160	12	191	12	222	12	250	12	281	12	311	12	342
13	8	13	39	13	70	13	100	13	131	13	161	13	192	13	223	13	251	13	282	13	312	13	343
14	9	14	40	14	71	14	101	14	132	14	162	14	193	14	224	14	252	14	283	14	313	14	344
15	10	15	41	15	72	15	102	15	133	15	163	15	194	15	225	15	253	15	284	15	314	15	345
16	11	16	42	16	73	16	103	16	134	16	164	16	195	16	226	16	254	16	285	16	315	16	346
17	12	17	43	17	74	17	104	17	135	17	165	17	196	17	227	17	255	17	286	17	316	17	347
18	13	18	44	18	75	18	105	18	136	18	166	18	197	18	228	18	256	18	287	18	317	18	348
19	14	19	45	19	76	19	106	19	137	19	167	19	198	19	229	19	257	19	288	19	318	19	349
20	15	20	46	20	77	20	107	20	138	20	168	20	199	20	230	20	258	20	289	20	319	20	350
21	16	21	47	21	78	21	108	21	139	21	169	21	200	21	231	21	259	21	290	21	320	21	351
22	17	22	48	22	79	22	109	22	140	22	170	22	201	22	232	22	260	22	291	22	321	22	352
23	18	23	49	23	80	23	110	23	141	23	171	23	202	23	233	23	261	23	292	23	322	23	353
24	19	24	50	24	81	24	111	24	142	24	172	24	203	24	234	24	262	24	293	24	323	24	354
25	20	25	51	25	82	25	112	25	143	25	173	25	204	25	235	25	263	25	294	25	324	25	355
26	21	26	52	26	83	26	113	26	144	26	174	26	205	26	236	26	264	26	295	26	325	26	356
27	22	27	53	27	84	27	114	27	145	27	175	27	206	27	237	27	265	27	296	27	326	27	357
28	23	28	54	28	85	28	115	28	146	28	176	28	207	28	238	28	266	28	297	28	327	28	358
29	24	29	55	29	86	29	116	29	147	29	177	29	208			29	267	29	298	29	328	29	359
30	25	30	56	30	87	30	117	30	148	30	178	30	209			30	268	30	299	30	329	30	360
31	26	31	57			31	118			31	179	31	210			31	269			31	330		

TABLE 38.

TABLE to convert gallons into barrels, and contra, barrels into gallons; each barrel containing 34 gallons.

Gal.	Bar.Gal	Gal.	Bar.Gal	Gal.	Bar.Gal	Gal.	Bar.Gal	Gal.	Bar.Gal
8,5	¼ 0,0	33,5	¼ 8	58,5	1½ 7,5	83,5	2¼ 7	108,5	3 6,5
9	¼ 0,5	34	1 0,0	59	1½ 8	84	2¼ 7,5	109	3 7
9,5	¼ 1	34,5	1 0,5	59,5	1¾ 0,0	84,5	2¼ 8	109,5	3 7,5
10	¼ 1,5	35	1 1	60	1¾ 0,5	85	2½ 0,0	110	3 8
10,5	¼ 2	35,5	1 1,5	60,5	1¾ 1	85,5	2½ 0,5	110,5	3¼ 0,0
11	¼ 2,5	36	1 2	61	1¾ 1,5	86	2½ 1	111	3¼ 0,5
11,5	¼ 3	36,5	1 2,5	61,5	1¾ 2	86,5	2½ 1,5	111,5	3¼ 1
12	¼ 3,5	37	1 3	62	1¾ 2,5	87	2½ 2	112	3¼ 1,5
12,5	¼ 4	37,5	1 3,5	62,5	1¾ 3	87,5	2½ 2,5	112,5	3¼ 2
13	¼ 4,5	38	1 4	63	1¾ 3,5	88	2½ 3	113	3¼ 2,5
13,5	¼ 5	38,5	1 4,5	63,5	1¾ 4	88,5	2½ 3,5	113,5	3¼ 3
14	¼ 5,5	39	1 5	64	1¾ 4,5	89	2½ 4	114	3¼ 3,5
14,5	¼ 6	39,5	1 5,5	64,5	1¾ 5	89,5	2½ 4,5	114,5	3¼ 4
15	¼ 6,5	40	1 6	65	1¾ 5,5	90	2½ 5	115	3¼ 4,5
15,5	¼ 7	40,5	1 6,5	65,5	1¾ 6	90,5	2½ 5,5	115,5	3¼ 5
16	¼ 7,5	41	1 7	66	1¾ 6,5	91	2½ 6	116	3¼ 5,5
16,5	¼ 8	41,5	1 7,5	66,5	1¾ 7	91,5	2½ 6,5	116,5	3¼ 6
17	½ 0,0	42	1 8	67	1¾ 7,5	92	2½ 7	117	3¼ 6,5
17,5	½ 0,5	42,5	1¼ 0,0	67,5	1¾ 8	92,5	2½ 7,5	117,5	3¼ 7
18	½ 1	43	1¼ 0,5	68	2 0,0	93	2½ 8	118	3¼ 7,5
18,5	½ 1,5	43,5	1¼ 1	68,5	2 0,5	93,5	2¾ 0,0	118,5	3¼ 8
19	½ 2	44	1¼ 1,5	69	2 1	94	2¾ 0,5	119	3½ 0,0
19,5	½ 2,5	44,5	1¼ 2	69,5	2 1,5	94,5	2¾ 1	119,5	3½ 0,5
20	½ 3	45	1¼ 2,5	70	2 2	95	2¾ 1,5	120	3½ 1
20,5	½ 3,5	45,5	1¼ 3	70,5	2 2,5	95,5	2¾ 2	120,5	3½ 1,5
21	½ 4	46	1¼ 3,5	71	2 3	96	2¾ 2,5	121	3½ 2
21,5	½ 4,5	46,5	1¼ 4	71,5	2 3,5	96,5	2¾ 3	121,5	3½ 2,5
22	½ 5	47	1¼ 4,5	72	2 4	97	2¾ 3,5	122	3½ 3
22,5	½ 5,5	47,5	1¼ 5	72,5	2 4,5	97,5	2¾ 4	122,5	3½ 3,5
23	½ 6	48	1¼ 5,5	73	2 5	98	2¾ 4,5	123	3½ 4
23,5	½ 6,5	48,5	1¼ 6	73,5	2 5,5	98,5	2¾ 5	123,5	3½ 4,5
24	½ 7	49	1¼ 6,5	74	2 6	99	2¾ 5,5	124	3½ 5
24,5	½ 7,5	49,5	1¼ 7	74,5	2 6,5	99,5	2¾ 6	124,5	3½ 5,5
25	½ 8	50	1¼ 7,5	75	2 7	100	2¾ 6,5	125	3½ 6
25,5	¾ 0,0	50,5	1¼ 8	75,5	2 7,5	100,5	2¾ 7	125,5	3½ 6,5
26	¾ 0,5	51	1½ 0,0	76	2 8	101	2¾ 7,5	126	3½ 7
26,5	¾ 1	51,5	1½ 0,5	76,5	2¼ 0,0	101,5	2¾ 8	126,5	3½ 7,5
27	¾ 1,5	52	1½ 1	77	2¼ 0,5	102	3 0,0	127	3½ 8
27,5	¾ 2	52,5	1½ 1,5	77,5	2¼ 1	102,5	3 0,5	127,5	3¾ 0,0
28	¾ 2,5	53	1½ 2	78	2¼ 1,5	103	3 1	128	3¾ 0,5
28,5	¾ 3	53,5	1½ 2,5	78,5	2¼ 2	103,5	3 1,5	128,5	3¾ 1
29	¾ 3,5	54	1½ 3	79	2¼ 2,5	104	3 2	129	3¾ 1,5
29,5	¾ 4	54,5	1½ 3,5	79,5	2¼ 3	104,5	3 2,5	129,5	3¾ 2
30	¾ 4,5	55	1½ 4	80	2¼ 3,5	105	3 3	130	3¾ 2,5
30,5	¾ 5	55,5	1½ 4,5	80,5	2¼ 4	105,5	3 3,5	130,5	3¾ 3
31	¾ 5,5	56	1½ 5	81	2¼ 4,5	106	3 4	131	3¾ 3,5
31,5	¾ 6	56,5	1½ 5,5	81,5	2¼ 5	106,5	4 4,5	131,5	3¾ 4
32	¾ 6,5	57	1½ 6	82	2¼ 5,5	107	3 5	132	3¾ 4,5
32,5	¾ 7	57,5	1½ 6,5	82,5	2¼ 6	107,5	3 5,5	132,5	3¾ 5
33	¾ 7,5	58	1½ 7	83	2¼ 6,5	108	3 6	133	3¾ 5,5

TABLE to convert gallons into barrels, and contra, continued.

Gal.	Bar.	Gal	Gal.	Bar.	Gal	Gal.	Barr.	Gal	Gal.	Bar.	Gal.	Bar.
133,5	3¾	6	158,5	4½	5,5	183,5	5¼	5	280,5	8¼	705,5	20¾
134	3¾	6,5	159	4½	6	184	5¼	5,5	289	8½	714	21
134,5	3¾	7	159,5	4½	6,5	184,5	5¼	6	297,5	8¾	722,5	21¼
135	3¾	7,5	160	4½	7	185	5¼	6,5	306	9	731	21½
135,5	3¾	8	160,5	4½	7,5	185,5	5¼	7	314,5	9¼	739,5	21¾
136	4	0,0	161	4½	8	186	5¼	7,5	323	9½	748	22
136,5	4	0,5	161,5	4¾	0.0	186,5	5¼	8	331,5	9¾	756,5	22¼
137	4	1	162	4¾	0,5	187	5½	0 0	340	10	765	22½
137,5	4	1,5	162,5	4¾	1	187,5	5½	0,5	348,5	10¼	773,5	22¾
138	4	2	163	4¾	1,5	188	5½	1	357	10½	782	23
138,5	4	2,5	163,5	4¾	2	188,5	5½	1,5	365,5	10¾	790,5	23¼
139	4	3	164	4¾	2,5	189	5½	2	374	11	799	23½
139,5	4	3,5	164,5	4¾	3	189,5	5½	2,5	382,5	11¼	807,5	23¾
140	4	4	165	4¾	3,5	190	5½	3	391	11½	816	24
140,5	4	4,5	165,5	4¾	4	190,5	5½	3,5	399,5	11¾	824,5	24¼
141	4	5	166	4¾	4,5	191	5½	4	408	12	833	24½
141,5	4	5,5	166,5	4¾	5	191,5	5½	4,5	416,5	12¼	841,5	24¾
142	4	6	167	4¾	5,5	192	5½	5	425	12½	850	25
142,5	4	6,5	167,5	4¾	6	192,5	5½	5,5	433,5	12¾	858,5	25¼
143	4	7	168	4¾	6,5	193	5½	6	442	13	867	25½
143,5	4	7,5	168,5	4¾	7	193,5	5½	6,5	450,5	13¼	875,5	25¾
144	4	8	169	4¾	7,5	194	5½	7	459	13½	884	26
144,5	4¼	0 0	169,5	4¾	8	194,5	5½	7,5	467,5	13¾	892,5	26¼
145	4¼	0,5	170	5	0,0	195	5½	8	476	14	901	26½
145,5	4¼	1	170,5	5	0,5	195,5	5¾	0,0	484,5	14¼	909,5	26¾
146	4¼	1,5	171	5	1	196	5¾	0,5	493	14½	918	27
146,5	4¼	2	171,5	5	1,5	196,5	5¾	1	501,5	14¾	926,5	27¼
147	4¼	2,5	172	5	2	197	5¾	1,5	510	15	935	27½
147,5	4¼	3	172,5	5	2,5	197,5	5¾	2	518,5	15¼	943.5	27¾
148	4¼	3,5	173	5	3	198	5¾	2,5	527	15½	952	28
148,5	4¼	4	173,5	5	3,5	198,5	5¾	3	535,5	15¾	960,5	28¼
149	4¼	4,5	174	5	4	199	5¾	3,5	544	16	969	28½
149,5	4¼	5	174,5	5	4,5	199,5	5¾	4	552,5	16¼	977,5	28¾
150	4¼	5,5	175	5	5	200	5¾	4,5	561	16½	986	29
150,5	4¼	6	175,5	5	5,5	200,5	5¾	5	569,5	16¾	994,5	29¼
151	4¼	6,5	176	5	6	201	5¾	5,5	578	17	1003	29½
151,5	4¼	7	176,5	5	6,5	201,5	5¾	6	586,5	17¼	1011,5	29¾
152	4¼	7,5	177	5	7	202	5¾	6,5	595	17½	1020	30
152,5	4¼	8	177,5	5	7,5	202,5	5¾	7	603,5	17¾	1028,5	30¼
153	4½	0 0	178	5	8	203	5¾	7,5	612	18	1037	30½
153,5	4½	0,5	178,5	5¼	0 0	203,5	5¾	8	620,5	18¼	1045,5	30¾
154	4½	1	179	5¼	0,5	204	6		629	18½	1054	31
154,5	4½	1,5	179,5	5¼	1	212,5	6¼		637,5	18¾	1062,5	31¼
155	4½	2	180	5¼	1,5	221	6½		646	19	1071	31½
155,5	4½	2,5	180,5	5¼	2	229,5	6¾		654,5	19¼	1079,5	31¾
156	4½	3	181	5¼	2,5	238	7		663	19½	1088	32
156,5	4½	3,5	181,5	5¼	3	246,5	7¼		671,5	19¾	1096,5	32¼
157	4½	4	182	5¼	3,5	255	7½		680	20	1105	32½
157,5	4½	4,5	182,5	5¼	4	263,5	7¾		688,5	20¼	1113,5	32¾
158	4½	5	183	5¼	4,5	272	8		697	20½	1122	33

TABLE 39. To find the Number of DAYS from one day to any other in the same year, or to any day of any month in the next year, by one subtraction only.

Days	January	February	March	April	May	June
1	730 , 365	699 , 334	671 , 306	640 , 275	610 , 245	579 , 214
2	729 , 364	698 , 333	670 , 305	639 , 274	609 , 244	578 , 213
3	728 , 363	697 , 332	669 , 304	638 , 273	608 , 243	577 , 212
4	727 , 362	696 , 331	668 , 303	637 , 272	607 , 242	576 , 211
5	726 , 361	695 , 330	667 , 302	636 , 271	606 , 241	575 , 210
6	725 , 360	694 , 329	666 , 301	635 , 270	605 , 240	574 , 209
7	724 , 359	693 , 328	665 , 300	634 , 269	604 , 239	573 , 208
8	723 , 358	692 , 327	664 , 299	633 , 268	603 , 238	572 , 207
9	722 , 357	691 , 326	663 , 298	632 , 267	602 , 237	571 , 206
10	721 , 356	690 , 325	662 , 297	631 , 266	601 , 236	570 , 205
11	720 , 355	689 , 324	661 , 296	630 , 265	600 , 235	569 , 204
12	719 , 354	688 , 323	660 , 295	629 , 264	599 , 234	568 , 203
13	718 , 353	687 , 322	659 , 294	628 , 263	598 , 233	567 , 202
14	717 , 352	686 , 321	658 , 293	627 , 262	597 , 232	566 , 201
15	716 , 351	685 , 320	657 , 292	626 , 261	596 , 231	565 , 200
16	715 , 350	684 , 319	656 , 291	625 , 260	595 , 230	564 , 199
17	714 , 349	683 , 318	655 , 290	624 , 259	594 , 229	563 , 198
18	713 , 348	682 , 317	654 , 289	623 , 258	593 , 228	562 , 197
19	712 , 347	681 , 316	653 , 288	622 , 257	592 , 227	561 , 196
20	711 , 346	680 , 315	652 , 287	621 , 256	591 , 226	560 , 195
21	710 , 345	679 , 314	651 , 286	620 , 255	590 , 225	559 , 194
22	709 , 344	678 , 313	650 , 285	619 , 254	589 , 224	558 , 193
23	708 , 343	677 , 312	649 , 284	618 , 253	588 , 223	557 , 192
24	707 , 342	676 , 311	648 , 283	617 , 252	587 , 222	556 , 191
25	706 , 341	675 , 310	647 , 282	616 , 251	586 , 221	555 , 190
26	705 , 340	674 , 309	646 , 281	615 , 250	585 , 220	554 , 189
27	704 , 339	673 , 308	645 , 280	614 , 249	584 , 219	553 , 188
28	703 , 338	672 , 307	644 , 279	613 , 248	583 , 218	552 , 187
29	702 , 337		643 , 278	612 , 247	582 , 217	551 , 186
30	701 , 336		642 , 277	611 , 246	581 , 216	550 , 185
31	700 , 335		641 , 276	——	580 , 215	——

The EXPLANATION and USE of this TABLE.

In this table, at each day, are two sets of figures; as for instance, against May 13, you have 598 and 233, parted with a comma; now suppose it was required to find how many days from May 13 to December 5? In this case, and all others, take the former figures, if they can be subtracted; if not, take the latter. Thus, against May 13 is - - - 598
And against Dec. 5 is - - - 392

Answer - - 206 Days.

Ex. 2. How many days are there from September 7, to December 31?
Against September 7 is - - - 481
Against December 31 is - - - 366

Answer - - 115 Days.

TABLE to find the Number of DAYS continued.

Days	July	August	September	October	November	December
1	549 , 184	518 , 153	487 , 122	457 , 92	426 , 61	396 , 31
2	548 , 183	517 , 152	486 , 121	456 , 91	425 , 60	395 , 30
3	547 , 182	516 , 151	485 , 120	455 , 90	424 , 59	394 , 29
4	546 , 181	515 , 150	484 , 119	454 , 89	423 , 58	393 , 28
5	545 , 180	514 , 149	483 , 118	453 , 88	422 , 57	392 , 27
6	544 , 179	513 , 148	482 , 117	452 , 87	421 , 56	391 , 26
7	543 , 178	512 , 147	481 , 116	451 , 86	420 , 55	390 , 25
8	542 , 177	511 , 146	480 , 115	450 , 85	419 , 54	389 , 24
9	541 , 176	510 , 145	479 , 114	449 , 84	418 , 53	388 , 23
10	540 , 175	509 , 144	478 , 113	448 , 83	417 , 52	387 , 22
11	539 , 174	508 , 143	477 , 112	447 , 82	416 , 51	386 , 21
12	538 , 173	507 , 142	476 , 111	446 , 81	415 , 50	385 , 20
13	537 , 172	506 , 141	475 , 110	445 , 80	414 , 49	384 , 19
14	536 , 171	505 , 140	474 , 109	444 , 79	413 , 48	383 , 18
15	535 , 170	504 , 139	473 , 108	443 , 78	412 , 47	382 , 17
16	534 , 169	503 , 138	472 , 107	442 , 77	411 , 46	381 , 16
17	533 , 168	502 , 137	471 , 106	441 , 76	410 , 45	380 , 15
18	532 , 167	501 , 136	470 , 105	440 , 75	409 , 44	379 , 14
19	531 , 166	500 , 135	469 , 104	439 , 74	408 , 43	378 , 13
20	530 , 165	499 , 134	468 , 103	438 , 73	407 , 42	377 , 12
21	529 , 164	498 , 133	467 , 102	437 , 72	406 , 41	376 , 11
22	528 , 163	497 , 132	466 , 101	436 , 71	405 , 40	375 , 10
23	527 , 162	496 , 131	465 , 100	435 , 70	404 , 39	374 , 9
24	526 , 161	495 , 130	464 , 99	434 , 69	403 , 38	373 , 8
25	525 , 160	494 , 129	463 , 98	433 , 68	402 , 37	372 , 7
26	524 , 159	493 , 128	462 , 97	432 , 67	401 , 36	371 , 6
27	523 , 158	492 , 127	461 , 96	431 , 66	400 , 35	370 , 5
28	522 , 157	491 , 126	460 , 95	430 , 65	399 , 34	369 , 4
29	521 , 156	490 , 125	459 , 94	429 , 64	398 , 33	368 , 3
30	520 , 155	489 , 124	458 , 93	428 , 63	397 , 32	367 , 2
31	519 , 154	488 , 123	———	427 , 62	———	366 , 1

Ex. 3. How many days are there from Sept. 7, 1782, to April 19, 1783?

Againſt September 7 is - - - 481
Againſt April 19 is - - - - 257

Anſwer - 224 Days.

Note. In this example, the two former numbers againſt September 7 and April 19, could not be ſubtracted, becauſe the number againſt April 19 was the largeſt; therefore I made uſe of the latter, as may be ſeen in the above example. The ſame rule is to be obſerved in all reſpects, where the former numbers cannot be ſubtracted.

How uſeful a table this is, in all parts of arithmetical ſcience relating to time, is ſufficiently evident to the ſkilful therein; but becauſe it is more particularly ſo for collectors clerks, ſupervifors, and officers of exciſe, I have prefixed it to the ſalary tables, in order to make them the more complete and uſeful.

TABLE 40. A TABLE of SALARIES, for collectors, supervisors, officers, &c. at 10l.—25l.—40l.—50l.—90l. and 120l. per annum, for the common year, or 365 days.

Days	At 10l. l.	s.	d.	q. 73p	Days	At 10l. l.	s.	d.	q. 73p	Days	At 25l. l.	s.	d.	q. 73p	Days	At 25l. l.	s.	d.	q. 73p
1	0	0	6	2,22	52	1	8	5	3,49	1	0	1	4	1,55	52	3	11	2	3,13
2	0	1	1	0,44	53	1	9	0	1,71	2	0	2	8	3,37	53	3	12	7	0,68
3	0	1	7	2,66	54	1	9	7	0,20	3	0	4	1	1,19	54	3	13	11	2,50
4	0	2	2	1,15	55	1	10	1	2,42	4	0	5	5	3,01	55	3	15	4	0,32
5	0	2	8	3,37	56	1	10	8	0,64	5	0	6	10	0,56	56	3	16	8	2,14
6	0	3	3	1,59	57	1	11	2	3,13	6	0	8	2	2,38	57	3	18	0	3,69
7	0	3	10	0,08	58	1	11	9	1,35	7	0	9	7	0,20	58	3	19	5	1,51
8	0	4	4	2,30	59	1	12	3	3,57	8	0	10	11	2,02	59	4	0	9	3,33
9	0	4	11	0,52	60	1	12	10	2,06	9	0	12	3	3,57	60	4	2	2	1,15
10	0	5	5	3,01	61	1	13	5	0,28	10	0	13	8	1,39	61	4	3	6	2,70
11	0	6	0	1,23	62	1	13	11	2,50	11	0	15	0	3,21	62	4	4	11	0,52
12	0	6	6	3,45	63	1	14	6	0,72	12	0	16	5	1,03	63	4	6	3	2,34
13	0	7	1	1,67	64	1	15	0	3,21	13	0	17	9	2,58	64	4	7	8	0,16
14	0	7	8	0,16	65	1	15	7	1,43	14	0	19	2	0,40	65	4	9	0	1,71
15	0	8	2	2,38	66	1	16	1	3,65	15	1	0	6	2,22	66	4	10	4	3,53
16	0	8	9	0,60	67	1	16	8	2,14	16	1	1	11	0,04	67	4	11	9	1,35
17	0	9	3	3,09	68	1	17	3	0,36	17	1	3	3	1,59	68	4	13	1	3,17
18	0	9	10	1,31	69	1	17	9	2,58	18	1	4	7	3,41	69	4	14	6	0,72
19	0	10	4	3,53	70	1	18	4	1,07	19	1	6	0	1,23	70	4	15	10	2,54
20	0	10	11	2,02	71	1	18	10	3,29	20	1	7	4	3,05	71	4	17	3	0,36
21	0	11	6	0,24	72	1	19	5	1,51	21	1	8	9	0,60	72	4	18	7	2,18
22	0	12	0	2,46	73	2	0	0	0,00	22	1	10	1	2,42	73	5	0	0	0,00
23	0	12	7	0,68	74	2	0	6	2,22	23	1	11	6	0,24	74	5	1	4	1,55
24	0	13	1	3,17	75	2	1	1	0,44	24	1	12	10	2,06	75	5	2	8	3,37
25	0	13	8	1,39	76	2	1	7	2,66	25	1	14	2	3,61	76	5	4	1	1,19
26	0	14	2	3,61	77	2	2	2	1,15	26	1	15	7	1,43	77	5	5	5	3,01
27	0	14	9	2,10	78	2	2	8	3,37	27	1	16	11	3,25	78	5	6	10	0,56
28	0	15	4	0,32	79	2	3	3	1,59	28	1	18	4	1,07	79	5	8	2	2,38
29	0	15	10	2,54	80	2	3	10	0,08	29	1	19	8	2,62	80	5	9	7	0,20
30	0	16	5	1,03	81	2	4	4	2,30	30	2	1	1	0,44	81	5	10	11	2,02
31	0	16	11	3,25	82	2	4	11	0,52	31	2	2	5	2,26	82	5	12	3	3,57
32	0	17	6	1,47	83	2	5	5	3,01	32	2	3	10	0,08	83	5	13	8	1,39
33	0	18	0	3,69	84	2	6	0	1,23	33	2	5	2	1,63	84	5	15	0	3,21
34	0	18	7	2,18	85	2	6	6	3,45	34	2	6	6	3,45	85	5	16	5	1,03
35	0	19	2	0,40	86	2	7	1	1,67	35	2	7	11	1,27	86	5	17	9	2,58
36	0	19	8	2,62	87	2	7	8	0,16	36	2	9	3	3,09	87	5	19	2	0,40
37	1	0	3	1,11	88	2	8	2	2,38	37	2	10	8	0,64	88	6	0	6	2,22
38	1	0	9	3,33	89	2	8	9	0,60	38	2	12	0	2,46	89	6	1	11	0,04
39	1	1	4	1,55	90	2	9	3	3,09	39	2	13	5	0,28	90	6	3	3	1,59
40	1	1	11	0,04	91	2	9	10	1,31	40	2	14	9	2,10	91	6	4	7	3,41
41	1	2	5	2,26	92	2	10	4	3,53	41	2	16	1	3,65	92	6	6	0	1,23
42	1	3	0	0,48	93	2	10	11	2,02	42	2	17	6	1,47	93	6	7	4	3,05
43	1	3	6	2,70	94	2	11	6	0,24	43	2	18	10	3,29	94	6	8	9	0.60
44	1	4	1	1,19	95	2	12	0	2,46	44	3	0	3	1,11	95	6	10	1	2,42
45	1	4	7	3,41	96	2	12	7	0,68	45	3	1	7	2,66	96	6	11	6	0,24
46	1	5	2	1,63	97	2	13	1	3,17	46	3	3	0	0,48	97	6	12	10	2,06
47	1	5	9	0,12	98	2	13	8	1,39	47	3	4	4	2,30	98	6	14	2	3,61
48	1	6	3	2,34	99	2	14	2	3,61	48	3	5	9	0,12	99	6	15	7	1,43
49	1	6	10	0,56	100	2	14	9	2,10	49	3	7	1	1,67	100	6	16	11	3,25
50	1	7	4	3,05	200	5	9	7	0,20	50	3	8	5	3,49	200	13	13	11	2,50
51	1	7	11	1,27	300	8	4	4	2,30	51	3	9	10	1,31	300	20	10	11	2,02

TABLE of SALARIES for the common year, continued.

Days	At 40l. l.	s.	d.	q. 73p.	Days	At 40l. l.	s.	d.	q. 73p.	Days	At 50l. l.	s.	d.	q. 73p.	Days	At 50l. l.	s.	d.	q. 73p.
1	0	2	2	1,15	52	5	13	11	2,50	1	0	2	8	3,37	52	7	2	5	2,26
2	0	4	4	2,30	53	5	16	1	3,65	2	0	5	5	3,01	53	7	5	2	1,63
3	0	6	6	3,45	54	5	18	4	1,07	3	0	8	2	2,38	54	7	7	11	1,27
4	0	8	9	0,60	55	6	0	6	2,22	4	0	10	11	2,02	55	7	10	8	0,64
5	0	10	11	2,02	56	6	2	8	3,37	5	0	13	8	1,39	56	7	13	5	0,28
6	0	13	1	3,17	57	6	4	11	0,52	6	0	16	5	1,03	57	7	16	1	3,65
7	0	15	4	0,32	58	6	7	1	1,67	7	0	19	2	0,40	58	7	18	10	3,29
8	0	17	6	1,47	59	6	9	3	3,09	8	1	1	11	0,04	59	8	1	7	2,66
9	0	19	8	2,62	60	6	11	6	0,24	9	1	4	7	3,41	60	8	4	4	2,30
10	1	1	11	0,04	61	6	13	8	1,39	10	1	7	4	3,05	61	8	7	1	1,67
11	1	4	1	1,19	62	6	15	10	2,54	11	1	10	1	2,42	62	8	9	10	1,31
12	1	6	3	2,34	63	6	18	0	3,69	12	1	12	10	2,06	63	8	12	7	0,68
13	1	8	5	3,49	64	7	0	3	1,11	13	1	15	7	1,43	64	8	15	4	0,32
14	1	10	8	0,64	65	7	2	5	2,26	14	1	18	4	1,07	65	8	18	0	3,69
15	1	12	10	2,06	66	7	4	7	3,41	15	2	1	1	0,44	66	9	0	9	3,33
16	1	15	0	3,21	67	7	6	10	0,56	16	2	3	10	0,08	67	9	3	6	2,60
17	1	17	3	0,36	68	7	9	0	1,71	17	2	6	6	3,45	68	9	6	3	2,34
18	1	19	5	1,51	69	7	11	2	3,13	18	2	9	3	3,09	69	9	9	0	1,71
19	2	1	7	2,66	70	7	13	5	0,28	19	2	12	0	2,46	70	9	11	9	1,35
20	2	3	10	0,08	71	7	15	7	1,43	20	2	14	9	2.10	71	9	14	6	0,72
21	2	6	0	1,23	72	7	17	9	2,58	21	2	17	6	1,47	72	9	17	3	0,36
22	2	8	2	2,38	73	8	0	0	0,00	22	3	0	3	1,11	73	10	0	0	0,00
23	2	10	4	3,53	74	8	2	2	1,15	23	3	3	0	0,48	74	10	2	8	3,37
24	2	12	7	0,68	75	8	4	4	2,30	24	3	5	9	0,12	75	10	5	5	3,01
25	2	14	9	2,10	76	8	6	6	3,45	25	3	8	5	3,49	76	10	8	2	2,38
26	2	16	11	3,25	77	8	8	9	0,60	26	3	11	2	3,13	77	10	10	11	2,02
27	2	19	2	0,40	78	8	10	11	2,02	27	3	13	11	2,50	78	10	13	8	1,39
28	3	1	4	1,55	79	8	13	1	3,17	28	3	16	8	2,14	79	10	16	5	1,03
29	3	3	6	2,70	80	8	15	4	0,32	29	3	19	5	1,51	80	10	19	2	0,40
30	3	5	9	0,12	81	8	17	6	1,47	30	4	2	2	1,15	81	11	1	11	0,04
31	3	7	11	1,27	82	8	19	8	2,62	31	4	4	11	0,52	82	11	4	7	3,41
32	3	10	1	2,42	83	9	1	11	0,04	32	4	7	8	0,16	83	11	7	4	3,05
33	3	12	3	3,57	84	9	4	1	1,19	33	4	10	4	3,53	84	11	10	1	2,42
34	3	14	6	0,72	85	9	6	3	2,34	34	4	13	1	3,17	85	11	12	10	2,06
35	3	16	8	2,14	86	9	8	5	3,49	35	4	15	10	2,54	86	11	15	7	1,43
36	3	18	10	3,29	87	9	10	8	0,64	36	4	18	7	2,18	87	11	18	4	1,07
37	4	1	1	0,44	88	9	12	10	2,06	37	5	1	4	1,55	88	12	1	1	0,44
38	4	3	3	1,59	89	9	15	0	3,21	38	5	4	1	1,19	89	12	3	10	0,08
39	4	5	5	3,01	90	9	17	3	0,36	39	5	6	10	0,56	90	12	6	6	3,45
40	4	7	8	0,16	91	9	19	5	1,51	40	5	9	7	0,20	91	12	9	3	3,09
41	4	9	10	1,31	92	10	1	7	2,66	41	5	12	3	3,57	92	12	12	0	2,46
42	4	12	0	2,46	93	10	3	10	0,08	42	5	15	0	3,21	93	12	14	9	2,10
43	4	14	2	3,61	94	10	6	0	1,23	43	5	17	9	2,58	94	12	17	6	1,47
44	4	16	5	1,03	95	10	8	2	2,38	44	6	0	6	2,22	95	13	0	3	1,11
45	4	18	7	2,18	96	10	10	4	3,53	45	6	3	3	1,59	96	13	3	0	0,48
46	5	0	9	3,33	97	10	12	7	0,68	46	6	6	0	1,23	97	13	5	9	0,12
47	5	3	0	0,48	98	10	14	9	2,10	47	6	8	9	0,60	98	13	8	5	3,49
48	5	5	2	1,63	99	10	16	11	3,25	48	6	11	6	0,24	99	13	11	2	3,13
49	5	7	4	3,05	100	10	19	2	0,40	49	6	14	2	3,61	100	13	13	11	2,50
50	5	9	7	0,20	200	21	18	4	1,07	50	6	16	11	3,25	200	27	7	11	1,27
51	5	11	9	1,35	300	32	17	6	1,47	51	6	19	8	2,62	300	41	1	11	0,04

TABLE of SALARIES for the common year, continued.

Days	At 90l. l.	s.	d.	q. .73p	Days	At 90l. l.	s.	d.	q. .73p	Days	At 120l. l.	s.	d.	q. .73p	Days	At 120l. l.	s.	d.	q. .73p
1	0	4	11	0,52	52	12	16	5	1,03	1	0	6	6	3,45	52	17	1	11	0,04
2	0	9	10	1,31	53	13	1	4	1,55	2	0	13	1	3,17	53	17	8	5	3,49
3	0	14	9	2,10	54	13	6	3	2,34	3	0	19	8	2,62	54	17	15	0	3,21
4	0	19	8	2,62	55	13	11	2	3,13	4	1	6	3	2,34	55	18	1	7	2,66
5	1	4	7	3,41	56	13	16	1	3,65	5	1	12	10	2,06	56	18	8	2	2,38
6	1	9	7	0,20	57	14	1	1	0,44	6	1	19	5	1,51	57	18	14	9	2,10
7	1	14	6	0,72	58	14	6	0	1,23	7	2	6	0	1,23	58	19	1	4	1,55
8	1	19	3	1,51	59	14	10	11	2,02	8	2	12	7	0,68	59	19	7	11	1,27
9	2	4	4	2,30	60	14	15	10	2,54	9	2	19	2	0,40	60	19	14	6	0,72
10	2	9	3	3,09	61	15	0	9	3,33	10	3	5	9	0,12	61	20	1	1	0,44
11	2	14	2	3,61	62	15	5	9	0,12	11	3	12	3	3,57	62	20	7	8	0,16
12	2	19	2	0,40	63	15	10	8	0,64	12	3	18	10	3,29	63	20	14	2	3,61
13	3	4	1	1,19	64	15	15	7	1,43	13	4	5	5	3,01	64	21	0	9	3,33
14	3	9	0	1,71	65	16	0	6	2,22	14	4	12	0	2,46	65	21	7	4	3,05
15	3	13	11	2,50	66	16	5	5	3,01	15	4	18	7	2,18	66	21	13	11	2,50
16	3	18	10	3,29	67	16	10	4	3,53	16	5	5	2	1,63	67	22	0	6	2,22
17	4	3	10	0,08	68	16	15	4	0,32	17	5	11	9	1,35	68	22	7	1	1,67
18	4	8	9	0,60	69	17	0	3	1,11	18	5	18	4	1,07	69	22	13	8	1,39
19	4	13	8	1,39	70	17	5	2	1,63	19	6	4	11	0,52	70	23	0	3	1,11
20	4	18	7	2,18	71	17	10	1	2,42	20	6	11	6	0,24	71	23	6	10	0,56
21	5	3	6	2,70	72	17	15	0	3,21	21	6	18	0	3,69	72	23	13	5	0,28
22	5	8	5	3,49	73	18	0	0	0,00	22	7	4	7	3,41	73	24	0	0	0,00
23	5	13	5	0,28	74	18	4	11	0,52	23	7	11	2	3,13	74	24	6	6	3,45
24	5	18	4	1,07	75	18	9	10	1,31	24	7	17	9	2,58	75	24	13	1	3,17
25	6	3	3	1,59	76	18	14	9	2,10	25	8	4	4	2,30	76	24	19	8	2,62
26	6	8	2	2,38	77	19	19	8	2,62	26	8	10	11	2,02	77	25	6	3	2,34
27	6	13	1	3,17	78	19	4	7	3,41	27	8	17	6	1,47	78	25	12	10	2,06
28	6	18	0	3,69	79	19	9	7	0,20	28	9	4	1	1,19	79	25	19	5	1,51
29	7	3	0	0,48	80	19	14	6	0,72	29	9	10	8	0,64	80	26	6	0	1,23
30	7	7	11	1,27	81	19	19	5	1,51	30	9	17	3	0,36	81	26	12	7	0,68
31	7	12	10	2,06	82	20	4	4	2,30	31	10	3	10	0,08	82	26	19	2	0,40
32	7	17	9	2,58	83	20	9	3	3,09	32	10	10	4	3,53	83	27	5	9	0,12
33	8	2	8	3,37	84	20	14	2	3,61	33	10	16	11	3,25	84	27	12	3	3,57
34	8	7	8	0,16	85	20	19	2	0,40	34	11	3	6	2,70	85	27	18	10	3,29
35	8	12	7	0,68	86	21	4	1	1,19	35	11	10	1	2,42	86	28	5	5	3,01
36	8	17	6	1,47	87	21	9	0	1,71	36	11	16	8	2,14	87	28	12	0	2,46
37	9	2	5	2,26	88	21	13	11	2,50	37	12	3	3	1,59	88	28	18	7	2,18
38	9	7	4	3,05	89	21	18	10	3,29	38	12	9	10	1,31	89	29	5	2	1,63
39	9	12	3	3,57	90	22	3	10	0,08	39	12	16	5	1,03	90	29	11	9	1,35
40	9	17	3	0,36	91	22	8	9	0,60	40	13	3	0	0,48	91	29	18	4	1,07
41	10	2	2	1,15	92	22	13	8	1,39	41	13	9	7	0,20	92	30	4	11	0,52
42	10	7	1	1,67	93	22	18	7	2,18	42	13	16	1	3,65	93	30	11	6	0,24
43	10	12	0	2,46	94	23	3	6	2,70	43	14	2	8	3,37	94	30	18	0	3,69
44	10	16	11	3,25	95	23	8	5	3,49	44	14	9	3	3,09	95	31	4	7	3,41
45	11	1	11	0,04	96	23	13	5	0,28	45	14	15	10	2,54	96	31	11	2	3,13
46	11	6	10	0,56	97	23	18	4	1,07	46	15	2	5	2,26	97	31	17	9	2,58
47	11	11	9	1,35	98	24	3	3	1,59	47	15	9	0	1,71	98	32	4	4	2,30
48	11	16	8	2,14	99	24	8	2	2,38	48	15	15	7	1,43	99	32	10	11	2,02
49	12	1	7	2,66	100	24	13	1	3,17	49	16	2	2	1,15	100	32	17	6	1,47
50	12	6	6	3,45	200	49	6	3	2,34	50	16	8	9	0,60	200	65	15	0	3,21
51	12	11	6	0,24	300	73	19	5	1,51	51	16	15	4	0,32	300	98	12	7	0,68

TABLE 41. A TABLE of SALARIES, for collectors, supervisors, officers, &c. at 10l.—25l.—40l.—50l.—90l. and 120l. per annum, for the leap year, or 366 days.

Days	At 10l. l.	s.	d.	q. 61p	Days	At 10l. l.	s.	d.	q. 61p	Days	At 25l. l.	s.	d.	q. 61p	Days	At 25l. l.	s.	d.	q. 61p
1	0	0	6	2,14	52	1	8	4	3,57	1	0	1	4	1,35	52	3	11	0	1,51
2	0	1	1	0,28	53	1	8	11	2,10	2	0	2	8	3,09	53	3	12	4	3,25
3	0	1	7	2,42	54	1	9	6	0,24	3	0	4	1	0,44	54	3	13	9	0,60
4	0	2	2	0 56	55	1	10	0	2,38	4	0	5	5	2,18	55	3	15	1	2,34
5	0	2	8	3,09	56	1	10	7	0,52	5	0	6	9	3,53	56	3	16	6	0,08
6	0	3	3	1,23	57	1	11	1	3,05	6	0	8	2	1,27	57	3	17	10	1,43
7	0	3	9	3,37	58	1	11	8	1,19	7	0	9	6	3,01	58	3	19	2	3,17
8	0	4	4	1,51	59	1	12	2	3,33	8	0	10	11	0,36	59	4	0	7	0,52
9	0	4	11	0,04	60	1	12	9	1,47	9	0	12	3	2,10	60	4	1	11	2,26
10	0	5	5	2,18	61	1	13	4	0,00	10	0	13	7	3,45	61	4	3	4	0,00
11	0	6	0	0,32	62	1	13	10	2,14	11	0	15	0	1,19	62	4	4	8	1,35
12	0	6	6	2,46	63	1	14	5	0,28	12	0	16	4	2,54	63	4	6	0	3,09
13	0	7	1	0,60	64	1	14	11	2,42	13	0	17	9	0,28	64	4	7	5	0,44
14	0	7	7	3,13	65	1	15	6	0,56	14	0	19	1	2,02	65	4	8	9	2,18
15	0	8	2	1,27	66	1	16	0	3,09	15	1	0	5	3,37	66	4	10	1	3,53
16	0	8	8	3,41	67	1	16	7	1,23	16	1	1	10	1,11	67	4	11	6	1,27
17	0	9	3	1,55	68	1	17	1	3,37	17	1	3	2	2,46	68	4	12	10	3,01
18	0	9	10	0,08	69	1	17	8	1,51	18	1	4	7	0,20	69	4	14	3	0,36
19	0	10	4	2,22	70	1	18	3	0,04	19	1	5	11	1,55	70	4	15	7	2,10
20	0	10	11	0,36	71	1	18	9	2,18	20	1	7	3	3,29	71	4	16	11	3,45
21	0	11	5	2,50	72	1	19	4	0,32	21	1	8	8	1,03	72	4	18	4	1,19
22	0	12	0	1,03	73	1	19	10	2,46	22	1	10	0	2,38	73	4	19	8	2,54
23	0	12	6	3,17	74	2	0	5	0,60	23	1	11	5	0,12	74	5	1	1	0,28
24	0	13	1	1,31	75	2	0	11	3,13	24	1	12	9	1,47	75	5	2	5	2,02
25	0	13	7	3,45	76	2	1	6	1,27	25	1	14	1	3,21	76	5	3	9	3,37
26	0	14	2	1,59	77	2	2	0	3,41	26	1	15	6	0,56	77	5	5	2	1,11
27	0	14	9	0,12	78	2	2	7	1,55	27	1	16	10	2,30	78	5	6	6	2,46
28	0	15	3	2,26	79	2	3	2	0,08	28	1	18	3	0,04	79	5	7	11	0,20
29	0	15	10	0,40	80	2	3	8	2,22	29	1	19	7	1,39	80	5	9	3	1,55
30	0	16	4	2,54	81	2	4	3	0,36	30	2	0	11	3,13	81	5	10	7	3,29
31	0	16	11	1,07	82	2	4	9	2,50	31	2	2	4	0,48	82	5	12	0	1,03
32	0	17	5	3,21	83	2	5	4	1,03	32	2	3	8	2,22	83	5	13	4	2,38
33	0	18	0	1,35	84	2	5	10	3,17	33	2	5	0	3,57	84	5	14	9	0,12
34	0	18	6	3,49	85	2	6	5	1,31	34	2	6	5	1,31	85	5	16	1	1,47
35	0	19	1	2,02	86	2	6	11	3,45	35	2	7	9	3,05	86	5	17	5	3,21
36	0	19	8	0,16	87	2	7	6	1,59	36	2	9	2	0,40	87	5	18	10	0,56
37	1	0	2	2,30	88	2	8	1	0,12	37	2	10	6	2,14	88	6	0	2	2,30
38	1	0	9	0,44	89	2	8	7	2,26	38	2	11	10	3,49	89	6	1	7	0,04
39	1	1	3	2,58	90	2	9	2	0,40	39	2	13	3	1,23	90	6	2	11	1,39
40	1	1	10	1,11	91	2	9	8	2,54	40	2	14	7	2,58	91	6	4	3	3,13
41	1	2	4	3,25	92	2	10	3	1,07	41	2	16	0	0,32	92	6	5	8	0,48
42	1	2	11	1,39	93	2	10	9	3,21	42	2	17	4	2,06	93	6	7	0	2,22
43	1	3	5	3,53	94	2	11	4	1,35	43	2	18	8	3,41	94	6	8	4	3,57
44	1	4	0	2,06	95	2	11	10	3,49	44	3	0	1	1,15	95	6	9	9	1,31
45	1	4	7	0,20	96	2	12	5	2,02	45	3	1	5	2,50	96	6	11	1	3,05
46	1	5	1	2,31	97	2	13	0	0,16	46	3	2	10	0,24	97	6	12	6	0,40
47	1	5	8	0,48	98	2	13	6	2,30	47	3	4	2	1,59	98	6	13	10	2,14
48	1	6	2	3,01	99	2	14	1	0,44	48	3	5	6	3,33	99	6	15	2	3,49
49	1	6	9	1,15	100	2	14	7	2,58	49	3	6	11	1,07	100	6	16	7	1,23
50	1	7	3	3,29	200	5	19	3	1,55	50	3	8	3	2,42	200	13	13	2	2,46
51	1	7	10	1,43	300	8	3	11	0,52	51	3	9	8	0,16	300	20	9	10	0,08

TABLE of SALARIES for leap year, continued.

Days	At 40l. l.	s.	d.	q. 61p	Days	At 40l. l.	s.	d.	q. 61p	Days	At 50l. l.	s.	d.	q. 61p	Days	At 50l. l.	s.	d.	q. 61p
1	0	2	2	0,56	52	5	13	7	3,45	1	0	2	8	3,09	52	7	2	0	3,41
2	0	4	4	1,51	53	5	15	10	0,40	2	0	5	5	2,18	53	7	4	9	2,50
3	0	6	6	2,46	54	5	18	0	1,35	3	0	8	2	1,27	54	7	7	6	1,59
4	0	8	8	3,41	55	6	0	2	2,30	4	0	10	11	0,36	55	7	10	3	1,07
5	0	10	11	0,30	56	6	2	4	3,25	5	0	13	7	3,45	56	7	13	0	0,16
6	0	13	1	1,31	57	6	4	7	0,20	6	0	16	4	2,54	57	7	15	8	3,25
7	0	15	3	2,26	58	6	6	9	1,15	7	0	19	1	2,02	58	7	18	5	2,34
8	0	17	5	3,21	59	6	8	11	2,10	8	1	1	10	1,11	59	8	1	2	1,43
9	0	19	8	0,16	60	6	11	1	3,05	9	1	4	7	0,20	60	8	3	11	0,52
10	1	1	10	1,11	61	6	13	4	0,00	10	1	7	3	3,29	61	8	6	8	0 00
11	1	4	0	2,06	62	6	15	6	0,56	11	1	10	0	2,38	62	8	9	4	3,09
12	1	6	2	3,01	63	6	17	8	1,51	12	1	12	9	1,47	63	8	12	1	2,18
13	1	8	4	3,57	64	6	19	10	2,46	13	1	15	6	0,56	64	8	14	10	1,27
14	1	10	7	0,52	65	7	2	0	3,41	14	1	18	3	0,04	65	8	17	7	0,36
15	1	12	9	1,47	66	7	4	3	0,36	15	2	0	11	3,13	66	9	0	3	3,45
16	1	14	11	2,42	67	7	6	5	1,31	16	2	3	8	2,22	67	9	3	0	2,54
17	1	17	1	3,37	68	7	8	7	2,26	17	2	6	5	1,31	68	9	5	9	2,02
18	1	19	4	0,32	69	7	10	9	3,21	18	2	9	2	0,40	69	9	8	6	1,11
19	2	1	6	1,27	70	7	13	0	0,16	19	2	11	10	3,49	70	9	11	3	0,20
20	2	3	8	2,22	71	7	15	2	1,11	20	2	14	7	2,58	71	9	13	11	3,29
21	2	5	10	3,17	72	7	17	4	2,06	21	2	17	4	2,06	72	9	16	8	2,38
22	2	8	1	0,12	73	8	19	6	3,01	22	3	0	1	1,15	73	9	19	5	1,47
23	2	10	3	1,07	74	8	1	8	3,57	23	3	2	10	0,24	74	10	2	2	0,56
24	2	12	5	2,02	75	8	3	11	0,52	24	3	5	6	3,33	75	10	4	11	0,04
25	2	14	7	2,58	76	8	6	1	1,47	25	3	8	3	2,42	76	10	7	7	3,13
26	2	16	9	3,53	77	8	8	3	2,42	26	3	11	0	1,51	77	10	10	4	2,22
27	2	19	0	0,48	78	8	10	5	3,37	27	3	13	9	0,60	78	10	13	1	1,31
28	3	1	2	1,43	79	8	12	8	0,32	28	3	16	6	0,08	79	10	15	10	0,40
29	3	3	4	2,38	80	8	14	10	1,27	29	3	19	2	3,17	80	10	18	6	3,49
30	3	5	6	3,33	81	8	17	0	2,22	30	4	1	11	2,26	81	11	1	3	2,58
31	3	7	9	0,28	82	8	19	2	3,17	31	4	4	8	1,35	82	11	4	0	2,06
32	3	9	11	1,23	83	9	1	5	0,12	32	4	7	5	0,44	83	11	6	9	1,15
33	3	12	1	2,18	84	9	3	7	1,07	33	4	10	1	3,53	84	11	9	6	0,24
34	3	14	3	3,13	85	9	5	9	2,02	34	4	12	10	3,01	85	11	12	2	3,33
35	3	16	6	0,08	86	9	7	11	2,58	35	4	15	7	2,10	86	11	14	11	2,42
36	3	18	8	1,03	87	9	10	1	3,53	36	4	18	4	1,19	87	11	17	8	1,51
37	4	0	10	1,59	88	9	12	4	0,48	37	5	1	1	0,28	88	12	0	5	0,60
38	4	3	0	2,54	89	9	14	6	1,43	38	5	3	9	3,37	89	12	3	2	0,08
39	4	5	2	3,49	90	9	16	8	2,38	39	5	6	6	2,46	90	12	5	10	3,17
40	4	7	5	0.44	91	9	18	10	3,33	40	5	9	3	1,55	91	12	8	7	2,26
41	4	9	7	1,39	92	10	1	1	0,28	41	5	12	0	1,03	92	12	11	4	1,35
42	4	11	9	2,34	93	10	3	3	1,23	42	5	14	9	0,12	93	12	14	1	0,44
43	4	13	11	3,29	94	10	5	5	2,18	43	5	17	5	3,21	94	12	16	9	3,53
44	4	16	2	0,24	95	10	7	7	3,13	44	6	0	2	2,30	95	12	19	6	3,01
45	4	18	4	1,19	96	10	9	10	0,08	45	6	2	11	1,39	96	13	2	3	2,10
46	5	0	6	2,14	97	10	12	0	1,03	46	6	5	8	0,48	97	13	5	0	1,19
47	5	2	8	3,09	98	10	14	2	1,59	47	6	8	4	3,57	98	13	7	9	0,28
48	5	4	11	0,04	99	10	16	4	2,54	48	6	11	1	3,95	99	13	10	5	3,37
49	5	7	1	0,60	100	10	18	6	3,49	49	6	13	10	2,14	100	13	13	2	2,46
50	5	9	3	1,55	200	21	7	1	3,37	50	6	16	7	1,23	200	27	6	5	1,31
51	5	11	5	2,50	300	32	15	8	3,25	51	6	19	4	0,32	300	40	19	8	0,16

TABLE of SALARIES for leap year, continued.

Days	At 90l. l.	s.	d.	q. 61	Days	At 90l. l.	s.	d.	q. 61	Days	At 120l. l.	s.	d.	q. 61	Days	At 120l. l.	s.	d.	q. 61
1	0	4	11	0,04	52	12	15	8	3,25	1	0	6	6	2,46	52	17	0	11	3,13
2	0	9	10	0,08	53	13	0	7	3,29	2	0	13	1	1,31	53	17	7	6	1,59
3	0	14	9	0,12	54	13	5	6	3,33	3	0	19	8	0,16	54	17	14	1	0,44
4	0	19	8	0,16	55	13	10	5	3,37	4	1	6	2	3,01	55	18	0	7	3,29
5	1	4	7	0,20	56	13	15	4	3,41	5	1	12	9	1,47	56	18	7	2	2,14
6	1	9	6	0,24	57	14	0	3	3,45	6	1	19	4	0,32	57	18	13	9	0,60
7	1	14	5	0,28	58	14	5	2	3,49	7	2	5	10	3,17	58	19	0	3	3,45
8	1	19	4	0,32	59	14	10	1	3,53	8	2	12	5	2,02	59	19	6	10	2,30
9	2	4	3	0,36	60	14	15	0	3,57	9	2	19	0	0,48	60	19	13	5	1,15
10	2	9	2	0,40	61	15	0	0	0,00	10	3	5	6	3,33	61	20	0	0	0,00
11	2	14	1	0,44	62	15	4	11	0,04	11	3	12	1	2,18	62	20	6	6	2,40
12	2	19	0	0,48	63	15	9	10	0,08	12	3	18	8	1,03	63	20	13	1	1,31
13	3	3	11	0,52	64	15	14	9	0,12	13	4	5	2	3,49	64	20	19	8	0,16
14	3	8	10	0,56	65	15	19	8	0,16	14	4	11	9	2,34	65	21	6	2	3,01
15	3	13	9	0,60	66	16	4	7	0,20	15	4	18	4	1,19	66	21	12	9	1,47
16	3	18	8	1,03	67	16	9	6	0,24	16	5	4	11	0,04	67	21	19	4	0,32
17	4	3	7	1,07	68	16	14	5	0,28	17	5	11	5	2,50	68	22	5	10	3,17
18	4	8	6	1,11	69	16	19	4	0,32	18	5	18	0	1,35	69	22	12	5	2,02
19	4	13	5	1,15	70	17	4	3	0,36	19	6	4	7	0,20	70	22	19	0	0,48
20	4	18	4	1,19	71	17	9	2	0,40	20	6	11	1	3,05	71	23	5	6	3,33
21	5	3	3	1,23	72	17	14	1	0,44	21	6	17	8	1,51	72	23	12	1	2,18
22	5	8	2	1,27	73	17	19	0	0,48	22	7	4	3	0,36	73	23	18	8	1,03
23	5	13	1	1,31	74	18	3	11	0,52	23	7	10	9	3,21	74	24	5	2	3,49
24	5	18	0	1,35	75	18	8	10	0,56	24	7	17	4	2,06	75	24	11	9	2,34
25	6	2	11	1,39	76	18	13	9	0,60	25	8	3	11	0,52	76	24	18	4	1,19
26	6	7	10	1,43	77	18	18	8	1,03	26	8	10	5	3,37	77	25	4	11	0,04
27	6	12	9	1,47	78	19	3	7	1,07	27	8	17	0	2,22	78	25	11	5	2,50
28	6	17	8	1,51	79	19	8	6	1,11	28	9	3	7	1,07	79	25	18	0	1,35
29	7	2	7	1,55	80	19	13	5	1,15	29	9	10	1	3,53	80	26	4	7	0,20
30	7	7	6	1,59	81	19	18	4	1,19	30	9	16	8	2,38	81	26	11	1	3,05
31	7	12	5	2,02	82	20	3	3	1,23	31	10	3	3	1,23	82	26	17	8	1,51
32	7	17	4	2,06	83	20	8	2	1,27	32	10	9	10	0,08	83	27	4	3	0,36
33	8	2	3	2,10	84	20	13	1	1,31	33	10	16	4	2,54	84	27	10	9	3,21
34	8	7	2	2,14	85	20	18	0	1,35	34	11	2	11	1,39	85	27	17	4	2,06
35	8	12	1	2,18	86	21	2	11	1,39	35	11	9	6	0,24	86	28	3	11	0,52
36	8	17	0	2,22	87	21	7	10	1,43	36	11	16	0	3,09	87	28	10	5	3,37
37	9	1	11	2,26	88	21	12	9	1,47	37	12	2	7	1,55	88	28	17	0	2,22
38	9	6	10	2,30	89	21	17	8	1,51	38	12	9	2	0,40	89	29	3	7	1,07
39	9	11	9	2,34	90	22	2	7	1,55	39	12	15	8	3,25	90	29	10	1	3,53
40	9	16	8	2,38	91	22	7	6	1,59	40	13	2	3	2,10	91	29	16	8	2,38
41	10	1	7	2,42	92	22	12	5	2,02	41	13	8	10	0,56	92	30	3	3	1,23
42	10	6	6	2,46	93	22	17	4	2,06	42	13	15	4	3,41	93	30	9	10	0,08
43	10	11	5	2,50	94	23	2	3	2,10	43	14	1	11	2,26	94	30	16	4	2,54
44	10	16	4	2,54	95	23	7	2	2,14	44	14	8	6	1,11	95	31	2	11	1,39
45	11	1	3	2,58	96	23	12	1	2,18	45	14	15	0	3,57	96	31	9	6	0,24
46	11	6	2	3,01	97	23	17	0	2,22	46	15	1	7	2,42	97	31	16	0	3,09
47	11	11	1	3,05	98	24	1	11	2,26	47	15	8	2	1,27	98	32	2	7	1,55
48	11	16	0	3,09	99	24	6	10	2,30	48	15	14	9	0,12	99	32	9	2	0,40
49	12	0	11	3,13	100	24	11	9	2,34	49	16	1	3	2,58	100	32	15	8	3,25
50	12	5	10	3,17	200	49	3	7	1,07	50	16	7	10	1,43	200	65	11	5	2,50
51	12	10	9	3,21	300	73	15	4	3,41	51	16	14	5	0,28	300	98	7	2	2,14

TABLE 42. For Auctions; likewise for Tax and Charity on salaries, at 6d. and 3d. in the pound.

Value or salary s.	d.	Tax or duty at 6d. d.	q. 10p	Charity or duty at 3d. d.	q. 10p	Total at 9d. d.	q. 10p
0	2	0	0,2	0	0,1	0	0,3
0	4	0	0,4	0	0,2	0	0,6
0	6	0	0,6	0	0,3	0	0,9
0	8	0	0,8	0	0,4	0	1,2
0	10	0	1,0	0	0,5	0	1,5
1	0	0	1,2	0	0,6	0	1,8
1	2	0	1,4	0	0,7	0	2,1
1	4	0	1,6	0	0,8	0	2,4
1	6	0	1,8	0	0,9	0	2,7
1	8	0	2,0	0	1,0	0	3,0
1	10	0	2,2	0	1,1	0	3,3
2	0	0	2,4	0	1,2	0	3,6
2	2	0	2,6	0	1,3	0	3,9
2	4	0	2,8	0	1,4	1	0,2
2	6	0	3,0	0	1,5	1	0,5
2	8	0	3,2	0	1,6	1	0,8
2	10	0	3,4	0	1,7	1	1,1
3	0	0	3,6	0	1,8	1	1,4
3	2	0	3,8	0	1,9	1	1,7
3	4	1	0,0	0	2,0	1	2,0
3	6	1	0,2	0	2,1	1	2,3
3	8	1	0,4	0	2,2	1	2,6
3	10	1	0,6	0	2,3	1	2,9
4	0	1	0,8	0	2,4	1	3,5
4	2	1	1,0	0	2,5	1	3,2
4	4	1	1,2	0	2,6	1	3,8
4	6	1	1,4	0	2,7	2	0,1
4	8	1	1,6	0	2,8	2	0,4
4	10	1	1,8	0	2,9	2	0,7
5	0	1	2,0	0	3,0	2	1,0
5	2	1	2,2	0	3,1	2	1,3
5	4	1	2,4	0	3,2	2	1,6
5	6	1	2,6	0	3,3	2	1,9
5	8	1	2,8	0	3,4	2	2,2
5	10	1	3,0	0	3,5	2	2,5
6	0	1	3,2	0	3,6	2	2,8
6	2	1	3,4	0	3,7	2	3,1
6	4	1	3,6	0	3,8	2	3,4
6	6	1	3,8	0	3,9	2	3,7
6	8	2	0,0	1	0,0	3	0,0
6	10	2	0,2	1	0,1	3	0,3
7	0	2	0,4	1	0,2	3	0,6
7	2	2	0,6	1	0,3	3	0,9
7	4	2	0,8	1	0,4	3	1,2
7	6	2	1,0	1	0,5	3	1,5
7	8	2	1,2	1	0,6	3	1,8
7	10	2	1,4	1	0,7	3	1,1
8	0	2	1,6	1	0,8	3	2,4
8	2	2	1,8	1	0,9	3	2,7
8	4	2	2,0	1	1,0	3	3,0
8	6	2	2,2	1	1,1	3	3,3
8	8	2	2,4	1	1,2	3	3,6
8	10	2	2,6	1	1,3	3	3,9
9	0	2	2,8	1	1,4	4	0,2
9	2	2	3,0	1	1,5	4	0,5

Value of salary s.	d.	Tax or duty at 6d. d.	q. 10p	Charity or duty at 3d. d.	q. 10p	Total at 9d. d.	q. 10p
9	4	2	3,2	1	1,6	4	0,8
9	6	2	3,4	1	1,7	4	1,1
9	8	2	3,6	1	1,8	4	1,4
9	10	2	3,8	1	1,9	4	1,7
10	0	3	0,0	1	2,0	4	2,0
10	2	3	0,2	1	2,1	4	2,3
10	4	3	0,4	1	2,2	4	2,6
10	6	3	0,6	1	2,3	4	2,9
10	8	3	0,8	1	2,4	4	3,2
10	10	3	1,0	1	2,5	4	3,5
11	0	3	1,2	1	2,6	4	3,8
11	2	3	1,4	1	2,7	5	0,1
11	4	3	1,6	1	2,8	5	0,4
11	6	3	1,8	1	2,9	5	0,7
11	8	3	2,0	1	3,0	5	1,0
11	10	3	2,2	1	3,1	5	1,3
12	0	3	2,4	1	3,2	5	1,6
12	2	3	2,6	1	3,3	5	1,9
12	4	3	2,8	1	3,4	5	2,2
12	6	3	3,0	1	3,5	5	2,5
12	8	3	3,2	1	3,6	5	2,8
12	10	3	3,4	1	3,7	5	3,1
13	0	3	3,6	1	3,8	5	3,4
13	2	3	3,8	1	3,9	5	3,7
13	4	4	0,0	2	0,0	6	0,0
13	6	4	0,2	2	0,1	6	0,3
13	8	4	0,4	2	0,2	6	0,6
13	10	4	0,6	2	0,3	6	0,9
14	0	4	0,8	2	0,4	6	1,2
14	2	4	1,0	2	0,5	6	1,5
14	4	4	1,2	2	0,6	6	1,8
14	6	4	1,4	2	0,7	6	2,1
14	8	4	1,6	2	0,8	6	2,4
14	10	4	1,8	2	0,9	6	2,7
15	0	4	2,0	2	1,0	6	3,0
15	2	4	2,2	2	1,1	6	3,3
15	4	4	2,4	2	1,2	6	3,6
15	6	4	2,6	2	1,3	6	3,9
15	8	4	2,8	2	1,4	7	0,2
15	10	4	3,0	2	1,5	7	0,5
16	0	4	3,2	2	1,6	7	0,8
16	2	4	3,4	2	1,7	7	1,1
16	4	4	3,6	2	1,8	7	1,5
16	6	4	3,8	2	1,9	7	1,7
16	8	5	0,0	2	2,0	7	2,0
16	10	5	0,2	2	2,1	7	2,3
17	0	5	0,4	2	2,2	7	2,6
17	2	5	0,6	2	2,3	7	2,9
17	4	5	0,8	2	2,4	7	3,2
17	6	5	1,0	2	2,5	7	3,5
17	8	5	1,2	2	2,6	7	3,8
17	10	5	1,4	2	2,7	8	0,1
18	0	5	1,6	2	2,8	8	0,4
18	2	5	1,8	2	2,9	8	0,7
18	4	5	2,0	2	3,0	8	1,0

TABLE for Auctions, &c. continued.

Value or salary			Tax or duty at 6d.			Charity or duty at 3d.			Total at 9d.				V. or salary	Tax d. at 6d.			Charity or duty at 3d			Total at 9d.		
l.	s.	d.	s.	d.	q. 10p	s.	d.	q 10p	l.	s.	d.	q. p	l.	l.	s.	d.	l.	s.	d.	l.	s.	d.
0	18	6	0	5	2,2	0	2	3,1	0	0	8	1,3	47	1	3	6	0	11	9	1	15	3
0	18	8	0	5	2,4	0	2	3,2	0	0	8	1,6	48	1	4	0	0	12	0	1	16	0
0	18	10	0	5	2,6	0	2	3,3	0	0	8	1,9	49	1	4	6	0	12	3	1	16	9
0	19	0	0	5	2,8	0	2	3,4	0	0	8	2,2	50	1	5	0	0	12	6	1	17	6
0	19	2	0	5	3,0	0	2	3,5	0	0	8	2,5	51	1	5	6	0	12	9	1	18	3
0	19	4	0	5	3,2	0	2	3,6	0	0	8	2,8	52	1	6	0	0	13	0	1	19	0
0	19	6	0	5	3,4	0	2	3,7	0	0	8	3,1	53	1	6	6	0	13	3	1	19	9
0	19	8	0	5	3,6	0	2	3,8	0	0	8	3,4	54	1	7	0	0	13	6	2	0	0
0	19	10	0	5	3,8	0	2	3,9	0	0	8	3,7	55	1	7	6	0	13	9	2	1	3
1	0	0	0	6	0,0	0	3	0,0	0	0	9	0,0	56	1	8	0	0	14	0	2	2	0
2	0	0	1	0	0	0	6	0	0	1	6	0	57	1	8	6	0	14	3	2	2	9
3	0	0	1	6	0	0	9	0	0	2	3	0	58	1	9	0	0	14	6	2	3	6
4	0	0	2	0	0	1	0	0	0	3	0	0	59	1	9	6	0	14	9	2	4	3
5	0	0	2	6	0	1	3	0	0	4	9	0	60	1	10	0	0	15	0	2	5	0
6	0	0	3	0	0	1	6	0	0	5	6	0	61	1	10	6	0	15	3	2	5	9
7	0	0	3	6	0	1	9	0	0	6	3	0	62	1	11	0	0	15	6	2	6	6
8	0	0	4	0	0	2	0	0	0	6	0	0	63	1	11	6	0	15	9	2	7	3
9	0	0	4	6	0	2	3	0	0	7	9	0	64	1	12	0	0	16	0	2	8	0
10	0	0	5	0	0	2	6	0	0	8	6	0	65	1	12	6	0	16	3	2	8	9
11	0	0	5	6	0	2	9	0	0	9	3	0	66	1	13	0	0	16	6	2	9	6
12	0	0	6	0	0	3	0	0	0	9	0	0	67	1	13	6	0	16	9	2	10	3
13	0	0	6	6	0	3	3	0	0	9	9	0	68	1	14	0	0	17	0	2	11	0
14	0	0	7	0	0	3	6	9	0	10	6	0	69	1	14	6	0	17	3	2	11	9
15	0	0	7	6	0	3	9	0	0	11	3	0	70	1	15	0	0	17	6	2	12	6
16	0	0	8	0	0	4	0	0	0	12	0	0	71	1	15	6	0	17	9	2	13	3
17	0	0	8	6	0	4	3	0	8	12	9	0	72	1	16	0	0	18	0	2	14	0
18	0	0	9	0	0	4	6	0	0	13	6	0	73	1	16	6	0	18	3	2	14	9
19	0	0	9	6	0	4	9	0	0	14	3	0	74	1	17	0	0	18	6	2	15	6
20	0	0	10	0	0	5	0	0	0	15	0	0	75	1	17	6	0	18	9	2	16	3
21	0	0	10	6	0	5	3	0	0	15	9	0	76	1	18	0	0	19	0	2	17	0
22	0	0	11	0	0	5	6	0	0	16	6	0	77	1	18	6	0	19	3	2	17	9
23	0	0	11	6	0	5	9	0	0	17	3	0	78	1	19	0	0	19	6	2	18	6
24	0	0	12	0	0	6	0	0	0	18	0	0	79	1	19	6	0	19	9	2	19	3
25	0	0	12	6	0	6	3	0	0	18	9	0	80	2	0	0	1	0	0	3	0	0
26	0	0	13	0	0	6	6	0	0	19	6	0	81	2	0	6	1	0	3	3	0	9
27	0	0	13	6	0	6	9	0	1	0	3	0	82	2	1	0	1	0	6	3	1	6
28	0	0	14	0	0	7	0	0	1	1	0	0	83	2	1	6	1	0	9	3	2	3
29	0	0	14	6	0	7	3	0	1	1	9	0	84	2	2	0	1	1	0	3	3	0
30	0	0	15	0	0	7	6	0	1	2	6	0	85	2	2	6	1	1	3	3	3	9
31	0	0	15	6	0	7	9	0	1	3	3	0	86	2	3	0	1	1	6	3	4	6
32	0	0	16	0	0	8	0	0	1	4	0	0	87	2	3	6	1	1	9	3	5	3
33	0	0	16	6	0	8	3	0	1	4	9	0	88	2	4	0	1	2	0	3	6	0
34	0	0	17	0	0	8	6	0	1	5	6	0	89	2	4	6	1	2	3	3	6	9
35	0	0	17	6	0	8	9	0	1	6	3	0	90	2	5	0	1	2	6	3	7	6
36	0	0	18	0	0	9	0	0	1	7	0	0	91	2	5	6	1	2	9	3	8	3
37	0	0	18	6	0	9	3	0	1	7	9	0	92	2	6	0	1	3	0	3	9	0
38	0	0	19	0	0	9	6	0	1	8	6	0	93	2	6	6	1	3	3	3	9	9
39	0	0	19	6	0	9	9	0	1	9	3	0	94	2	7	0	1	3	6	3	10	6
40	0	0	20	0	0	10	0	0	1	10	0	0	95	2	7	6	1	3	9	3	11	3
41	0	0	20	6	0	10	3	0	1	10	9	0	96	2	8	0	1	4	0	3	12	0
42	0	0	21	0	0	10	6	0	1	11	6	0	97	2	8	6	1	4	3	3	12	9
43	0	0	21	6	0	10	9	0	1	12	3	0	98	2	9	0	1	4	6	3	13	6
44	0	0	22	0	0	11	0	0	1	13	0	0	99	2	9	6	1	4	9	3	14	3
45	0	0	22	6	0	11	3	0	1	13	9	0	100	2	10	0	1	5	0	3	15	0
46	0	0	23	0	0	11	6	0	1	14	6	0	200	5	0	0	2	10	0	7	10	0

TABLE 43. Of the areas of circles in *Mash Tun Gallons*, to all diameters in inches, and inches and tenths, from 1 to 120 inches.

Diam. in inches.	0	,1	,2	,3	,4
1	,0034	,0041	,0049	,0058	,0067
2	,0138	,0152	,0167	,0183	,0199
3	,0311	,0332	,0354	,0376	,0400
4	,0553	,0581	,0610	,0639	,0669
5	,0865	,0900	,0935	,0971	,1008
6	,1245	,1287	,1330	,1373	,1417
7	,1695	,1744	,1793	,1843	,1894
8	,2214	,2270	,2326	,2383	,2441
9	,2802	,2865	,2928	,2993	,3957
10	,3460	,3529	,3600	,3670	,3742
	0	,1	,2	,3	,4
11	,4186	,4263	,4340	,4418	,4496
12	,4982	,5066	,5150	,5234	,5320
13	,5847	,5938	,6029	,6120	,6213
14	,6782	,6879	,6977	,7075	,7175
15	,7785	,7889	,7994	,8100	,8206
16	,8858	,8969	,9080	,9193	,9306
17	1,000	1,011	1,023	1,035	1,047
18	1,121	1,133	1,146	1,158	1,171
19	1,249	1,262	1,275	1,288	1,302
20	1,384	1,397	1,411	1,425	1,440
	0	,1	,2	,3	,4
21	1,525	1,540	1,555	1,569	1,584
22	1,674	1,690	1,705	1,720	1,736
23	1,830	1,846	1,862	1,878	1,894
24	1,993	2,009	2,026	2,043	2,060
25	2,162	2,179	2,197	2,214	2,232
26	2,339	2,357	2,375	2,393	2,411
27	2,522	2,541	2,560	2,578	2,597
28	2,712	2,732	2,751	2,771	2,790
29	2,910	2,930	2,950	2,970	2,990
30	3,114	3,134	3,155	3,176	3,197
	0	,1	,2	,3	,4
31	3,325	3,346	3,368	3,389	3,411
32	3,543	3,565	3,587	3,610	3,632
33	3,768	3,791	3,813	3,836	3,860
34	4,000	4,023	4,047	4,070	4,094
35	4,238	4,263	4,287	4,311	4,336
36	4,484	4,509	4,534	4,559	4,584
37	4,737	4,762	4,788	4,814	4,840
38	4,996	5,022	5,049	5,075	5,102
39	5,262	5,290	5,317	5,344	5,371
40	5,530	5,564	5,591	5,619	5,647

TABLE of the areas of circles in *Mash Tun Gallons*, to all diameters in inches, and inches and tenths, from 1 to 120 inches.

Diam. in inches.	,5	,6	,7	,8	,9
1	,0077	,0088	,0099	,0112	,0124
2	,0216	,0233	,0252	,0271	,0290
3	,0423	,0448	,0473	,0499	,0526
4	,0700	,0732	,0764	,0797	,0830
5	,1046	,1085	,1124	,1164	,1204
6	,1461	,1507	,1553	,1600	,1647
4	,1946	,1998	,2051	,2105	,2159
8	,2500	,2559	,2619	,2679	,2740
9	,3122	,3188	,3255	,3323	,3391
10	,3814	,3887	,3961	,4035	,4111
	,5	,6	,7	,8	,9
11	,4576	,4656	,4736	,4817	,4900
12	,5406	,5493	,5580	,5669	,5758
13	,6306	,6400	,6494	,6589	,6685
14	,7275	,7375	,7477	,7579	,7682
15	,8313	,8420	,8529	,8638	,8747
16	,9420	,9534	,9650	,9766	,9882
17	1,059	1,071	1,084	1,096	1,108
18	1,184	1,197	1,219	1,222	1,236
19	1,315	1,329	1,342	1,356	1,370
20	1,454	1,468	1,482	1,497	1,511
	,5	,6	,7	,8	,9
21	1,599	1,614	1,629	1,644	1,659
22	1,751	1,767	1,783	1,798	1,814
23	1,910	1,927	1,943	1,960	1,976
24	2,076	2,093	2,111	2,128	2,145
25	2,259	2,267	2,285	2,303	2,321
26	2,429	2,448	2,466	2,485	2,503
27	2,616	2,635	2,654	2,674	2,693
28	2,810	2,830	2,850	2,870	2,898
29	3,011	3,031	3,052	3,072	3,093
30	3,218	3,240	3,261	3,282	3,303
	,5	,6	,7	,8	,9
31	3,433	3,455	3,477	3,499	3,521
32	3,654	3,677	3,699	3,722	3,745
33	3,883	3,906	3,929	3,953	3,976
34	4,118	4,142	4,166	4,190	4,214
35	4,360	4,385	4,410	4,434	4,459
36	4,609	4,635	4,660	4,685	4,711
37	4,865	4,891	4,917	4,944	4,970
38	5,128	5,155	5,182	5,209	5,236
39	5,398	5,426	5,453	5,481	5,508
40	5,675	5,703	5,731	5,760	5,788

TABLE of the areas of circles in *Mash Tun Gallons*, continued.

Diam. in inches.	0	,1	,2	,3	,4
41	5,816	5,845	5,873	5,902	5,930
42	6,103	6,132	6,162	6,191	6,220
43	6,397	6,427	6,457	6,487	6,517
44	6,698	6,729	6,760	6,790	6,821
45	7,006	7,038	7,069	7,100	7,132
46	7,321	7,353	7,385	7,417	7,449
47	7,643	7,676	7,708	7,741	7,774
48	7,972	8,005	8,038	8,072	8,105
49	8,307	8,341	8,375	8,410	8,444
50	8,650	8,685	8,719	8,754	8,789
	0	,1	,2	,3	,4
51	9,000	9,035	9,070	9,106	9,141
52	9,356	9,392	9,428	9,464	9,500
53	9,719	9,756	9,793	9,830	9,866
54	10,089	10,127	10,164	10,202	10,240
55	10,467	10,505	10,543	10,581	10,619
56	10,851	10,890	10,928	10,967	11,006
57	11,242	11,281	11,321	11,360	11,400
58	11,640	11,680	11,720	11,760	11,801
59	12,044	12,085	12,126	12,167	12,208
60	12,456	12,498	12,539	12,581	12,623
	0	,1	,2	,3	,4
61	12,875	12,917	12,960	13,002	13,044
62	13,301	13,343	13,386	13,430	13,473
63	13,733	13,777	13,820	13,864	13,908
64	14,173	14,217	14,261	14,306	14,350
65	14,619	14,664	14,709	14,754	14,799
66	15,072	15,118	15,164	15,210	15,255
67	15,532	15,579	15,625	15,672	15,718
68	16,000	16,047	16,094	16,141	16,188
69	16,474	16,521	16,569	16,617	16,665
70	16,955	17,003	17,052	17,100	17,149
	0	,1	,2	,3	4,
71	17,442	17,492	17,541	17,590	17,640
72	17,937	17,987	18,037	18,087	18,137
73	18,439	18,490	18,540	18,591	18,642
74	18,948	18,999	19,050	19,102	19,153
75	19,463	19,515	19,567	19,619	19,671
76	19,986	20,038	20,091	20,144	20,197
77	20,515	20,568	20,622	20,675	20,729
78	21,051	21,105	21,160	21,214	21,268
79	21,595	21,649	21,704	21,759	21,814
80	22,145	22,200	22,256	22,311	22,367

TABLE of the areas of circles in *Mash Tun Gallons*, continued.

Diam. in inches.	,5	,6	,7	,8	,9
41	5,959	5,988	6,016	6,045	6,074
42	6,250	6,279	6,308	6,338	6,368
43	6,547	6,577	6,607	6,638	6,668
44	6,852	6,882	6,913	6,944	6,975
45	7,163	7,195	7,226	7,258	7,290
46	7,481	7,514	7,546	7,578	7,611
47	7,807	7,840	7,872	7,906	7,939
48	8,139	8,172	8,206	8,240	8,274
49	8,478	8,512	8,547	8,581	8,615
50	8,824	8,859	8,894	8,929	8,964
	,5	,6	,7	,8	,9
51	9,177	9,213	9,248	9,284	9,320
52	9,537	9,573	9,610	9,646	9,683
53	9,903	9,941	9,978	10,015	10,052
54	10,277	10,315	10,353	10,391	10,429
55	10,658	10,696	10,735	10,773	10,812
56	11,045	11,084	11,124	11,163	11,202
57	11,440	11,480	11,520	11,560	11,600
58	11,841	11,882	11,922	11,963	12,004
59	12,250	12,291	12,332	12,373	12,415
60	12,665	12,707	12,749	12,791	12,833
	,5	,6	,7	,8	,9
61	13,087	13,129	13,172	13,215	13,258
62	13,516	13,559	13,603	13,646	13,690
63	13,952	13,996	14,040	14,084	14,128
64	14,395	14,440	14,484	14,529	14,574
65	14,845	14,890	14,935	14,981	15,027
66	15,301	15,347	15,394	15,440	15,486
67	15,765	15,812	15,859	15,906	15,952
68	16,236	16,283	16,331	16,378	16,426
69	16,713	16,761	16,810	16,858	16,906
70	17,198	17,246	17,295	17,344	17,393
	,5	,6	,7	,8	,9
71	17,689	17,738	17,788	17,838	17,887
72	18,187	18,237	18,288	18,338	18,388
73	18,692	18,743	18,794	18,845	18,896
74	19,205	19,256	19,308	19,360	19,411
75	19,724	19,776	19,828	19,881	19,933
76	20,250	20,302	20,356	20,409	20,462
77	20,782	20,836	20,890	20,944	20,997
78	21,322	21,377	21,431	21,485	21,540
79	21,869	21,924	21,979	22,034	22,090
80	22,423	22,478	22,534	22,590	22,646

TABLE of the areas of circles in *Mash Tun Gallons* continued.

Diam. in inches.	0	,1	,2	,3	,4
81	22,702	22,758	22,814	22,836	22,927
82	23,266	23,323	23,380	23,436	23,493
83	23,802	23,899	23,952	24,010	24,067
84	24,415	24,473	24,531	24,589	24,648
85	25,000	25,058	25,117	25,176	25,259
86	25,591	25,651	25,710	25,770	25,830
87	26,190	26,250	26,310	26,371	26,431
88	26,795	26,860	26,917	26,978	27,040
89	27,408	27,469	27,531	27,593	27,655
90	28,027	28,089	28,152	28,214	28,277
	0	,1	,2	,3	,4
91	28,655	28,716	28,780	28,843	28,906
92	29,287	29,350	29,414	29,478	29,542
93	39,907	29,991	30,056	30,120	30,185
94	30,574	30,639	30,704	30,769	30,835
95	31,193	31,294	31,360	31,425	31,491
96	31,889	31,955	32,022	32,088	32,155
97	32,557	32,624	32,691	32,758	32,826
98	33,231	33,299	33,367	33,435	33,503
99	33,913	33,982	34,050	34,119	34,188
100	34,602	34,671	34,740	34,810	34,879
	0	,1	,2	,3	,4
101	35,297	35,367	35,437	35,507	35,577
102	36,000	36,070	36,141	36,212	36,282
103	36,709	36,746	36,852	36,923	36,995
104	37,425	37,497	37,569	37,607	37,714
105	38,148	38,221	38,294	38,367	38,440
106	38,880	38,952	39,025	39,099	39,172
107	39,615	39,690	39,763	39,838	39,912
108	40,359	40,434	40,509	40,580	40,659
109	41,110	41,182	41,261	41,337	41,413
110	41,868	41,979	42,020	42,097	42,173
	0	,1	,2	,3	,4
111	42,633	42,710	42,786	42,863	42,941
112	43,404	43,482	43,560	43,637	43,715
113	44,183	44,261	44,339	44,418	44,496
114	44,968	45,047	45,126	45,205	45,284
115	45,761	45,840	45,920	46,000	46,080
116	46,560	46,640	46,721	46,801	46,847
117	47,366	47,482	47,528	47,610	47,691
118	48,181	48,261	48,343	48,425	48,507
119	49,000	49,082	49,164	49,247	49,329
120	49,826	49,910	49,993	50,076	50,159

TABLE of the areas of circles in *Mash Tun Gallons*, continued.

Diam. in inches.	,5	,6	,7	,8	,9
81	22,983	23,040	23,096	23,160	23,209
82	23,551	23,608	23,664	23,722	23,779
83	24,125	24,183	24,241	24,299	24,359
84	24,706	24,765	24,823	24,882	24,941
85	25,294	25,354	25,413	25,472	25,532
86	25,890	25,950	26,010	26,070	26,130
87	26,492	26,552	26,613	26,674	26,734
88	27,101	27,162	27,223	27,285	27,346
89	27,717	27,779	27,841	27,903	27,961
90	28,339	28,402	28,465	28,528	28,587
	,5	,6	,7	,8	,9
91	28,969	29,029	29,096	29,159	29,232
92	29,606	29,670	29,734	29,798	29,863
93	30,200	30,314	30,379	30,444	30,509
94	30,900	30,965	31,031	31,097	31,162
95	31,557	31,624	31,655	31,756	31,822
96	32,222	32,289	32,356	32,422	32,490
97	32,893	32,961	33,028	33,096	33,164
98	33,571	33,640	33,708	33,776	33,845
99	34,256	34,325	34,394	34,463	34,532
100	34,948	35,018	35,088	35,157	35,227
	,5	,6	,7	,8	,9
101	35,647	35,718	35,788	35,858	35,929
102	36,353	36,424	36,495	36,566	36,638
103	37,066	37,138	37,210	37,281	37,353
104	37,786	37,858	37,930	38,003	38,076
105	38,512	38,586	38,659	38,732	38,805
106	39,246	39,320	39,394	39,467	39,541
107	39,987	40,061	40,135	40,210	40,285
108	40,734	40,809	40,884	40,960	41,035
109	41,488	41,564	41,640	41,716	41,792
110	42,250	42,326	42,403	42,479	42,556
	,5	,6	,7	,8	,9
111	43,018	43,095	43,172	43,249	43,327
112	43,793	43,871	43,949	44,027	44,105
113	44,575	44,653	44,732	44,811	44,890
114	45,364	45,443	45,522	45,602	45,681
115	46,160	46,240	46,320	46,400	46,480
116	46,962	47,043	47,089	47,204	47,285
117	47,772	47,850	47,935	48,016	48,098
118	48,589	48,671	48,753	48,835	48,917
119	49,412	49,495	49,578	49,661	49,743
120	50,243	50,326	50,410	50,493	50,577

TABLE 44. Of the areas of circles in *Ale Gallons*, to all diameters, in inches, and inches and tenths, from 1 to 120 inches.

Diam. in inches.	0	,1	,2	,3	,4
1	,0027	,0033	,0040	,0047	,0054
2	,0111	,0112	,0134	,0147	,0160
3	,0250	,0267	,0285	,0303	,0321
4	,0445	,0468	,0491	,0514	,0539
5	,0696	,0724	,0753	,0782	,0812
6	,1002	,1036	,1070	,1105	,1140
7	,1364	,1403	,1443	,1484	,1525
8	,1782	,1827	,1872	,1918	,1965
9	,2255	,2306	,2357	,2408	,2460
10	,2785	,2841	,2897	,2954	,3012
	0	,1	,2	,3	,4
11	,3369	,3431	,3493	,3556	,3619
12	,4010	,4077	,4145	,4213	,4282
13	,4706	,4779	,4852	,4926	,5000
14	,5458	,5537	,5615	,5695	,5775
15	,6266	,6350	,6434	,6519	,6605
16	,7129	,7219	,7309	,7399	,7490
17	,8048	,8143	,8239	,8335	,8432
18	,9023	,9124	,9225	,9327	,9429
19	1,0054	1,0160	1,0266	1,0374	1,0482
20	1,1140	1,1252	1,1364	1,1477	1,1590
	0	,1	,2	,3	,4
21	1,2282	1,2399	1,2517	1,2635	1,2754
22	1,3479	1,3602	1,3726	1,3850	1,3974
23	1,4733	1,4861	1,4990	1,5120	1,5250
24	1,6042	1,6176	1,6310	1,6445	1,6581
25	1,7406	1,7546	1,7686	1,7827	1,7968
26	1,8827	1,8972	1,9118	1,9264	1,9411
27	2,0303	2,0454	2,0605	2,0757	2,0909
28	2,1835	2,1991	2,2148	2,2305	2,2463
29	2,3422	2,3584	2,3746	2,3909	2,4073
30	2,5065	2,5233	2,5401	2,5569	2,5738
	0	,1	,2	,3	,4
31	2,6764	2,6937	2,7111	2,7285	2,7459
32	2,8519	2,8697	2,8877	2,9056	2,9236
33	3,0329	3,0513	3,0698	3,0883	3,1069
34	3,2195	3,2385	3,2575	3,2766	3,2957
35	3,4117	3,4312	3,4508	3,4704	3,4901
36	3,6094	3,6295	3,6497	3,6698	3,6901
37	3,8128	3,8334	3,8541	3,8748	3,8956
38	4,0216	4,0428	4,0641	4,0854	4,1067
39	4,2361	4,2578	4,2796	4,3015	4,3234
40	4,4562	4,4784	4,5008	4,5232	4,5457

TABLE of the areas of circles in *Ale Gallons*, to all diameters, in inches, and inches and tenths, from 1 to 120 inches.

Diam. in inches.	,5	,6	,7	,8	,9
1	,0062	,0071	,0080	,0090	,0100
2	,0174	,0188	,0203	,0218	,0234
3	,0341	,0360	,0381	,0402	,0423
4	,0563	,0589	,0615	,0641	,0668
5	,0842	,0873	,0904	,0936	,0969
6	,1176	,1213	,1250	,1287	,1325
7	,1566	,1608	,1651	,1694	,1738
8	,2012	,2059	,2108	,2156	,2206
9	,2503	,2566	,2620	,2674	,2729
10	,3070	,3129	,3188	,3248	,3308
	,5	,6	,7	,8	,9
11	,3683	,3747	,3812	,3877	,3943
12	,4351	,4421	,4492	,4563	,4634
13	,5075	,5151	,5227	,5303	,5381
14	,5855	,5936	,6018	,6100	,6183
15	,6691	,6777	,6864	,6952	,7041
16	,7582	,7674	,7767	,7860	,7954
17	,8529	,8627	,8725	,8824	,8923
18	,9532	,9635	,9739	,9843	,9948
19	1,0590	1,0699	1,0808	1,0918	1,1029
20	1,1704	1,1818	1,1933	1,2049	1,2165
	,5	,6	,7	,8	,9
21	1,2874	1,2994	1,3114	1,3235	1,3357
22	1,4099	1,4225	1,4351	1,4478	1,4605
23	1,5380	1,5511	1,5643	1,5775	1,5908
24	1,6717	1,6854	1,6991	1,7129	1,7267
25	1,8110	1,8252	1,8395	1,8538	1,8682
26	1,9558	1,9706	1,9854	2,0003	2,0153
27	2,1062	2,1215	2,1369	2,1524	2,1679
28	2,2621	2,2781	2,2940	2,3100	2,3261
29	2,4237	2,4401	2,4567	2,4732	2,4899
30	2,5908	2,6078	2,6249	2,6420	2,6592
	,5	,6	,7	,8	,9
31	2,7635	2,7810	2,7987	2,8164	2,8341
32	2,9417	2,9598	2,9780	2,9963	3,0146
33	3,1255	3,1442	3,1630	3,1818	3,2006
34	3,3149	3,3342	3,3535	3,3728	3,3922
35	3,5099	3,5297	3,5495	3,5694	3,5894
36	3,7104	3,7308	3,7512	3,7716	3,7922
37	3,9165	3,9374	3,9584	3,9794	4,0005
38	4,1282	4,1496	4,1712	4,1928	4,2144
39	4,3454	4,3674	4,3895	4,4117	4,4339
40	4,5682	4,5908	4,6134	4,6361	4,6589

TABLE of the areas of circles in *Ale Gallons*, continued.

Diam. in inches.	0	,1	,2	,3	4,
41	4,6817	4,7046	4,7275	4,7505	4,7735
42	4,9129	4,9363	4,9598	4,9833	5,0069
43	5,1490	5,1736	5,1976	5,2217	5,2459
44	5,3919	5,4164	5,4410	5,4657	5,4904
45	5,6398	5,6649	5,6900	5,7152	5,7405
46	5,8932	5,9189	5,9446	5,9703	5,9962
47	6,1522	6,1784	6,2047	6,2310	6,2574
48	6,4168	6,4436	6,4704	6,4973	6,5242
49	6,6870	6,7143	6,7417	6,7691	6,7966
50	6,9627	6,9906	7,0185	7,0465	7,0745
	0	,1	,2	,3	,4
51	7,2440	7,2724	7,3009	7,3295	7,3581
52	7,5309	7,5599	7,5889	7,6180	7,6472
53	7,8233	7,8528	7,8825	7,9121	7,9418
54	8,1213	8,1514	8,1816	8,2118	8,2421
55	8,4249	8,4555	8,4863	8,5170	8,5479
56	8,7340	8,7652	8,7965	8,8279	8,8592
57	9,0487	9,0805	9,1124	9,1442	9,1762
58	9,3690	9,4014	9,4338	9,4662	9,4987
59	9,6949	9,7278	9,7607	9,7937	9,8268
60	10,0263	10,0598	10,0933	10,1268	10,1604
	0	,1	,2	,3	,4
61	10,3633	10,3973	10,4314	10,4655	10,4997
62	10,7059	10,7404	10,7751	10,8097	10,8445
63	11,0540	11,0891	11,1243	11,1595	11,1948
64	11,4077	11,4434	11,4791	11,5149	11,5508
65	11,7670	11,8032	11,8395	11,8759	11,9123
66	12,1318	12,1686	12,2055	12,2424	12,2793
67	12,5023	12,5396	12,5770	12,6145	12,6520
68	12,8783	12,9162	12,9541	12,9921	13,0302
69	13,2598	13,2983	13,3368	13,3754	13,4140
70	13,6469	13,6860	13,7250	13,7642	13,8034
	0	,1	,2	,3	,4
71	14,0396	14,0792	14,1188	14,1585	14,1983
72	14,4379	14,4780	14,5182	14,5585	14,5988
73	14,8417	14,8824	14,9232	14,9640	15,0048
74	15,2512	15,2924	15,3337	15,3751	15,4165
75	15,6661	15,7079	15,7498	15,7917	15,8337
76	16,0867	16,1290	16,1715	16,2139	16,2565
77	16,5128	16,5557	16,5987	16,6417	16,6848
78	16,9445	16,9880	17,0315	17,0751	17,1187
79	17,3818	17,4258	17,4699	17,5140	17,5582
80	17,8246	17,8692	17,9138	17,9585	18,0033

TABLE of the areas of circles in *Ale Gallons*, continued.

Diam. in inches.	,5	,6	,7	,8	,9
41	4,7966	4,8197	4,8429	4,8662	4,8895
42	5,0305	5,0542	5,0780	5,1018	5,1257
43	5,2701	5,2943	5,3186	5,3430	5,3674
44	5,5151	5,5400	5,5648	5,5898	5,6147
45	5,7658	5,7912	5,8166	5,8421	5,8676
46	6,0220	6,0480	6,0739	6,1000	6,1261
47	6,2838	6,3103	6,3369	6,3635	6,3901
48	6,5512	6,5782	6,6053	6,6325	6,6597
49	6,8241	6,8517	6,8794	6,9071	6,9349
50	7,1027	7,1308	7,1590	7,1873	7,2156
	,5	,6	,7	,8	,9
51	7,3867	7,4154	7,4442	7,4730	7,5019
52	7,6764	7,7057	7,7350	7,7644	7,7938
53	7,9716	8,0014	8,0313	8,0613	8,0913
54	8,2724	8,3028	8,3332	8,3637	8,3943
55	8,5788	8,6097	8,6407	8,6717	8,7029
56	8,8907	8,9222	8,9537	8,9854	9,0170
57	9,2082	9,2402	9,2724	9,3045	9,3367
58	9,5313	9,5639	9,5965	9,6293	9,6620
59	9,8599	9,8931	9,9263	9,9596	9,9929
60	10,1941	10,2278	10,2616	10,2955	10,3294
	,5	,6	,7	,8	,9
61	10,5339	10,5682	10,6025	10,6369	10,6714
62	10,8792	10,9141	10,9490	10,9839	11,0189
63	11,2302	11,2656	11,3010	11,3365	11,3721
64	11,5867	11,6226	11,6586	11,6947	11,7308
65	11,9487	11,9852	12,0218	12,0584	12,0951
66	12,3164	12,3534	12,3906	12,4277	12,4650
67	12,6896	12,7272	12,7649	12,8026	12,8404
68	13,0683	13,1065	13,1448	13,1831	13,2214
69	13,4527	13,4914	13,5302	13,5691	13,6080
70	13,8426	13,8819	13,9212	13,9607	14,0001
	,5	,6	,7	,8	,9
71	14,2381	14,2779	14,3178	14,3578	14,3978
72	14,6391	14,6795	14,7200	14,7605	14,8011
73	15,0458	15,0867	15,1277	15,1688	15,2100
74	15,4580	15,4995	15,5411	15,5827	15,6244
75	15,8757	15,9178	15,9599	16,0021	16,0444
76	16,2991	16,3417	16,3844	16,4271	16,4699
77	16,7280	16,7712	16,8144	16,8577	16,9011
78	17,1624	17,2062	17,2500	17,2939	17,3378
79	17,6025	17,6468	17,6912	17,7356	17,7801
80	18,0481	18,0930	18,1379	18,1829	18.2279

TABLE of the areas of circles in *Ale Gallons* continued.

Diam. in inches.	0	,1	,2	,3	,4
81	18,2730	18,3181	18,3633	18,4086	18,4539
82	18,7270	18,7727	18,8184	18,8642	18,9101
83	19,1865	19,2328	19,2791	19,3255	19,3719
84	19,6516	19,6984	19,7453	19,7922	19,8392
85	20,1223	20,1697	20,2171	20,2646	20,3121
86	20,5985	20,6465	20,6945	20,7425	20,7906
87	21,0804	21,1289	21,1774	21,2260	21,2747
88	21,5678	21,6168	21,6659	21,7151	21,7643
89	22,0607	22,1103	22,1600	22,2097	22,2595
90	22,5593	22,6094	22,6596	22,7099	22,7602
	0	,1	,2	,3	,4
91	23,0634	23,1141	23,1649	23,2157	23,2666
92	23,5730	23,6243	23,6756	23,7270	23,7785
93	24,0883	24,1401	24,1920	24,2439	24,2959
94	24,6091	24,6615	24,7139	24,7664	24,8190
95	25,1355	25,1884	25,2414	25,2945	25,3476
96	25,6674	25,7209	25,7745	25,8281	25,8818
97	26,2050	26,2590	26,3131	26,3673	26,4215
98	26,7481	26,8027	26,8573	26,9121	26,9668
99	27,2967	27,3519	27,4071	27,4624	27,5177
100	27,8510	27,9067	27,9625	28,0183	28,0742
	0	,1	,3	,0	,4
101	28,4108	28,4670	28,5234	28,5798	28,6362
102	28,9761	29,0330	29,0899	29,1468	29,2038
103	29,5471	29,6045	29,6619	29,7194	29,7770
104	30,1236	30,1815	30,2396	30,2976	30,3558
105	30,7057	30,7642	30,8228	30,8814	30,9401
106	31,2933	31,3524	31,4115	31,4707	31,5300
107	31,8866	31,9462	32,0059	32,0656	32,1254
108	32,4854	32,5455	32,6058	32,6661	32,7264
109	33,0897	33,1505	33,2113	33,2721	33,3330
110	33,6997	33,7610	33,8223	33,8837	33,9452
	0	,1	,3	,3	,4
111	34,3152	34,3770	34,4389	34,5009	34,5629
112	34,9362	34,9987	35,0611	35,1237	35,1862
113	35,5629	35,6259	35,6889	35,7520	35,8151
114	36,1951	36,2586	36,3222	36,3859	36,4496
115	36,8329	36,8970	36,9611	37,0253	37,0896
116	37,4763	37,5409	37,6056	37,6703	37,7352
117	38,1252	38,1904	38,2556	38,3209	38,3863
118	38,7797	38,8454	38,9113	38,9771	39,0430
119	39,4398	39,5061	39,5724	39,6389	39,7053
120	40,1054	40,1723	40,2392	40,3062	40,3732

TABLE of the areas of circles in *Ale Gallons*, continued.

Diam. in inches.	,5	,6	7,	,8	,9
81	18,4993	18,5447	18,5902	18,6357	18,6813
82	18,9560	19,0020	19,0481	19,0941	19,1403
83	19,4184	19,4649	19,5115	19,5581	19,6049
84	19,8863	19,9334	19,9805	20,0277	20,0750
85	20,3597	20,4074	20,4551	20,5029	20,5507
86	20,8388	20,8870	20,9352	20,9836	21,0319
87	21,3234	21,3721	21,4210	21,4698	21,5188
88	21,8135	21,8629	21,9123	21,9617	22,0112
89	22,3093	22,3592	22,4091	22,4591	22,5092
90	22,8106	22,8611	22,9115	22,9621	23,0127
	,5	,6	,7	,8	,9
91	23,3175	23,3685	23,4195	23,4707	23,5218
92	23,8300	23,8815	23,9331	23,9848	24,0365
93	24,3480	24,4001	24,4523	24,5045	24,5568
94	24,8716	24,9243	24,9770	25,0296	25,0826
95	25,4008	25,4540	25,5073	25,5606	25,6140
96	25,9355	25,9893	26,0431	26,0970	26,1510
97	26,4758	26,5301	26,5845	26,6390	26,6935
98	27,0217	27,0766	27,1315	27,1865	27,2416
99	27,5731	27,9286	27,6841	27,7397	27,7953
100	28,1302	28,1862	28,2422	28,2983	28,3545
	,5	,6	,7	,8	,9
101	28,6927	28,7493	28,8059	28,8626	28,9193
102	29,2609	29,3180	29,3752	29,4324	29,4897
103	29,8346	29,8923	29,9501	30,0078	30,0657
104	30,4139	30,4722	30,5305	30,5888	30,6472
105	30,9988	31,0576	31,1165	31,1754	31,2343
106	31,5893	31,6486	31,7080	31,7675	31,8270
107	32,1853	32,2452	32,3051	32,3652	32,4252
178	32,7868	32,8473	32,9078	32,9684	33,0290
109	33,3940	33,4550	33,5161	33,5772	33,6384
110	34,0067	34,0683	34,1299	34,1916	34,2534
	,5	,6	,7	,8	,9
111	34,6250	34,6871	34,7493	34,8116	34,8739
112	35,2489	35,3116	35,3743	35,4371	35,5000
113	35,8783	35,9416	36,0049	36,0682	36,1316
114	36,5133	36,5771	36,6410	36,7049	36,7689
115	37,1539	37,2182	37,2827	37,3471	37,4117
116	37,8000	37,8649	37,9299	37,9950	38,0600
117	38,4517	38,5172	38,5827	38,6483	38,7140
118	39,1090	39,1751	39,2411	39,3073	39,3735
119	39,7719	39,8385	39,9051	39,9718	40,0386
120	40,4403	40,5075	40,5746	40,6420	40,7093

The Explanation and Use of the foregoing TABLES.

THE first table for common brewers strong beer begins with $\frac{1}{4}$ of a barrel, and ends with 50,000; therefore you must look for the given number of barrels and firkins (if any) in the column under barrels, and right against it, in the next column to the right hand, you will find the duty, in pounds, shillings, pence, farthings, and 23 parts of a farthing.

E. 1. What is the duty of 21$\frac{1}{2}$ barrels of common brewers strong beer, at 8*s.* per barrel?

Against 21$\frac{1}{2}$ barrels is 7*l.* 13*s.* 3*d.* 2*q.* 14*p.* the duty required.

E. 2. What is the duty of 96$\frac{1}{4}$ barrels of common brewers strong beer, at 8*s.* per barrel?

	Bar.		*£.*	*s.*	*d.*	*q.*	*p.*	
Against	96	is	34	4	6	1	1	
Against	$\frac{1}{4}$	is	0	1	9	1	13	
Answer	96$\frac{1}{4}$	is	34	6	3	2	14	the duty required.

The use of table 2 and 3 being exactly the same, they need no example.

Table 4, for victuallers strong beer, begins with $\frac{1}{4}$ of a barrel, and ends with 50,000, and increaseth only $\frac{1}{4}$ of a barrel to 100, whereby the duty of any number of barrels, fourths, &c. from $\frac{1}{4}$ to 100, may be found by inspection.

EXAMPLE. What is the duty of 84$\frac{1}{2}$ barrels of victuallers strong beer, at 8*s.* per barrel?

Against 84$\frac{1}{2}$ barrels is 33*l.* 16*s.* 0*d.* the duty required.

Table 5, for victuallers small beer, is exactly the same as for strong, therefore examples would be unnecessary.

Table 6, for the drawback on victuallers strong beer begins, increaseth, and ends the same as the cash tables for victuallers strong and small beer; therefore you must look for the given number of barrels, fourths, &c. in the column under barrels, and against it in the next column to the right hand, you have the drawback.

EXAMPLE. Suppose a victualler be charged with 60$\frac{1}{2}$ barrels of strong beer, at 8*s.* per barrel, what must be allowed him for the drawback, at 1*s.* 8d. per barrel?

Against 60$\frac{1}{2}$ barrels is 5*l.* 0*s.* 10d. the allowance for drawback.

Table 7, for the drawback on victuallers small beer, is the same as for strong, both with regard to its beginning, increase, and end, therefore I shall omit giving any examples.

Table 8, to money couch bushels, begins with one-tenth of a bushel, and ends with 1000 bushels, shewing at one view all the separate duties by inspection, agreeable to the several acts of parliament for laying additional duties thereon, with a total column of the whole.

Table 9, to money floor bushels, begins, increaseth, and ends the same as table 8, and likewise shews all the separate duties by inspection, with a total column of the whole, both of which are so plain, that to give any examples would be looked upon as tautology.

Table

Table 10, of 5*l.* and 15*l.* per cent. begins with one farthing, and increaseth by one farthing, to twenty shillings; therefore you have nothing more to do but look for the value in the column under value, and right against it, and under the rate per cent. you have the duty.

E. 1. Suppose the value was 11*s.* 6¼*d.* what is the duty at 5 per cent.

Against 11*s.* 6¼*d.* and under 5 per cent. is 6*d.* 3*q.* 65*p.* the duty required.

E. 2. What is the duty of 16*s.* 11¾*d.* at 15*l.* per cent.

Against 16*s.* 11¾*d.* and under 15*l.* per cent. is 10*d.* 0*q.* 75*p.* the duty required.

Table 11, to money the old drawback on soap, at ⅓ of a penny per *lb.* this table begins with 1 and ends with 6,000, and right against every respective number, in the next column to the right hand, you find the value, duty, or amount.

E. 1. What is the drawback on 900*lb.* of soap, at ⅓ of a penny per *lb.*

Against 900*lb.* is 1*l.* 5*s.* 0*d.* the duty or drawback required.

E. 2. Required the duty of 2640*lb.* of soap, at ⅓ of a penny per *lb.*

	lb.		£.	*s.*	*d.*	*q.*	*p.*	
Against	2000	is	2	15	6	2	2	
Against	600	is	0	16	8	0	0	
Against	40	is	0	1	1	1	1	
	Answer		3	13	4	0	0	the duty required.

Table 12, cash table for the additional duty on soap, at ½ per *lb.*

Table 13, additional duty cash table to money drawbacks upon soap, at ¾ per *lb.*

Table 14, cash table for the duty on tallow, candles, and hops, at 1*d.* per *lb.*

Table 15, cash table, for the duty on tawed sheep and lamb skins, at 1¼*d.* per *lb.*

Table 16, cash table for the duty on sheep, lamb, &c. tanned; calf and kid tawed; and soap, at 1½*d.* per *lb.*

Table 17, cash table for the duty on roans, tanned skins, pieces in oil, and starch, at 2*d.* per *lb.*

Table 18, cash table for the duty on sheep and lamb in oil, &c.

Table 19, cash table for the duty on goats, tanned with shumack, at 4*d.* per *lb.*

Table 20, cash table for the duty on silver wire, buck and doe skins tawed; hides, deer, goat, and beaver, in oil, at 6*d.* per *lb.*

Table 21, cash table for the duty on wax candles, gilt wire, and calf skins, in oil, at 8*d.* per *lb.*

Table 22, cash table for the duty on common bottle glass, at 3*s.* 6*d.* per *cwt.*

Table 23, cash table for the duty on malt cyder, at 4*s.* per *hhd.*

Table 24, cash table for the duty on cyder, in the year 1766, at 6*s.* per *hhd.*

Table 25, cash table for the excise duty on cyder, at 6*s.* 8*d.* per *hhd.* and verjuice, at 6*s.* 8*d.* per *barrel.*

Table

Table 26, caſh table for the duty on broad glaſs, at 7*s.* per *cwt.*

Table 27, caſh table for the duty on vinegar, at 8*s.* 9*d.* per *barrel.*

Table 28, caſh table for the duty on crown glaſs, at 14*s.* per *cwt.*

Table 29, caſh table for the duty on ſweets, at 18*s.* per *barrel*, of 31½ gallons to the barrel.

Table 30, caſh table for the duty on plate or flint glaſs, at 18*s.* 8*d.* per *cwt.*

Though theſe tables are adapted for the uſe of the exciſe officers, for their ready aſcertaining the duties therein mentioned, yet they are equally as uſeful to all traders, in buying and ſelling all ſorts of goods, wholeſale or retail, ſhewing at one view the amount or value of any number or quantity of goods or merchandize, according to the price fixed at the head of the table.

Table 31, of the rates of hides. In this table you have every ſpecies at one view, likewiſe how they are dreſſed and charged; and to ſave the officer and trader the trouble of computing the amount of the duty of thoſe goods which pay duty ad valorum, after the rates of 15*l.* and 30*l.* per cent. I have added table 32, at 15*l.* per cent. and table 33, at 30*l.* per cent. whereby the duty may be found by inſpection.

Table 34, contains the value, dimenſions, and duty on all ſorts of paper, agreeable to the act of parliament made in the year 1781, for laying additional duties thereon.

Table 35, is an ad valorum table, of equal parts, at 18*l.* per cent. very uſeful to find the duty on all ſorts of paper that pay duty ad valorum.

Table 36, for reducing hundreds, quarters, and pounds, into pounds groſs, and pounds groſs into pounds net; the allowance for the bag or tare, which is 1 pound in 10, being deducted.

EXAMPLE. In 18 *cwt.* 3 *qrs.* 20 *lb.* of bag hops, I deſire to know the number of pounds groſs, net, and likewiſe how much the exciſe will amount to, at 1*d.* per *lb.*

Firſt, for the pounds groſs, by table 36.

	C.	q.	lb.		lb.
Againſt	18	0	0	is	2016
———	0	3	20	is	104
				Anſwer	2120 pounds groſs.

Secondly, for the pounds net, by the ſame table.

	lbs. groſs.		lbs. net.
Againſt	2016	is	1814,4
———	104	is	93,6
Anſwer	2120	is	1908,0 pounds net.

Thirdly, for the amount of the exciſe, by table 14.

	lb.		£.	s.	d.
Againſt	1000	is	4	3	4
———	900	is	3	15	0
———	8	is	0	0	8
Anſwer	1908	is	7	19	0 the amount of the exciſe.

Note, If you want to turn pounds net into pounds groſs, or into hundreds, quarters, and pounds; find the pounds net (in table 36) and in the next column, to the left hand, you have the pounds groſs, and next to that the hundreds, quarters, and pounds that are equal to it.

Table

Table 37, is particularly adapted for the uſe of collectors, ſupervifors, &c. whereby the number of days, from the 5th of July to any given day of any month may be found by inſpection.

E. 1. Suppoſe an officer be removed to another collection the 4th day of September, how many days ſalary has he to receive, ſuppoſing he has received ſalary to the 5th of July incluſive?

Againſt the 4th day of September you find 61, the number of days required.

E. 2. It is required to find the number of days from the 5th of July to the 31ſt of March?

Look for the 31ſt of March, and right againſt it you find 269, the number of days required.

Note, In leap year when February is included, you muſt add one day to the number found in the table.

Table 38, is to convert gallons into barrels, and contra, barrels into gallons, this table begins with 8½ gallons, and ends with 1122, and increaſeth by half a gallon, to 204 gallons; therefore, againſt the given number of gallons, in the column under gallons, in the next column to the right hand, under barrels and gallons, you have the anſwer in barrels, fourths, and gallons.

E. 1. Suppoſe it was required to reduce 194 gallons to barrels, &c.
Againſt 194 *gall.* is 5½ *bar.* 7 *gall.* the anſwer.

E. 2. In 1045,5 gallons, how many barrels?
Againſt 1045,5 *gall.* is 30¾ *bar.* the anſwer.

Table 39, to find the number of days from any one day to any other in the ſame year, or to any day of any month in the next year, by one ſubtraction only. This table is very uſeful in all parts of arithmetical ſcience, relating to time; and likewiſe in computing the time for ſervants wages, and all other ſalaries, of what time or rate ſoever. See the explanation and uſe page 428 and 429.

Table 40. Though this table is particularly adapted for the uſe of collectors, ſupervifors, officers of exciſe, &c. yet it is very uſeful in computing ſervants wages, and all other ſalaries that come within the limits of the table.

To find the ſalary at any rate per annum, look for the number of days in the columns under days, and right againſt it, and under the yearly ſalary, you will find the anſwer, in pounds, ſhillings, pence, farthings, and decimal parts of a farthing; whoſe denominator in a common year is 365, the number of days in the ſaid year, which denominator reduced to its loweſt terms = 73, therefore all the decimal parts in this table are ſo many 73 parts of a farthing.

E. 1. What ſalary has an officer due to him for 80 days, at 50*l.* per annum?

Againſt 80 *days*, and under 50*l.* is 10*l.* 19*s.* 2*d.* 0*q.* 40*p.* the ſalary required.

E. 2. How much would be due to a ſupervifor for 260 days, at 90*l.* per annum?

	Days.		*£.*		*£.*	*s.*	*d.*	*q.* *p.*
Againſt	200	and under	90	is	49	6	3	2,34
	60			is	14	15	10	2,54
Anſwer	260	at 90*l.* per ann.		is	64	2	2	1,15 the ſalary required.

Table

Table 41, is the salaries for leap year, or 366 days; therefore look for the given number of days in the column under days, and right against it, and under the yearly salary is the answer, in pounds, shillings, pence, farthings, and decimal parts of a farthing; whose denominator in a leap year is 366, the number of days in the said year, which denominator reduced to its lowest terms = 61; therefore all the decimal parts in this table are so many 61 parts of a farthing.

E. 1. How much would be due to an officer for 60 days, at 50*l.* per annum, leap year?

Against 60 *days*, and under 50*l.* is 8*l.* 3*s.* 11*d.* 0*q.* 52*p.* the salary required.

E. 2. What salary has an officer due to him for 120 days, at 40*l.* per annum, leap year?

	Days.		£.		£.	*s.*	*d.*	*q.*	*p.*
Against	100	and under	40	is	10	18	6	3,49	
———	20	———	40	is	2	3	8	2,22	
Answer	120	days, at	40 per ann.	=	13	2	3	2,10	the salary requir.

Table 42, for the use of auctioneers, also for tax and charity. This table is divided into four columns, the first of which contains the value or salary, the second, tax or duty, at 6*d.* the third, the charity or duty, at 3*d.* the fourth, the total, at 9*d.* per pound. Therefore look for the value or salary in the column so tituled at the head, and right against it, in the next three columns to the right hand, you will find the duty at 6*d.* and 3*d.* and the total at 9*d.* in the pound.

E. 1. Suppose an auctioneer hath sold goods, &c. to the value of 32*l.* what duty has he to pay for the same, at 6*d.* and 3*d.* in the pound?

Against 32*l.* you will find, in the two next columns, 16*s.* and 8*s.* = 1*l.* 4*s.* the duty required.

E. 2. Suppose an officer's gross salary amounted to 18*l.* for 73 days, what must be deducted therefrom for tax and charity?

Against 18*l.* the gross salary, is 9*s.* tax, and 4*s.* 6*d.* charity, = 13*s.* 6*d.* to be deducted. ∴ 18*l.* — 13*s.* 6*d.* = 17*l.* 6*s.* 6*d.* the neat salary.

And in this manner the deduction for tax and charity, and also the neat salary may be found.

Table 43, of the areas of circles in mash tun gallons. In the 1st column, under the words diameter in inches, seek for the even inches of any given diameter, from 1 to 120 inches, and right against it, under 0 in the 2d, you will find the area to the thousandth part of a gallon. But if the given diameter consists of inches and tenths of an inch, then seek the even inches of the diameter in either the 1st or 7th columns, and the tenths in any of the other 9 columns at the head of the table, and at the angle of meeting you will find the area as before.

E. 1. Suppose it were required to find the area of a circle in mash tun gallons, whose diameter was 70 inches?

I look for 70 in the first column, and right against it, in the second column, under 0 I find 16,955, the area required.

E. 2. Suppose the given diameter were 75,4 inches, what is the area in mash tun gallons?

I find 75 in the first column, and the four-tenths at the head of the table, then at the angle of meeting I find 19,671, the area in mash tun gallons.

E. 3.

E. 3. Suppose the given diameter were 40,8 inches, what is the area in mash tun gallons?

I find 40 in the 7th column, and the eight-tenths at the head of the table, then at the angle of meeting is 5,760, the area in mash tun gallons.

And thus may the area of any other diameter within the compass of the table be found. But if you want to know the area of a circle of a greater diameter than any contained in the table, you may find it by the following directions: Thus, when the given diameter exceeds 120, seek for the area of half that diameter, in the table, and multiply it by 4, and that product will be equal to the area sought.

EXAMPLE. Suppose the given diameter were 160 inches, what is the area thereof in mash tun gallons?

I look in the table for 80, half the given diameter, whose area is 22,145, which multiplied by 4 is = 88,580, the area required.

And in this manner the area of any diameter that exceeds the bounds of the table, may be found.

Table 44, of ale areas. The explanation and use of the table of mash tun areas may serve to explain this also, because the diameters and areas are placed alike in both tables; for if you find any diameter from 1 to 120 inches in the 1st or 7th column, and the tenths (if any) at the head of the table, at the angle of meeting you will have the area required, to the ten thousandth part of an ale gallon.

In case the given diameter should exceed the limits of the table, then, according to the directions for mash tun areas, find the areas of half the given diameter, which multiply by 4, and the product will be equal to the area required.

The computation of tables of the areas of circles in mash tun, ale, and wine gallons, according to the common method, (page 354) being very laborious, and consequently liable to error, I apprehend the following exact and easy methods of computation, will be acceptable to such persons as are desirous of examining, or extending the said tables to larger diameters.

The method of operation by which the whole table of mash tun areas was computed, is as follows:—To ,00346, the area for 1 inch, add ,00692, and the sum ,07266 (called the *reserved sum*) being added to ,00346 (the area for 1 inch diameter) gives ,0041866 the area for 1,1 inches; again, to ,07266 (the *reserved sum*) add the common addend ,00692*, and this (*reserved*

---

* The reason of ,00692 being a common addend for a mash tun; ,000055702 a common addend for ale, and ,000068 a common addend for wine gallons, is very evident from the following

L E M M A.

If the terms of any arithmetical progression (either ascending or descending) be squared, and disposed of in a series, then will the differences of every two adjacent terms of that series, form another arithmetical progression, whereof the common difference will be expressed by twice the square of the common difference of the first progression.

For any arithmetical progression, whose first term is m, and the common difference n, will be expressed by m, $+ m \pm n$, $+ m \pm 2n$, $+ m \pm 3n$, $+ m \pm 4n$, $+$ &c. whereof the square of each term is, m^2, $+ m^2 \pm 2mn + n^2$, $+ m^2 \pm 4mn + 4n^2 + m^2 \pm 6mn + 9n^2$, $+ m^2 \pm 8mn + 16n^2$, $+$, &c.

The

(reserved sum) ,07958, being added to ,0041866, gives ,0049824, the area for 1,2. Proceed in this manner, still adding the last reserved sum, and the common addend ,00692 together, and then adding the sum of those two to the last area; *i. e.* when the diameter is increased by one-tenth of an inch.——See the following operation.

Inches.	*Gallons.*		
1 —	,00346	,00692	common addend.
add	007266	,00346	
1,1 —	,0041866	,007266	reserved sum.
add	,007958	,00692	
1,2 —	,0049824	,007958	
	&c.	&c.	

The table of ale areas being derived from the very same principle, therefore for the satisfaction of those who are inclined to proceed farther, proceed thus: To ,067315867 add ,000055702, and the sum ,067371569 (called the *reserved sum*) being added to 40,709272233, gives 40,776643802, the area for 121 inches, and so on; see the following operation.

Inches.	*Gallons.*		
120,9 —	40,709272233	,067315867	reserved sum, at 120,9.
	,067371569	,000055702	
121,0 —	40,776643802	,067371569	reserved sum.
	,067427271	,000055702	
121,1 —	40,844071073	,067427271	
	&c.	&c.	

The differences of the two adjacent terms will form the series, viz. $\pm 2mn + n^2$, $\pm 2mn + 3n$, $\pm 2mn + 5n^2$, $\pm 2mn + 7n^2$, +, &c. the common difference of which is evidently $2n$. Q. E. I.

And in this case, $n = ,1$, $\therefore 2n^2 = \overline{,1}|^2 \times 2$ (or ,02;) which being multiplied by ,00346 for the mash tun; ,0027851 for ale, and ,0034 for wine (in order to have its measure in parts of a gallon) gives ,00692, the common addend for mash tuns, ,000055702, the common addend for ale, and ,000068, the common addend for wine gallons, when the diameter of the circle is constantly increased by one-tenth of an inch.

LXXVI. SURVEYING.

IN the following instructions for surveying, I shall make use of no other instrument but the chain, for that used properly, is the compleatest for the purpose both in accuracy and expedition.

The chain now most in use (commonly called Gunter's) contains 4 poles, or 22 yards in length, and is divided into 100 decimal parts, or links, each link containing 7,92 inches; and an acre contains 10 square chains, viz. 10 in length and 1 in breadth; there are likewise 100,000 square links in an acre.

And for the learner's better information, I have inserted the following new table of Square Measure.

A TABLE of SQUARE MEASURE.

Inches	Links	Feet	Yards	Paces	Perches	Chains	Acres	Mile
1								
62,7264	1							
144	2,295	1						
1296	20,75	9	1					
3600	57,38	25	2,7	1				
39204	625	272,25	30,25	10,89	1			
627264	10000	4356	484	174,24	16	1		
6272640	100000	43560	4840	1742,4	160	10	1	
4014489600	64000000	27878400	3097600	1115136	102400	6400	640	1

The above table may be read thus, viz. in one chain there are 16 square perches, 174, 24 square paces; 484 square yards; 4356 square feet; 10000 square links; and 627264 square inches; and so of the rest.

The

The laying down of plain figures being already taught in practical geometry, Part 4. sec. 66. it is quite unnecessary to enlarge upon that subject in this place: I shall therefore observe to the learner, that in measuring with the chain, he must be careful to get the shortest distance between any two objects, otherwise he will make more of the land than it really is.

In taking dimensions of a field, it is best to begin at some remarkable place, and from thence proceed according to the situation of the field.

It is proper for the learner, at his entrance into the field, to draw a figure of the same at random, which will enable him to plan the same with more certainty; but when he is ready in the practical part of surveying, such draughts will be unnecessary.

When the field is bounded by irregular hedges, care must be taken to streighten them by taking up off-sets, which must be as near the fence as possible; and every perpendicular must be taken, so, that if a right line was drawn from the end of any one to the next, that line would neither include your neighbour's ground, nor exclude any part of that you are measuring.

Prob. 1. *To measure and find the content of a square piece of land.*

RULE. Multiply the side by itself, the product is the content.

EXAMPLE, How many acres are contained in a square field, each of whose sides are 12 chains and 20 links, or 12,20?

```
       12,20 = side
       12,20
      ------
       24400
      2440
     1220
     -------
     14,88400
            4
     --------
      3,53600
           40
     --------
     21,44000
     A  R  P
Answer 14  3  21
```

Note. Having multiplied the side by itself the product is 1488400, which to reduce into acres, roods, and perches, point off five places of figures to the right hand, the remainder to the left hand are acres, then multiply the figures pointed off to the right hand by 4, and point off five figures to the right hand as before, and the left hand figure is roods; lastly multiply the figures pointed off to the right hand by 40, and point off five figures to the right as before, the remaining figures to the left are perches.

If you work the links as the decimal of a chain, and divide the product by 10, the square chains in an acre, the answer in acres, &c. will be the same.——See the last example worked by this method.

```
           12,20 = side
           12,20
          ------
           24400
          2440
         1220
        --------
     10) 148,8400
        --------       A  R  P
Answer  14,88400  =    4  3  21 the same as before
```

Prob. 2.

Prob. 2. *To find the content of a rectangular piece of land.*

RULE. Multiply the length by the breadth, the product is the content.

EXAMPLE. How many acres are contained in a rectangular field, whose length is 12,15, and breadth 5,40?

12,15 = length
5,40 = breadth
———
48600
6075
———
6,56100 Answer A 6, R 2, P 9
4
———
2,24400
40
———
9,76000

Prob. 3. *To find the content of a triangular piece of land.*

RULE. Multiply the base by half the perpendicular, the product is the content.

EXAMPLE. There is a triangular piece of land whose base measures 10,42, and perpendicular 5,22, what is the content in acres, &c.?

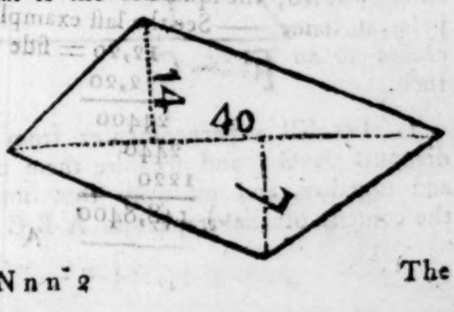
10,42 = base
2,61 = half perpendicular
———
1042
6252
2084
———
2,71962 Answer A 2, R 2, P 35
4
———
2,87848
40
———
35,13920

Prob. 4. *To find the content of a field that is comprehended under four unequal sides, called a trapezium.*

RULE. Multiply the diagonal by half the sum of the perpendiculars, the product is the content.

EXAMPLE. Let the following figure represent a four sided field, whose diagonal measures 40 chains, one perpendicular 7 chains, and the other 14 chains; what is the content in acres?

First 14 + 7 = 21 sum of the perpendiculars; and 21 ÷ 2 = 10,5 half sum of perpendiculars; then 10,5 × 40 = 420, and 420 ÷ 10 = 42 acres the Answer.



 The

The above dimensions being taken by a four pole chain, the product as above, is 420 square chains, which, divided by 10, the quotient is 42 acres, as appears by the work; therefore when one or any of your numbers consists of chains only, there is no necessity to prefix cyphers in the place of links.

The common way of measuring a field of four unequal sides, is by measuring from one corner to the opposite one, which divides it into two triangles, and in measuring this line perpendiculars must be erected to the other corners of the field, the places from whence these perpendiculars rise are found by the cross staff or triangle, fixed on your staff for that purpose, so this diagonal is a base line common to both perpendiculars, which may be measured by the preceding rule.

But you may plot the former field very expeditiously, if you measure round it, putting down each side separately, and likewise measure one diagonal, which divides it into two triangles, each of which may be truly plotted by prob. 14. sec. 66.

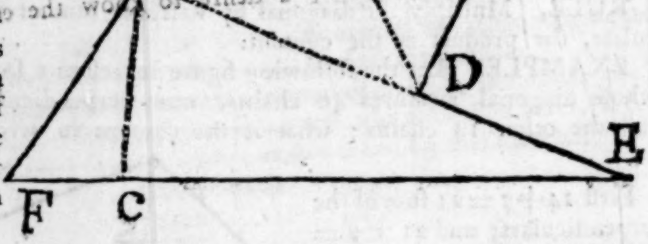
Prob. 5. *To find the content of any irregular field, consisting of any number of sides.*

RULE. Measure round it, and put down each side separately, then reduce it into trapeziums and triangles, and measure each separately, which several dimensions collected into one sum will be the superficial content of the field.

EXAMPLE. Admit a field, A B C D E F, consisting of 6 unequal sides, whose dimensions in chains and links are as follow, viz. from A to B 3,15; B to C 5,90; C to D 4,40; D to E 4,20; E to F 11,40; and from F to A 4,00; what is the content?

To plot this field, proceed as in the last example; then proceed to find the area, thus:

Note. This and the following figures are laid down by a scale of sixteen statute poles, or four chains to an inch.



1. Let fall a perpendicular from B to *a*, and D to *b*, to the diagonal A C; and measure them on your scale, which products add together, and multiply that sum by half A C, and you have the content of the trapezium A B C D.

2. Let

2. Let fall a perpendicular, from A to *c*, to the base F E, and measure it on your scale, which multiply by half F E, and the product is the content of the triangle F A E: then add the contents together, the sum will be the content of the whole field. See the work.

2,70 = B *a*	3,55 = A *c*
3,30 = D *b*	5,70 = ½ F E
6,00 = Sum	24850
3,55 = ½ AC	1775
3000	2,02350 = area of the triangle F A E
3000	
1800	

2,13000 = area of the trapezium A B C D.
2,02350

	A	R	P
4,15350 = area of the whole field =	4	0	24

4
,61400
40
24,56000

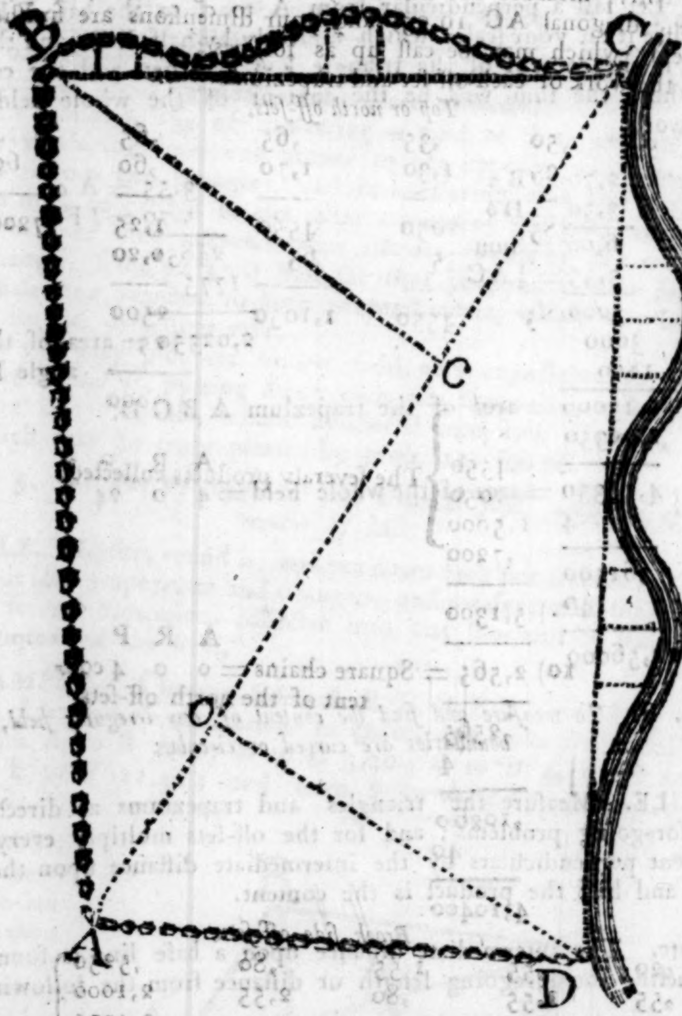
Prob. 6. *To measure and find the content of any irregular field, whose boundaries are curved or circular.*

RULE. Measure the triangles and trapeziums as directed in the foregoing problems; and for the off-sets multiply every two adjacent perpendiculars by the intermediate distance upon the base line, and half the product is the content.

Note. The intermediate distance upon a base line is found by subtracting the foregoing length or distance from the following.

EXAMPLE. Let the following figure represent a field, bounded on one side by a small river; I desire to know the content in acres?

First,



First, suppose you enter the field at A; measure the side AB, which you will find is 16,00,

Secondly, measure the side BC, which is 10,60, and as you measure take up the off-sets, and enter them in your field-book, thus; when you have measured 1 chain from B towards C, take up the first off-set with your staff, which is 0,50; at 2 chains another off-set 0,35; and at 6,25 you will perceive it necessary to take up another off-set, which you will find 0,65; and at 7,45 another, which is 0,60.

Thirdly, measure the side CD, which you will find 16,30, and as you advance take up the off-sets; thus as you measure from C towards D at 3,65, you take up the first off-set = 0,80; at 4,60—1,45; at 6,00—1,00; at 9,40—0,60; at 10,20—1,10; and at 11,10—1,00.

Fourthly,

Fourthly, measure the side DA, which is 9,00; and lastly measure the diagonal AC 19,25, and your dimensions are finished in the field, which may be cast up as follows:

See the work of each off-set and trapezium.

Top or north off-sets.

,50	,50	,35	,65	,65	1,20
1,	,35	1,30	1,70	,60	60
,50	,85	1050	4550	1,25	,7200
	1	35	65	1,20	
	,85	,4550	1,1050	2500	
				125	
				1,5000	

,50
,85
,4550
1,1050
1,5000
,7200
The severaly products collected.

2) 5,1300

		A	R	P
10) 2,565 = Square chains =		0	0	4 content of the north off-sets

,2565
4
,10260
40
4,10400

Brook side off-sets.

,20	,55	1,55	,80	,5250
,55	1,55	,80	2,55	2,1000
,75	2,10	2,35	400	3,4075
,70	1,00	1,45	400	2,0400
,5250	2,1000	1175	160	,5000
		940	2,0400	1,6000
		235		1,7100
		3,4075		2,3400

The several products collected

2) 14,2225

10) 7,1112

	A	R	P
,71112 =	0	2	33

4
2,84448
40
33,77920

Content of the brook side off-sets.

,50	,50	1,00	,90
1,00	1,00	,90	2,60
,5000	1,50	1,90	5400
	1,00	,90	18
	1,6000	1,7100	2,3400

The

The trapezium.

8,70 = B c

8,00 = Ditto

16,70 = Sum of the perpendiculars B c Ditto

6,62 = ½ AC the diagonal

3340

10020

15030

A R P

16,06540 = 16 0 10, content of trapezium

4

,26160

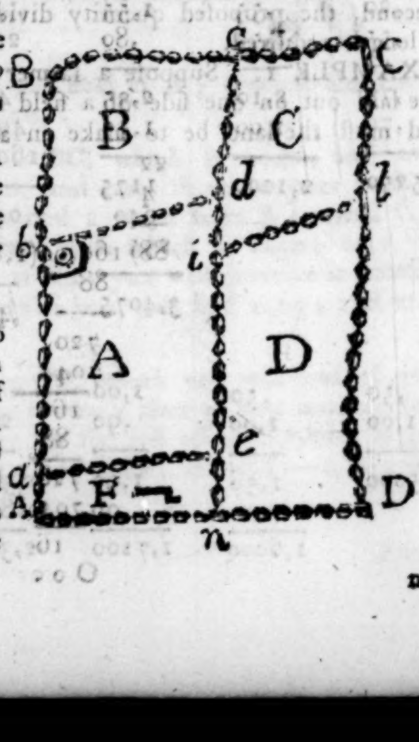
40

10,46400

A	R	P	
16	0	10	= area of trapezium
0	2	33	= area of brookside off-sets
0	0	4	= area of north off-sets
16	3	7	= area of the whole field.

Note. Some practitioners cast up off-sets by dividing the sum of the perpendiculars by the numbers thereof taken for a mean breadth, contained between the straight line and the hedge, which part they cast up as a parallelogram, this erroneous practice is, I fear, too much used, on account of ease and expedition.

EXAMPLE. Let the following figure represent the ground plot of a small estate, left by Mr. Geo. Fentham for charitable uses to the town of Birmingham, and it is required to measure the same with the chain only, so that the dimensions thereof may be cast up and planned; what is the content of the whole, and each field separately?



Suppose you enter the estate at A, 1st, measure from A to a 1 chain, from a to b 4 chains, and from b to B 3 chains, 15 links. Having chained from A to B, proceed to measure the other sides; but as you advance take care to lay down the off-sets, as before directed.

2d. Measure from B to c 3 chains 20 links, from c to d 2,35, from d to i 1,15, from d to b 3,26, the dimensions of the field B being finished, walk to c, and measure from c to C 2,65, from C to b 3,05, and from l to i 2,90, and as c i was measured before, there will be

no

no need of meaſuring or ſtraightening that ſide again, ſo the dimenſions of the field C being finiſhed, proceed from *i* to *e* 4,45, from *e* to *n* 1 chain, from *n* to D 2,90, and from D to *l* 530; the dimenſions in the field D being finiſhed, I walk to *e* and meaſure *ea* 3,10, and as *di* and *ie* was meaſured before, there is no need to meaſure that over again; therefore the dimenſions in the field A are finiſhed.

Laſtly, meaſure from *n* to A 3,25, and your dimenſions in the field are finiſhed, which may be caſt up and planned at pleaſure.

Note. As the off-ſets, &c. are meaſured and caſt up as before directed, I think it will be unneceſſary to inſert the operations at length, as it will give the learner an opportunity of caſting up the ſame, and if the reſult of his work ſhould agree with the following contents of each field, his work is undoubtedly true.

		A	R	P
Houſe, garden, &c. —	F ——	0	1	13
Well-piece —	A — —	1	1	14
Hill-piece — —	B —	0	3	18
Calf's croft ——	C — —	1	2	23
Pit-piece ——	D — —	1	1	37
Total of the whole eſtate		5	2	25

Prob. 7. *To meaſure woods or large pools of water.*

RULE. Meaſure round it, and at every bending or turn take the angle, and meaſure the diſtance from one turn to another, and enter all down in your field-book, and when you come round to the place where you began, if your plot cloſe, your work is right; when you have plotted your work, take off the baſes and perpendiculars from the ſame ſcale you laid down the plot, and from thence caſt up the content in acres, as taught before.

Prob. 8. *To lay down any quantity of land in a field, when the quantity and either the length or breadth are given.*

RULE. Firſt, divide the area of the given quantity, by the length of the field, and the quotient will be the breath required.

Second, the propoſed quantity divided by the breadth will give the length required.

EXAMPLE 1. Suppoſe a farmer lets an acre of meadowing, to be laid out on one ſide of a field that is 22 chains long, how broad muſt the land be to make an acre?

```
 22
  4
 ——                (4
 88) 160,00(1,818
     88      ——
     ——     ,454 links anſwer
     720
     704
     ———
      160
       88
      ———
      720
      704
      ———
       16
```

O o o But

But for the aſſiſtance of thoſe who do not underſtand decimal arithmetic, I have inſerted the following table.

The uſe of the TABLE.

Breadth C	Length C	L	P
1	10	00	,0
2	5	00	,0
3	3	33	,33
4	2	02	,50
5	2	00	,0
6	1	66	,6
7	1	42	,28
8	1	25	,0
9	1	11	,11
Length	Breadth		

If the length or breadth be given in chains, links, &c. the other may be found by inſpection, viz. if the length be even chains, look on the contrary dimenſions and you will ſee how many chains, links, &c. muſt be meaſured for an acre.

EXAMPLE. Suppoſe the length of a field was 5 chains, how much in breadth will make an acre? againſt five chains in the table you will find two chains; the breadth required to make an acre; and ſo of the reſt.

Prob. 9. *To reduce a large plot of land or map into a leſſer compaſs, or, on the contrary, to enlarge it.*

RULE. If it be a field or two it is the beſt way to plot it over again by a greater or leſſer ſcale; but if it be large, as the map of a county or manor, &c. the readieſt way is to circumſcribe it with a geometric ſquare, and divide that ſquare into ſeveral other leſſer ſquares, and by this means every field, houſe, &c. in one will fall in the ſame ſquare in the other.

Prob. 10. *To find the exact diſtance to any viſible object without any inſtrument, or actually meaſuring the ſame.*

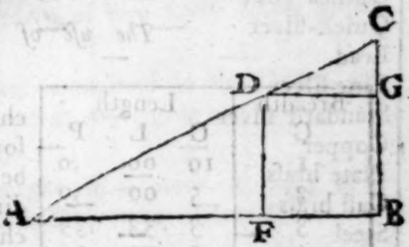
RULE. Firſt, get four ſtraight ſticks, of any length you pleaſe, then let it be required to find the diſtance A B upon level ground, at B put down one of your ſticks, there ſtand and order an aſſiſtant to put down another at F; ſo that ſtanding at B you may ſee the ſtaff F and the object at A, both in a ſtraight line (now it matters not at what diſtance the ſtaff B is from the ſtaff F, but if your diſtance required be far, then the further F is from B the better), then take a third ſtaff and go from F any number of yards, chains, or any other meaſure to D, ſo that the line F D may be at right angles with B A, and at D put down the third ſtaff. Laſtly, take the other ſtaff, and go from B (ſquare-wiſe, as before) ſo far till you can ſee the ſtaff D and the object A in a right line; which ſuppoſe at C; here make a mark, and meaſure the diſtance C G, 25 feet, &c. and G B 39,1; then C G 25 + G B 39,1 = B C, 64,1 feet; by meaſuring F D = B G = 39,1; the truth of this problem is grounded upon ſimilar triangles, for the triangle C G D is ſimilar to the triangle C B A. Therefore it will always hold.

As

As CG 25 is to GD 39, so is CB 64,1 to BA 100 the answer.

The work at length.

As 25 : 39 :: 64,1
39
———
5769
1923
———
25) 2499,9 (99,9 = 100 nearly, the distance required.
225
———
249
225
———
249
225
———
24



N. B. As the narrow limits of this treatise would not admit of room to give the whole art of surveying, I have given such examples as I thought would be the most useful to the young *Tyro*, as well as the farmer and grazier, which examples carefully considered and duly regarded, will enable the learner to find the content, and plan a single field, or estate with certainty and expedition.

LXXVII. SPECIFIC GRAVITY.

THE specific gravity of a body, is the relation that the weight of a body of one kind hath to the weight of an equal magnitude of a body of another kind; the knowledge of which is of great use, not only in natural philosophy, but also in common life, in computing the weights of such bodies as are too unweildy to have their weights discovered by other means.

The following table shews the specific gravity to rain water, of metals, and other bodies; and the weight of a cubic inch of each, in parts of a pound avoirdupoise and of ounces troy.

N. B. If the specific gravity of any solid in the table be less than 1000, it will swim in water; but if greater than 1000 it will sink.

Bodies.	sp. gr.	wt. lb. avoir.	wt. oz. troy.
Fine gold	19.639	0.7103587	10.359273
Standard gold	18.887	0.7060185	9.962625
Guinea gold	17.793	0.6828703	9.911707
Quick-ſilver	13.762	0.4976574	7.384411
Lead	11.313	0.4091696	5.984010
Fine ſilver	11.091	0.4011501	5.850035
Standard ſilver	10.629	0.3844400	5.556769
Copper	8.769	0.3171058	4.747121
Plate braſs	8.350	0.2942593	4.404273
Caſt braſs	8.104	0.2929832	4.272409
Steel	7.850	0.2839265	4.142127
Bar iron	7.764	0.2808159	4.031361
Block tin	7.238	0.2417901	3.861519
Caſt iron	7.135	0.2380647	3.806568
Load ſtone	5.106	0.1846788	2.724083
Blue ſlate	3.500	0.1264914	1.867272
Veined marble	2.702	0.0977286	1.429411
Common glaſs	2.600	0.0940393	1.360841
Flint ſtone	2.582	0.0933883	1.351419
Portland ſtone	2.570	0.0929543	1.345139
Free ſtone	2.352	0.0915788	1.231038
Brick	2.000	0.0723379	1.046801
Alabaſter	1.888	0.0683061	0.988456
Ivory } Horn }	1.832	0.0662606	0.958489
Brimſtone	1.800	0.0651042	0.949424
Clay	1.712	0.0619213	0.902498
Lignum-vitæ	1.327	0.0479862	0.699936
Coal	1.255	0.0553921	0.661959
Pitch	1.150	0.0415943	0.606759
Mahogany wood	1.063	0.0384475	0.560691
Dry box wood	1.030	0.0372530	0.543282
Milk } Sea water }	1.033	0.0372530	0.543742
Rain water	1.000	0.0361690	0.527458
Red wine	0.993	0.0359158	0.523766
Bees wax	0.996	0.0359881	0.524820
Linſed oil	0.932	0.0337095	0.491591
Proof ſpirits or brandy	0.927	0.0335503	0.489268
Dry oak	0.915	0.0330946	0.489008
Olive oil	0.913	0.0330222	0.481569
Beech	0.854	0.0308883	0.450449
Dry elm } Dry aſh }	0.800	0.0289352	0.421966
Dry wainſcot	0.747	0.0270182	0.394011
Dry yellow fir	0.657	0.0237630	0.346539
Man's body	1.111	0.041310	0.600354
Cedar	0.613	0.0221715	0.323332
Dry white deal	0.569	0.0205801	0.300123
Cork	0.240	0.0186805	0.126590
Air	0.0012	0.0000434	0.000633

Note

Note. If you take away the points from the numbers, in the second column, and reckon them to be whole numbers, they will shew how many avoirdupoise ounces are contained in a cubic foot of each of the above bodies, in the table.

CASE 1. The dimensions, or solidity of any body being given to find its weight.

RULE. Multiply the cubic inches contained in that body, by the tabular weight corresponding, and it will give the weight in pounds avoirdupoise, or ounces troy.

EXAMPLE 1. What is the weight of a piece of oak, of a rectangular form, whose solidity is 11880 cubic inches?

```
      .0330946
         11880
      --------
      26475680
     2647568
     330946
    330946
  ------------
112) 393.1638480 (3.5104 cwt the answer.
```

E. 2. There is a bar of iron, in length 156 inches, and 1 inch square; I desire to know how many pounds avoirdupoise it doth weigh?

```
       ,2808159
            156
     ----------
       16848954
      14040795
      2808159
     ----------
     43.8072804 lb. the answer.
```

E. 3. What is the weight of a piece of fir, whose girt is 20 inches, and length 40 feet?

First, $20 \div 5 = 4$, also $4 \times 4 = 16$ square of $\frac{1}{5}$ girt. And 40 feet $= 480$ inches. Then $480 \times 2 \times 16 = 960 \times 16 = 15360$ cubic inches (per rule 2. sect. 73).

```
         Therefore  .0237630
                       15360
                    --------
                    14257800
                    712890
                   1188150
                   237630
                 -----------              c.  qr.  lb.
          Answer 364.9997800 lbs. = 3   1    1
```

EX-

E. 4. What is the weight of iron ſhot, of 8 inches diameter?

Firſt, $8 \times 8 \times 8 \times .5236 = 268.0832$ ſolid inches, (per prob. 10, ſect. 68.)

Therefore - 268.0832
.2580647

18765824
10723328
16084992
214466560
13404160
5361664

69.18281058304 *lb.* the anſwer.

E. 5. What is the diameter of an iron ſhot, weighing 69,18281058304 *lb.* avoirdupoiſe?

Firſt, .2580647) 69.18281058304 (268.0832 ſolid inches.

Then .5236) 268.0832 ($=$ 512 cube of the diameter.

Therefore $\sqrt[3]{512} = 8$ the diameter ſought.

EXAMPLE 6. What is the weight of an iron bomb ſhell, of 3 inches thick, the greateſt diameter being 14 inches.

Firſt, $14 - 6 = 8$, diameter of the concavity.

Then $14 \times 14 \times 14 \times .5236 = 1436.7584$ content of the whole.

And $8 \times 8 \times 8 \times .5256 = 268.0832$ do. of the concavity.

Solidity of the ſhell $= 1168.6752$ inches.

Therefore $1168.6752 \times .2580647 = 301.593814885 44$ *lb.* the weight required.

E. 7. In the walls of Balbeck, in Turkey, there are three ſtones laid end to end, now in ſight, that meaſure in length 61 yards; one of which, in particular, is 63 feet long, 12 feet thick, and 4 yards over: now, if this ſtone was marble, what power would balance it, ſo as to prepare it for moving?

Firſt, $63 \times 12 \times 12 = 9072$ ſolid feet.

Then $9072 \times 1728 = 15676416$ cubic inches.

Therefore 15676416
.0977286

94058496
125411328
31352832
109734912
141087744

2240) 1532034.1886976 (683.9438 $=$ 683 tons, 18 cwt. 97 lb. the anſwer.

CASE

CASE 2. The weight of any body being given to find the solidity, and the specific gravity thereof.

RULE. Divide the given weight by the tabular weight, corresponding to the name of the same kind, and the quotient will be the solidity in cubic inches.

EXAMPLE 1. How many solid feet are there in a block of marble that weighs 8 tons; and what will it come to at 5 shillings per foot solid?

First, 8 tons = 17920 lb.

Then .0977286) 17920.00000000 (183328,45 inches.

Now 1728) 183328,45 (106,09 cubic feet (nearly) at 5*s.* or ,25*l.*

```
        106,09
           ,25
        ------
         53045
        21218
        ------
       26,5225 = 26 l. 10 s. 5¼ d. answer.
```

E. 2. In the Spectators club of fat people, it is said that each person weighed no less than 4 cwt. how many solid inches was there in one of their bodies?

First, 4 cwt. = 448 lb.

Then .041310) 448.000000 (10844.8 solid inches the answer.

EXAMPLE 3. Suppose that a man of war, with all its ordnance, rigging, and appointment, draws so much water as to displace 1300 tons of sea water, London beer measure: the weight of the vessel is required?

First, 1300 × 4 = 5200 hhds. and a hhd = 282 × 54 = 15228 cubic inches.

Therefore 15228 × 5200 = 79185600 cubic inches displaced.

```
Then   79185600
        .037253
      ---------
       2375568
      3959280
     1583712
    5542992
   2375568
   ------------                              T.  cwt. lb.
2240) 2949901.156800 (1316.92015 tons = 1316  18  17 the weight
                                                       required.
```

E. 4. Hiero, king of Scilly, ordered his jeweller to make him him a crown, containing 63 ounces of gold; the workmen thought by substituting part silver therein, to have a proper perquisite, which taking air, Archimedes was appointed to examine it, who, on putting it into a vessel of water, found it raised the fluid, or that itself contained 8.2245 cubic inches of metal, and having discovered

covered that the cubic inch of gold more critically weighed 10.36 ounces, and that of silver but 5.85 ounces, he, by calculation, found what part of his Majesty's gold had been changed, and you are desired to repeat the process?

First, 10.36) 63.00 (6.0810 8, had it been all gold.
Also, 5.85) 63.00 (10.76923, if all silver
Then by sect. 28.

$$\text{Mean rate } 8.2245 - \begin{cases} 6.08108 = 2.54473 \\ 10.76903 = 2.14342 \end{cases}$$

Sum 4,68815

4.68815) 2.54473 (,5428, oz. part gold.
4.68815) 2.14342 (,4572, oz. part silver.

$$\text{Then } \begin{Bmatrix} .5428 \\ .4572 \end{Bmatrix} \times 63 = \begin{cases} \overset{oz.}{34.1964} = \overset{oz.}{34}\ \overset{dwts.}{3}\ \overset{grs.}{22.272}, \text{ G} \\ 28.8036 = 28\ 16\ 1.728, \text{ S} \end{cases}$$

Proof 63 00 00,000

CASE 3. The weight and magnitude being given, to find the specific gravity.

RULE. Divide the weight in ounces, by the solidity in cubic feet, the quotient will be the specific gravity.

EXAMPLE 1. I have a piece of marble that contains 4 solid feet; and weighs 675 lb. what is its specific gravity?

First, $675 \times 16 = 10800$ ounces
Then $10800 \div 4 = 2700$ the specific gravity.

E. 2. I have a piece of timber that contains 6 feet, and weighs 300 lb. what wood is it?

First, $300 \times 16 = 4800$ ounces.
Then $48000 \div 6 = 800$ the specific gravity.

Now, in the table of specific gravity, against 800 you will find dry ash or elm, the wood required.

N. B. All bodies of what nature or kind soever, being weighed in open air, and balanced by those whose specific gravity is greatest; those bodies whose specific gravity is least, will weigh the heaviest in vacuo.

Thus, if a piece of lead, at the end of a nice balance, and a piece of cork at the other end, are in equilibro in the air, and thus placed under the receiver of an air-pump, as soon as the air begins to be exhausted, the equilibrum will begin to be destroyed, till at last when all the air is taken away, the cork will descend and shew itself really heavier than the lead.—(And for the same reason, a pound of feathers is heavier than a pound of lead, which may seem a paradox to some); but the reason is very evident from the

the laws of Hydroſtatics; for both bodies being weighed in air, each would loſe the weight of an equal bulk of air, conſequently the cork will loſe a greater weight than the lead, becauſe it is of greater bulk, therefore when the air is taken away from both, the weight that is reſtored to the cork, being the greateſt, will cauſe it to preponderate or weigh down the lead in vacuo.

CASE 4. The ſolidity of any piece of timber being given to find how far it will ſink.

RULE. Divide the ſpecific gravity of the timber by the ſpecific gravity of the water; multiply this queſtion by the depth of the timber, and that gives the inches under water.

EXAMPLE 1. How many inches will a cubic foot of elm ſink in common water?

First, 1,000) ,8000 (,800
12
9,600 inches the anſwer.

E. 2. How many inches will a cubic foot of deal ſink in common water?

First, 1,000) ,6570 (,657
12
7,884 inches under water.

CASE 5. The ſolidity of any timber being given, to find how much it will carry.

RULE. Subſtract the ſpecific gravity of the timber from that of the water, the remainder is the number of ounces that one ſolid foot will carry.

EXAMPLE 1. How much weight is juſt neceſſary to ſink a cubic foot of deal in common water?

1,000
,657
16) ,343 (21,437 lb. the anſwer.

E. 2. How much will a raft, made of 12 pieces of yellow dale, carry in ſea water, if each piece be a foot ſquare, and 20 feet long?

First, $1033 - 657 = 376$, and $12 \times 20 = 240$
Then $240 \times 376 = ,90240$
$\therefore$ $90240 \div 16 = 5840$ lb. the anſwer.

To make a deceitful balance, or pair of ſcales, whoſe beam will hang in equilibrio without the ſcales, or with the empty ſcales; and yet ſhall alſo be in equilibrio when unequal weights are placed

 in

in the ſcales; ſo as to cheat in any proportion intended, in making the balance at firſt. See plate 1. fig. 32.

To the beam A B = 23 inches long, whoſe arm C B, of 11 inches in length, keeps in equilibrio about the point C, the arm A C, of 12 inches in length, by being made ſo much thicker, or having ſo much more matter, as may make amends for its being ſhorter; hang the ſcales D, E, in ſuch a manner, that D, which weighs one part in twelve leſs than E, ſhall hang at the longeſt end of the beam, and they will keep each other in equilibrio; then placing 12 pounds weight at G, in the ſcale E, it will keep in equilibro no more than 11 pounds of F, the commodity to be ſold, if placed in the ſcale D: becauſe then, F will be to G in a reciprocal proportion of B C to A C.

Now, though ſuch a balance may be ſo nicely made as to deceive the eye, the cheat is immediately diſcovered by changing the weights, and the commodity F, from one ſcale to another; for then, the owner of the balance, muſt either confeſs the fraud, or add to the commodity he ſells, &c. not only what was wanting, but alſo as much as he intended to cheat him of; and a fraction of the added weight, proportionable to the inequality of the arms of the balance, that is, in this caſe, the buyer, inſtead of 11 pounds offered him for 12, his due, will have, by changing the ſcales, $13\frac{1}{11}$ pounds. For whereas, in the firſt poſition of the balance $F = 11 \times 12 = AC$ was equal to $G = 12 \times 11 = BC$ when G or 12 pounds is placed in the ſcale D, then 12×12 will be equal to no leſs than $CB = 11 \times 13\frac{1}{11} = G$. Or,

As the arm C B, 11 inches long, is to the arm C A, 12 inches long, ſo is F, or the weight 12, placed in the ſcale D; to G $13\frac{1}{11}$, or the weight of the commodity keeping the weight in equilibrio.

And therefore, as this analogy gives a reciprocal proportion between the weights and their velocities, the momenta will be equal; which, with contrary directions, deſtroy one another.

N B. In all theſe caſes, we ſuppoſe the weight to hang freely from thoſe ends of the balance to which they are faſtened.

A Table,

A table by which the quantity and weight of water in a cylindrical pipe of any given diameter of bore, and perpendicular height may be found: and conseqently, the power may be known that will be sufficient to raise the water to the top of the pipe, in any pump, or any other hydraulic machine.

Feet high.	Diameter of the Cylindric Bore, 1 Inch.		
	Quantity of water in cubic inches.	Weight of water in troy ounces.	In avoirdupoise ounces.
1	9,4247781	4,971234	5,4541539
2	18,8495562	9,942468	10,9083078
3	28,2743343	14,913702	16,3624617
4	37,6991124	19,884936	21,8166156
5	47,1238905	24,856170	27,2707695
6	56,5486686	29,827404	32,7249234
7	65,9734467	34,798638	38,1790773
8	75,3982248	39,769872	43,6332312
9	84,8230029	44,741106	49,0872851

For tens of feet high, remove the decimal points one place forward; for hundreds of feet, two places, for thousands, three places, and so on. Then multiply the sums by the square of the diameter of the given bore, and the products will be the answer.

EXAMPLE. The quantity and weight of water in a cylindric pipe, 85 feet high, and 10 inches diameter.

The square of 10 = 100.

Feet high.	Cubic inches.	Troy ounces.	Avoird-ounces.
80	753,982248	397,698720	436,332312
5	47,123890	24,856170	27,270769
85	801,106138	422,554890	463,603081
	×100	×100	×100
Answer	80110,613800	42255,489000	46360,308100

Which number 80110,6 of cubic inches being divided by 231, the cubic inches in a wine gallon, gives 342,6 for the number of gallons: and the respective weights 42255,489, and 46360,3, being divided, the former by 12, and the latter by 16, give 3521,29 for the number of troy pounds, and 2897,5 for the number of avoirdupoise pounds, that the water in the pipe weighs. So much power would be required to balance or support the water in the pipe, and as much more to work the engine, as the friction thereof amounts to.

In all pumps, the pressure of the column of water, or its weight felt by the working power, when raised to any given height above

the surface of the well, is in proportion to the height of the column, considered throughout, as if it were equal in diameter to that part of the bore, in which the piston or bucket works.

The advantage or power gained by the handle of the pump, is the same as in the common lever; that is, as great as the length from the axes of the handle to its end, where the power is applied, exceeds the length of the other part of the handle, from the axes on which it turns, to the pump rod, wherein it is fixed, for lifting the piston and water.

In the making of pumps, the diameter of the bore, where the bucket works, should be proportioned to the height which the pump raises water above the surface of the well, as that a man of ordinary strength might work all pumps equally easy, let their heights be what they will. The annexed table shews how this may be done, and what quantities of water may be raised in a minute by one man, supposing the handle of the pump to be a lever, increasing the power five times.

The use of the Table.

Find the given height of the pump, in the first column of the table; and against it in the second column, you have the diameter, which the bore must be of, in inches, and hundredth parts of an inch; and in the third column, you have the quantity of water in gallons and pints, that a man of common strength can raise to that height in a minute.

A Table for Pump-makers.

Height of the pump above the surface of the well.	Diameter of the bore, where the piston works.	Water discharged in a minute in gallons and pints.	
Feet.	Inches.	Gallons.	Pints.
10	6,93	81	6
15	5,65	54	4
20	4,90	40	7
25	4,38	32	6
30	4,00	27	2
35	3,70	23	3
40	3,47	20	4
45	3,26	18	1
50	3,10	16	3
55	2,95	14	7
60	2,83	13	5
65	2,71	12	4
70	2,62	11	5
75	2,53	10	7
80	2,44	10	2

With respect to the power required to work the pump, or the quantity of water discharged thereby, it matters not what the diameter of the bore be in any other part, than that wherein the piston or bucket works.

Before I quit this subject, I shall observe one thing more to my reader, concerning the pressure of fluids, which is this:

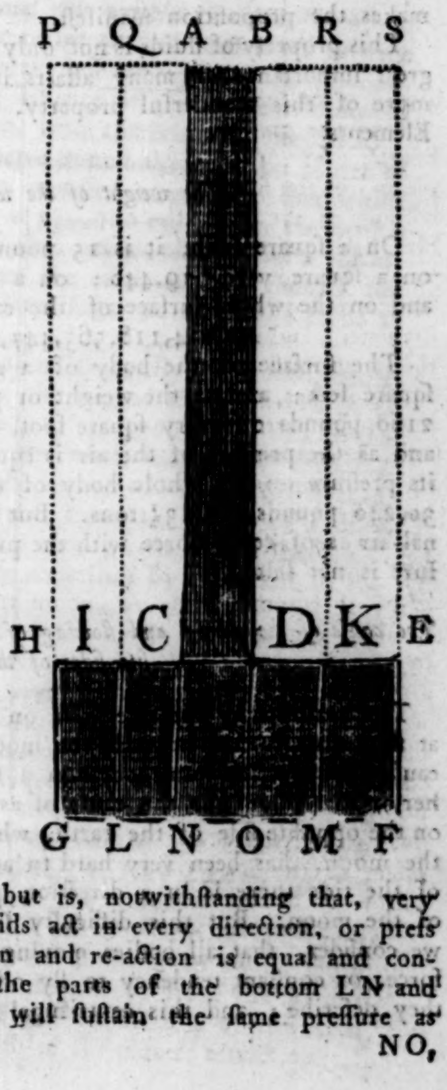
Let a body be ever so heavy, it may be made to swim in liquids, by knowing the specific gravity.

Thus,

Thus, becauſe the ſpecific gravity of gold is to that of water, as 19 to 1; therefore if you hold a guinea to the bottom of a tube of equal diameter (ſo as no water can get in) by means of a ſtring; then put the tube down in the water, above 19 times the thickneſs of the guinea in depth, and letting the ſtring go, the guinea will not ſink, but ride ſuſtained by the preſſure of the ſubadjacent water, which now is ſtronger than the force of gravity in the guinea; and thus you may make any body ſwim, let it be ever ſo large and weighty.

SCHOLIUM.

The writers on Hydroſtatics demonſtrate, that the preſſure of liquids on the bottom and ſides of veſſels, is always proportional to the height thereof, and every way equal at the ſame depth.



To illuſtrate this, let G E in the annexed figure, be a veſſel, from whoſe upper part H E proceeds a tall tube, A B C D, communicating therewith. Let this tube and veſſel be filled with water, then will the preſſure of the water on the bottom G F be as great, and every way the ſame, as it would be, were the veſſel itſelf as high as the tube, and filled with water to the level of P S; that is the column of water, A N O B in the preſent caſe has the ſame effect on the bottom of the veſſel G F, as the column of water P G F S would have.

This is no ſmall paradox, but is, notwithſtanding that, very eaſy to conceive; for ſince fluids act in every direction, or preſs every way equally; and action and re-action is equal and contrary; it muſt follow, that the parts of the bottom L N and G L (being equal to N O) will ſuſtain the ſame preſſure as NO,

N O, or as they would do, were the columns of water continued to the height of P Q A. For in the line C N, the force of the column of water A O, is exerted on each side equally, and has the same effect at I L as at D O, and therefore the lateral pressure being equal, the perpendicular pressures also on L N and N O will be equal.

Or thus: If the pressure of the part of I L were less than on the part D O, the fluid in the column C O would, by reason of its greater gravity, have a motion towards the part I L, and the surface A B would descend: but since there is a perfect quiescence of all the parts of the fluid, and that in the column C O is as much at rest as that in the column C L, it is evident their pressures and effects are every way the same, and consequently that the column C L presses as much on the part L N, as the column C O does on the part N O. What is thus proved of the column I N, is to be proved of all the rest, H L, D M, and K F, which makes the proposition manifest.

This property of fluids is not only in itself very curious, but of great importance in many affairs in life. They who would see more of this wonderful property, may consult Dr. Gravesande's Elements.

The weight of the whole Atmosphere.

On a square inch, it is 15 pounds; on a square foot, 2160; on a square yard, 19,440; on a square mile, 60,217,344,000; and on the whole surface of the earth, and sea together,——

12,014,118,565,447,680,000 pounds.

The surface of the body of a middle sized man, is about 14 square feet; and as the weight or pressure of the air is equal to 2160 pounds on every square foot, on or near the earth's surface, and as the pressure of the air is equal in all manner of directions, its pressure on the whole body of a middle sized man is equal to 30,240 pounds, or $13\frac{1}{2}$ tons. But because the spring of the internal air is of equal force with the pressure of the external, the pressure is not felt.

The cause of the ebbing and flowing of the sea, at the same time, on opposite sides of the globe.

The reason why the tides rise on the side of the earth, which is at any time turned towards the moon, is plain to every one; because her attraction must occasion a swelling of the waters towards her on that side: but the cause of as great a swell, at the same time, on the opposite side of the earth, which is then turned away from the moon, has been very hard to account for; because the rising of the tide there is in a direction quite contrary to the attraction of the moon. But this difficulty is immediately removed, when we consider, that all bodies moving in circles, have a centrifugal force, or constant tendency to fly off from the centers of the circles they describe; and this centrifugal force is always in proportion

to the diſtance of the body from the center of its orbit, and the velocity with which it moves therein.

When the body is large, the ſide of it which is fartheſt from the center of its orbit will have a greater degree of centrifugal force than the center of the body has; and the ſide of it which is neareſt the center of its orbit, will have a leſs degree of centrifugal force than its center has.

As the moon goes round the earth every month in her orbit, the earth alſo goes round an orbit every month, which is as much leſs than the moon's orbit, as the quantity of matter in the moon is leſs than the quantity of matter in the earth, which is 40 times. For, by the laws of nature, when a ſmall body moves round a great one, in free and open ſpace, both theſe bodies muſt move round the common center of gravity between them.

The moon's main diſtance from the earth's center is 240,000 miles; divide therefore this diſtance by 40, the difference between the quantity of matter in the earth and moon, and the quotient will be 6000 miles, which is the diſtance of the common center of gravity between the earth and moon, from the center of the earth.

Now, as the earth and moon move round the common center of gravity between them, once every month; it is plain, that whilſt the moon moves round her orbit, at 240,000 miles from the earth's center, the center of the earth deſcribes a circle of 6000 miles radius, round the center of gravity between the earth and the moon; the moon's attraction balancing the centrifugal force of the earth at its center.

The diameter of the earth is 8000 miles (nearly) and conſequently its ſemi-diameter is 4000: ſo that the ſide of the earth which is at any time turned towards the moon, is 4000 miles nearer the common center of gravity between the earth and moon, than the earth's center is; and the ſide of the earth, which is then fartheſt from the moon, is 4000 miles farther from the center of gravity between the earth and moon, than the earth's center is at that time.

Therefore, the radius of the circle deſcribed by the parts of the earth which come about towards the moon, by the earth's diurnal motion, is 2000 miles; the radius of the circle deſcribed by the earth's center is 6000; and the radius of the circle deſcribed by thoſe parts of the earth which, in revolving on its axis, are fur-theſt from the moon, is 10,000 miles.

The centrifugal forces of the different parts of the earth being directly as their diſtances from the above-mentioned common center of gravity, round which both the earth and moon move, theſe forces may be expreſſed by 2000 for the ſide of the earth neareſt the moon, by 6000 for the earth's center, and by 10,000 for the ſide of the earth which is fartheſt from the moon.

But the moon's attraction is greateſt on the ſide of the earth next her, where the centrifugal force or tendency to fly off from the common center of gravity (and conſequently, from the moon) is leaſt; and therefore, the tides muſt riſe on the ſide of the earth which is neareſt the moon, by the exceſs of the moon's attraction.

A

As her attraction balances the centrifugal force at the earth's center, it is plain that the centrifugal force of the side of the earth which is farthest from the moon is greater than her attraction; and therefore, the tides will rise as high upon that side from the moon, by the excess of the centrifugal force, as they rise on the side next her by the excess of her attraction. And as the earth is in constant motion on its axis, so as that any given meridian revolves from the moon to the moon again in 24 hours $50\frac{1}{2}$ minutes, each place will come to the two eminences of water, under and opposite to the moon, in 24 hours $50\frac{1}{2}$ minutes, or have two tides of flood and two of ebb in that time. For, as much as the waters rise above the common level of the surface of the sea, under and opposite to the moon, so much they must fall below that level half way between the highest places, or at 90 degrees from them.

On these principles, it is equally easy to account for the rising of the tides, at the same time, on both sides of the earth; and this rising is made evident to sight in Mr. Ferguson's Lecture on the Central Forces.

CHRONOLOGY.

PART V.

SECTION LXXVIII.

CHRONOLOGY is the art of estimating and comparing together the times when any memorable transaction hath happened.

It also takes a view of the various tracts, calendars, and methods of computing time, practised by different nations; compares them together, and settles such order among them, that the exact time in which any remarkable event happened may be certainly known.

RULES for finding the corresponding years of the Julian period, with the years of the world, and years before and since the birth of Christ; supposing that the creation of the world was in the 706th year of the Julian period; and that the birth of Christ was (according to the vulgar æra thereof) in the 4713th year of the Julian period.

From any given year of the Julian period subtract 706, and the remainder will be the year of the world's age.

If the number of the given year of the Julian period be less than 4713, subtract it from 4713; and the remainder will be the number of years before the year of Christ's birth.

If the given year of the Julian period is greater than 3967, subtract 3967 from it; and the remainder will be the number of years after the famous æra of Nabonassar.

Sub-

Subtract 1 from any given year of the Julian period, and divide the remainder by 4; if nothing remains, the given year is a leap-year; but if 1, 2, or 3 remains, it is the first, second, or third year after leap-year, in the old stile.

If any year before the year of Christ's birth be given, subtract its number from 4713, and the remainder will be the year of the Julian period: and if you subtract the said year from 4007, the remainder will be the year of the world's age.

If any year after the year of Christ's birth be given, add 4713 to it, and the sum will be the year of the Julian period; or if you add 4007 to it, the sum will be the years of the world's age.

If any year of the world's age is given, add 706 to it, and the sum will be the year of the Julian period. If the given year of the world be less than 4007, subtract it from 4007, and the remainder will be the number of years before the year of Christ's birth. But, if the given year of the world be more than 4007, subtract 4007 from it; and the remainder will be the number of years after the year of Christ's birth.

A TABLE of remarkable ÆRAS and EVENTS.

		Julian period.	World's age.	Before Christ.
1	The creation of the world —	706		4007
2	The flood . — —	2362	1656	2351
3	The Assyrian monarchy founded by Nimrod	2537	1831	2176
4	The birth of Abraham —	2714	2008	1999
5	The destruction of Sodom and Gomorrah	2816	2110	1897
6	The kingdom of Athens founded by Cecrops	3157	2451	1556
7	Moses receives the ten commandments from God — —	3222	2516	1491
8	The Israelites enter Canaan	3262	2556	1451
9	The destruction of Troy	3529	2823	1184
10	The beginning of King David's reign	3650	2944	1063
11	The founding of Solomon's temple	3701	2995	1012
12	The Argonautic expedition	3776	3070	937
13	Lycurgus formed his excellent laws	3829	3103	884
14	Arbaces, first king of the Medes	3838	3132	875
15	Mandaucus, the second —	3865	3159	848
16	Sosarmus, the third —	3915	3029	798
17	The beginning of the Greek Olimpiades	3938	3232	775
18	Artica, the fourth king of the Medes	3945	3239	768
19	The Catonian epocha of the building of Rome — —	3961	3255	752
20	The æra of Nabonasiar —	3967	3261	746
21	The destruction of Samaria by Salmaneser	3992	3286	721
22	The first eclipse of the moon on record	3993	3287	720
23	Cardicea, the fifth king of the Medes	3996	3290	717
24	Phraortes, the sixth —	4058	3352	655
25	Cyaxares, the seventh —	4080	3374	633
26	The first Babylonish captivity by Nebuchadnezzar —	4107	3401	606

		Julian period.	World's age.	Before Christ.
27	The long war ended between the Medes and Lydians — —	4111	3405	602
28	The second Babylonish captivity, and birth of Cyrus —	4114	3408	599
29	The destruction of Solomon's temple	4125	3419	588
30	Nebuchadnezzar struck with madness	4144	3438	569
31	Daniel's vision of the four monarchies	4158	3452	555
32	Cyrus begins to reign —	4177	3471	536
33	The battle of Marathon —	4223	3517	490
34	Artaxerxes Longimanus begins to reign	4249	3543	464
35	The beginning of Daniel's seventy weeks of years — —	4256	3350	457
36	The beginning of the Peloponnesian war	4282	3576	434
37	Alexander's victory at Arbela	438[illegible]	3677	330
38	His death — —	439[illegible]	[illegible]684	323
39	The captivity of 100,000 Jews by King Ptolomy — —	43[illegible]	[illegible]687	320
40	The Colossus of Rhodes thrown down by an earthquake — —	4491	3875	222
41	Antiochus defeated by Ptolemy Philopater	4496	3700	217
42	The famous Archimedes murdered at Syracuse — —	4596	3800	207
43	Jason butchered the inhabitants of Jerusalem	4543	3837	170
44	Corinth taken and plundered by the Consul Mummius — —	4567	3861	146
45	Julius Cæsar invaded Britain	4659	3953	54
46	He corrects the calendar —	4667	3961	46
47	Is killed in the senate-house	4671	3965	42
48	Herod made king of Judea —	4671	3967	40
49	The battle at Actium	4683	3977	30
50	Agrippa builds the Pantheon at Rome	4668	3982	25
51	The true æra of Christ's birth	4709	4003	4
52	The death of Herod —	4710	4004	3
				Since Christ
53	The Dionysian, or vulgar æra of Christ's birth — —	4713	4007	
54	The true year of his crucifixion	4746	4040	33
55	The destruction of Jerusalem	4783	4077	70
56	Adrian built the long wall in Britain	4833	4127	120
57	Constantius defeated the Picts in Britain	5019	4313	306
58	The Council of Nice —	5038	4332	725
59	The death of Constantine the Great	5050	4344	337
60	The Saxons invited to Britain	5158	4452	445
61	The Arabian Hegira, or flight of Mahomed	5335	4629	662
62	The death of Mahomed —	5343	4637	630
63	The Persian Yesdegird —	4344	4638	631
64	The art of printing discovered	6153	5447	1440
65	The reformation begun by Martin Luther	6230	5524	1517
66	Oliver Cromwell died —	6371	5665	1658

	Julian period.	World's age.	Since Christ
67 Sir Isaac Newton born at Woolstrope, in Lincolnshire, December 25	6355	5649	1642
Went to Trinity college, in Cambridge	6373	5667	1660
Was elected Fellow of that college	6380	5674	1667
Invented the Fluxions —	6382	5676	1669
Made Professor of Mathematics, in the room of Dr. Barrow —	6382	5676	1669
Published his principia —	6400	5684	1687
Exerted himself for religion	6401	5685	1688
Made President of the Royal Society	6416	5700	1703
Knighted by Queen Anne —	6418	5702	1705
Died, March 20 —	6440	5734	1727

In this table, the years both before and since Christ are reckoned exclusive from the year of his birth.

By the following tables (pages 484, 485, and 486) the day of the month answering to any given day of the week, and the day of the week answering to any given day of the month, may be found, in the old stile, within the limits of 5500 years, before the year of Christ's birth, and 5500 years after it; and, in the new stile, from A.D. 1752, to 1821, inclusive.

1. For any given year before Christ, look for the complete hundreds of that year (when its number amounts to hundreds) at the head of the table on page 484, and for the years below or less than an hundred, to make up the number of the given year, at the left hand; and where the columns meet, you have the dominical letter for the given year.

EXAMPLE. What was the dominical letter for the 584th year before Christ's birth?

Under 500, at the head of the table, and against 84, at the left hand, I find F E, the dominical letter required, and shews the said year to have been leap-year, as every leap-year has two dominical letters; the first of which serves for January and February, and the last for the rest of the year.

EXAMPLE 2. For the year 1741, I demand the dominical letter?

Under 1700, at the head of the table, and downwards thence, in that column against 41, at the left hand, I find D; the dominical letter required.

Note, These two tables shew the dominical letter for the old stile, and the table on page 486 shews it for the new stile.

A TABLE, shewing the Dominical Letters for Years before the Year of Christ's Birth.

Old Stile.				Hundreds of Years.						
Years less than an hundred.				0	100	200	300	400	500	600
				700	800	900	1000	1100	1200	1300
				1400	1500	1600	1700	1800	1900	2000
				2100	2200	2300	2400	2500	2600	2700
				2800	2900	3000	3100	3200	3300	3400
				3500	3600	3700	3800	3900	4000	4100
				4200	4300	4400	4500	4600	4700	4800
				4900	5000	5100	5200	5300	5400	5500
0	28	56	84	D C	C B	B A	A G	G F	F E	E D
1	29	57	85	E	D	C	B	A	G	F
2	30	58	86	F	E	D	C	B	A	G
3	31	59	87	G	F	E	D	C	B	A
4	32	60	88	B A	A G	G F	F E	E D	D C	C B
5	33	61	89	C	B	A	G	F	E	D
6	34	62	90	D	C	B	A	G	F	E
7	35	63	91	E	D	C	B	A	G	F
8	36	64	92	G F	F E	E D	D C	C B	B A	A G
9	37	65	93	A	G	F	E	D	C	B
10	38	66	94	B	A	G	F	E	D	C
11	39	67	95	C	B	A	G	F	E	D
12	40	68	96	E D	D C	C B	B A	A G	G F	F E
13	41	69	97	F	E	D	C	B	A	G
14	42	70	98	G	F	E	D	C	B	A
15	43	71	99	A	G	F	E	D	C	B
16	44	72		C B	B A	A G	G F	F E	E D	D C
17	45	73		D	C	B	A	G	F	E
18	46	74		E	D	C	B	A	G	F
19	47	75		F	E	D	C	B	A	G
20	48	76		A G	G F	F E	E D	D C	C B	B A
21	49	77		B	A	G	F	E	D	C
22	50	78		C	B	A	G	F	E	D
23	51	79		D	C	B	A	G	F	E
24	52	80		F E	E D	D C	C B	B A	A G	G F
25	53	81		G	F	E	D	C	B	A
26	54	82		A	G	F	E	D	C	B
27	55	83		B	A	G	F	E	D	C

A TABLE, shewing the Dominical Letters for Years after the Year of Christ's Birth.

Old Stile.				Hundreds of Years.						
Years less than an hundred.				0 700 1400 2100 2800 3500 4200 4900	100 800 1500 2200 2900 3600 4300 5000	200 900 1600 2300 3000 3700 4400 5100	300 1000 1700 2400 3100 3800 4500 5200	400 1100 1800 2500 3200 3900 4600 5300	500 1200 1900 2600 3300 4000 4700 5400	600 1300 2000 2700 3400 4100 4800 5500
0	28	56	84	D C	E D	F E	G F	A G	B A	C B
1	29	57	85	B	C	D	E	F	G	A
2	30	58	86	A	B	C	D	E	F	G
3	31	59	87	G	A	B	C	D	E	F
4	32	60	88	F E	G F	A G	B A	C B	D C	E D
5	33	61	89	D	E	F	G	A	B	C
6	34	62	90	C	D	E	F	G	A	B
7	35	63	91	B	C	D	E	F	G	A
8	36	64	92	A G	B A	C B	D C	E D	F E	G F
9	37	65	93	F	G	A	B	C	D	E
10	38	66	94	E	F	G	A	B	C	D
11	39	67	95	D	E	F	G	A	B	C
12	40	68	96	C B	D C	E D	F E	G F	A G	B A
13	41	69	97	A	B	C	D	E	F	G
14	42	70	98	G	A	B	C	D	E	F
15	43	71	99	F	G	A	B	C	D	E
16	44	72		E D	F E	G F	A G	B A	C B	D C
17	45	73		C	D	E	F	G	A	B
18	46	74		B	C	D	E	F	G	A
19	47	75		A	B	C	D	E	F	G
20	48	76		G F	A G	B A	C B	D C	E D	F E
21	49	77		E	F	G	A	B	C	D
22	50	78		D	E	F	G	A	B	C
23	51	79		C	D	E	F	G	A	B
24	52	80		B A	C B	D C	E D	F E	G F	A G
25	53	81		G	A	B	C	D	E	F
26	54	82		F	G	A	B	C	D	E
27	55	83		E	F	G	A	B	C	D

Dominical Letters for the New Stile.

1752	B A	1787	G
1753	G	1788	F E
1754	F	1789	D
1755	E	1790	C
1756	D C	1791	B
1757	B	1792	A G
1758	A	1793	F
1759	G	1794	E
1760	F E	1795	D
1761	D	1796	C B
1762	C	1797	A
1763	B	1798	G
1764	A G	1799	F
1765	F	1800	E
1766	E	1801	D
1767	D	1802	C
1768	C B	1803	B
1769	A	1804	A G
1770	G	1805	F
1771	F	1806	E
1772	E D	1807	D
1773	C	1808	C B
1774	B	1809	A
1775	A	1810	G
1776	G F	1811	F
1777	E	1812	E D
1778	D	1813	C
1779	C	1814	B
1780	B A	1815	A
1781	G	1816	G F
1782	F	1817	E
1783	E	1818	D
1784	D C	1819	C
1785	B	1820	B A
1786	A	1821	G

A Table, shewing the days of the months for ever, both in the old and new stile, by the dominical letters.

Months.	A	B	C	D	E	F	G
Jan. 31 Oct. 31	1	2	3	4	5	6	7
	8	9	10	11	12	13	14
	15	16	17	18	19	20	21
	22	23	24	25	26	27	28
	29	30	31	—	—	—	—
Feb. 28 Mar. 31 Nov. 30				1	2	3	4
	5	6	7	8	9	10	11
	12	13	14	15	16	17	18
	19	20	21	22	23	24	25
	26	27	28	29	30	31	—
April 30 July 31							1
	2	3	4	5	6	7	8
	9	10	11	12	13	14	15
	16	17	18	19	20	21	22
	23	24	25	26	27	28	29
	30	31	—	—	—	—	—
August 31			1	2	3	4	5
	6	7	8	9	10	11	12
	13	14	15	16	17	18	19
	20	21	22	23	24	25	26
	27	28	29	30	31	—	—
Sept. 30 Dec. 31						1	2
	3	4	5	6	7	8	9
	10	11	12	13	14	15	16
	17	18	19	20	21	22	23
	24	25	26	27	28	29	30
	31						
May 31		1	2	3	4	5	6
	7	8	9	10	11	12	13
	14	15	16	17	18	19	20
	21	22	23	24	25	26	27
	28	29	30	31	—	—	—
June 30					1	2	3
	4	5	6	7	8	9	10
	11	12	13	14	15	16	17
	18	19	20	21	22	23	24
	25	26	27	28	29	30	

The dominical letter for the new stile may be found by the following

RULE. To the given year add its fourth part, omitting fractions; divide that sum by 7; the remainder, taken from 7, leaves the index of the letter in the common year's reckoning.

1	2	3	4	5	6	7
A	B	C	D	E	F	G

But in leap-years, this letter and its preceding one are the dominical letters.

E. 1. For the year 1782, I demand the dominical letter?

$$\begin{array}{l} 4)1782 \\ \hline \quad 445 \\ +1782 \\ \hline 7)2227 \quad 7 \\ \hline \quad 318-1 \\ \qquad \overline{} \\ \quad 6 = \text{F domin. letter} \end{array}$$

E. 2. What is the dominical letter for the year 1783?

$$\begin{array}{l} 4)1783 \\ \hline \quad 445 \\ +1783 \\ \hline 7)2228 \quad 7 \\ \hline \quad 318-2 \\ \qquad \overline{} \\ \quad 5 = \text{E, domin. letter} \end{array}$$

To find whether any given year be leap-year.

RULE. Divide the given year by 4; if nothing remains, it is leap-year; but if 1, 2, or 3 remains, it is so many years after.

E. 1. Is the year of our Lord 1764 leap-year?

$$\begin{array}{l} 4)1764 \\ \hline \quad 441 \text{ and nothing remains, so it is leap-year.} \end{array}$$

E. 2. I desire to know whether 1782 be leap-year? $1782 \div 4 = 445$, and 2 remains, so it is the second year after leap-year.

To know on what day of the week any proposed day of the month will fall.

Having found the dominical letter for the given year, look for that letter at the top of the table shewing the days of the month (page 486) and under the said letter you have all the days of the months which are Sundays in that year, in the divisions of the months. Under the next, towards the right hand, all the days in the column are Mondays; those under the next are Tuesdays; and so on. When you are out at the right hand of the table, go back to the left, and so reckon on according to the order of the days of the week.

Thus, suppose for the year 584, before Christ, for which the dominical letter was FE; the first serving for January and February, and the last for all the rest of the year; in the table (page 486) I find, under F, the 6th, 13th, 20th, and 27th, of January; and the 3d, 10th, 17th, and 24th, of February; and then, under E, I find

I find the 2d, 9th, 16th, 23d, and 30th, of March and November; the 5th, 12th, 19th, and 26th of October; the 6th, 13th, 20th, and 27th of April and July; the 3d, 10th, 17th, 24th, and 31st, of August; the 7th, 14th, 21st, and 28th, of September and December; the 4th, 1th, 18th, and 25th, of May; and the 1st, 8th, 15th, 22d, and 29th, of June; which being all Sundays in that year, the rest of the days of the months, answering to given days of the week, are easily found.

E. 1. In the 584th year before the birth of Christ, I desire to know on what day of the week the 18th of May fell on?

Look for the 28th of May in the table, and you will find A stands at the top of the column in which that day is found: And as the 25th of May fell on Sunday, it is plain, that the 28th of May must have been on Wednesday.

E. 2. On what day of the week does Christmas-day fall on for the year 1783?

The dominical letter for this year is E. Then under E, in the the division for December, in the table (page 486) I find that the 7th, 14th, 21st and 28th, are Sundays; and consequently, as the 21st of December falls on Sunday, the 25th (or Christmas-day) must be on Thursday.

To find the solar cycles.

RULE. To the given year add 9, divide the sum by 28, the remainder is the cycle of the sun.

Note. The solar cycle, or cycle of the sun, is a period of 28 years; in which time all the varieties of the dominical letters will have happened, and the 29th year the cycle begins again, when the same order of the letters will return as they were 28 years before. At the birth of Christ, 9 years had passed in this cycle.

E. Required the year of the solar cycle, for the years 1781 and 1782?

$$28\begin{cases}4)\ \underline{1781 + 9} = 1790 \\ 7)\ 447-2 \\ \quad 63-6\end{cases} = 26 \text{ cycle of the sun}$$

$$28\begin{cases}4)\ \underline{1782 + 9} = 1791 \\ 7)\ 447-3 \\ \quad 63-6\end{cases} = 27 \text{ solar cycle}$$

To find the lunar cycle.

RULE. To the given year add 1, and divide this sum by 19; the remainder shews the cycle of the moon, or lunar cycle.

E. What

E. What is the golden number, or lunar cycle, for the year 1783 ?

$$\begin{array}{r} 1783 \\ +1 \\ \hline 19)1784(93 \text{ revolutions since the birth of Christ.} \\ 171 \\ \hline 74 \\ 57 \\ \hline \end{array}$$

Anfw. 17 lunar cycle, or golden number

Note. The lunar cycle, or golden number, is a period of 19 years, containing all the variations of the days on which the new and full moons happen; after which time they fall on the fame days they did 19 years before; and then fhe begins again with the fun.

But when a centiffimal or hundredth year falls in the cycle, the new and full moon, according to the new ftile, will fall a day later than otherwife. The birth of Chrift happened in the fecond year of this cycle.

To find the Roman indiction.

RULE. To the given year add 3, and divide the fum by 15; the remainder is the number of indiction.

E. What is the Roman indiction for the year 1783 ?

$$\begin{array}{r} 1783 \\ +3 \\ \hline 15)1786(119 \\ 15 \\ \hline 28 \\ 15 \\ \hline 136 \\ 135 \\ \hline \end{array}$$

1 Roman indiction.

N. B. The Roman indiction is a cycle of 15 years, which firft began the third year before Chrift.

To find the epact till the year 1900.

RULE. Multiply the golden number for the given year by 11, divide that product by 30, and from the remainder take 11, leaves the epact. If the remainder is lefs than 11, add 19 to it, and the fum will be the epact.

E. Required the epact for the year 1782 ?

First, the golden number for this year is 16.

Then per rule 17

$$\begin{array}{r} 11 \\ \hline 3|0)18|7 \\ \hline \end{array}$$

6—7+19=26 the epact.

R i r N. B.

N. B. The epact of any year is the moon's age at the beginning of that year; that is, the days past since the last new moon.

To find the moon's age.

RULE. To the epact add the month of the year, and the day of the month; their sum, if under 30, is the moon's age; but if that sum is greater, then 30 taken from it leaves the moon's age.

The moon's age taken from 30, leaves the day of the next change.

When the solar and lunar cycles begin together, the moon's age on the first of each month, or the monthly epacts, are called the numbers of the month, and are as follows, viz.

These,	0.	2.	1.	2.	3.	4.
For	Jan.	Feb.	Mar	April	May	June
These	5.	6.	8.	8.	10.	10.
For	July	Aug.	Sept.	Oct.	Nov.	Dec.

E. 1. Required the moon's age on January 1st, 1783?

First, the epact is 26
The month 0
The day 1

Answ. 27 days, the moon's age.

E. 2. The moon's age is 27; how many days is there to the day of her change, which age never exceeds 30 days?

30
27

Answ. 3 days to the change.

To find the time of the moon's southing.

RULE. Multiply the moon's age by 4, divide the product by 5, and the quotient gives the hours; the remainder, multiplied by 12, gives the additional minutes.

If this time is less than 12 hours, it is the time of southing after mid-day; but if greater, 12 hours taken from it, leaves the southing after midnight.

E. 1. Required the time of the moon's southing at London, on the 21st of May, 1776?

3 moon's age
4

5)12

2—2
12

24

Answ. 2h. 24 m. afternoon.

E. 2.

E. 2. Required the time of the moon's fouthing at Birmingham, on the 1ft day of January, 1783?

27 moon's age
4
—
5)108
—
21—$\frac{3}{5}$=36
12
—
9h. 36 m.

Anfwer 9h. 36m. after midnight.

To find the time of high water at any place.

RULE. To the time of the moon's fouthing, add the time the moon has paffed the meridian, to make high water at that place, and the fum will fhew the time of high water.

The diftance of the moon from the meridian, when high water at the following places, is; at London, ☽ bears N. E. or S. E. 3h. om. Briftol key, ☽ bears E. by S. and W. by N. 6h. 45m.

E. 1. On the 21ft of May, 1776, at what time was it high water at London?

	H.	M.	
The moon fouth's at -	2	24	P. M. N.
At London ☽ bears N. E. or S. E.	3	00	
Sum	5	24	

Anfwer, 24 minutes paft 5 in the afternoon.

High water is the ftate of the tide when higheft, or the time it ceafes to flow up.

ASTRONOMY.

PART VI.

SECTION LXXIX.

ASTRONOMY is derived from two Greek words, viz. After, a ftar, and Nomos, a law, or rule. It is a fcience, which, by infallible demonftration, teaches us the motions, diftances, and magnitudes of the heavenly bodies; their revolutions, anomalies, aplelions, eccentricities, enlongations, and parallaxes of the planets, eclipfes of the luminaries, occultations of the primary planets and fixed ftars, by the moon; as alfo, the rifing, culminating, fetting, and amplitudes; and, in fhort, whatever belongs to the right underftanding of the true fyftem of the world.

Of eclipses there are four sorts, viz. 1st, partial; 2d, total without continuance; 3d. total with continuance; and lastly, annular.

1st. Partial, is when part of the sun's diameter is obscured from some particular tract of the earth, and these happen more frequently than any other.

2d. Total without continuance, is when, at the time of the visible conjunction, the true latitude of the moon is equal to her parallax in latitude from the sun: that so the center of one is exactly seen in the center of the other; and then also their visible diameters are equal; that so the sun is no sooner hid from our sight by the dark body of the moon, but very speedily he is seen to recover his light on the other side.

3d. Total with continuance, is when, at the visible conjunction, the eclipse is central (which is always when the north latitude and parallax are equal) the moon is in perigeon, and the sun in apogeon; then the apparent diameter of the moon exceeds that of the sun, and this excess can never amount to one minute, so that the total darkness of any solar eclipse, can never exceed 4′ or 5′ in time.

4th. Lastly, annular, is when the visible central conjunction happeneth, the sun being in perigeon, and the moon in apogeon; here the diameter of the sun exceeds that of the moon, and consequently there will then be a ring of light round the moon.

That the sun's eclipse always begins on the west side, and goes off, or ends, on the east side of his body, is a most manifest truth; because the moon (who is always the cause of this obscurity) moving in her annual motion always in consequence, or according to the order of the signs, must, of necessity, first touch the sun's western limb, and last leave his eastern.

Astronomers have divided the sun's diameter into 12 equal parts, which they call digits; so that if we speak of a digit, or finger's breadth, it is no more than $\frac{1}{12}$ part of the sun's diameter.

Because most people are satisfied with knowing on what days of the months the moon is new and full, I shall here give a table of all the days of the months on which the mean changes of the moon fall, from A. D. 1781 to 1800, in the new stile. The days of full moons are then easily found; for when the change happens before the 15th day of the month, 15 days added to the day of change will give the day of full moon; and when the change is after the 15th day of the month, 15 days subtracted therefrom will give the day of full moon.

Within the above limits, the day of any month on which the moon changeth, in any given year, is found under that month, and right against the year. Thus, suppose it was required to find on what day of April the change happened in A. D. 1781; under April, at the head of the table, and against 1781 at the left hand, is 23; the day of the change required.

When the figures are double, as $\frac{1}{30}$ or $\frac{1}{31}$, against any year, and under any month; they shew, that the moon changes on the first day of that month, and also on the 30th, or 31st thereof.

A TABLE,

A TABLE, shewing on what days of the months the mean changes of the moon fall, from A. D. 1781, to A. D. 1800, new stile.

Years	Jan.	Feb.	March	April	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
1781	24	23	24	23	22	21	20	19	17	17	10	15
1782	14	12	14	12	12	10	10	8	7	6	5	4
1783	3	2	3	2	1	29	29	27	26	25	24	23
					31							
1784	22	20	21	19	19	18	17	16	14	14	12	12
1785	10	9	10	9	8	7	6	5	3	3	2	2
												31
1786	29	28	29	28	27	26	25	24	22	22	20	20
1787	18	17	19	17	17	15	15	13	12	11	10	9
1788	8	6	7	5	5	4	3	2	30	29	28	27
								31				
1789	26	24	26	24	24	22	22	21	19	19	17	17
1790	15	14	15	14	13	12	11	10	8	8	6	6
1791	5	3	5	3	3	1	1	29	27	27	25	25
							30					
1792	23	22	22	21	21	19	19	17	16	15	14	13
1793	12	10	12	10	10	8	8	6	5	5	3	2
1794	1	—	1	29	29	27	27	25	24	23	22	22
	31		31									
1795	20	19	28	19	18	17	16	15	13	13	11	11
1796	9	8	8	7	7	5	5	3	2	1	29	29
										31		
1797	27	26	27	26	25	24	23	22	21	20	19	18
1798	17	15	17	15	15	13	13	11	10	9	8	8
1799	6	5	6	5	4	3	2	1	29	28	27	26
								30				
1800	25	23	25	23	22	21	20	19	17	17	15	15

This table begins the day at midnight, which is according to the common way of reckoning.

Of the causes and times of eclipses.

An eclipse of the sun is caused by the moon's opaque body passing between the sun, and those parts of the earth from which she hides the whole or part of the sun; and this can never happen, but at the time of new moon.

An eclipse of the moon is caused by the whole, or part, of her body passing through the earth's shadow; which can never happen, but when the moon is full.

If the moon's orbit lay in the plane of the ecliptic (in which the earth always moves, and the sun appears to move) the sun would be eclipsed at the time of every new moon; and the moon would be eclipsed at the time of every full.

But

But one half of the moon's orbit lies on the north side of the ecliptic, and the other half on the south side of it; therefore, the moon's orbit intersects the ecliptic only in two opposite points, which are called the moon's nodes; and the angle which the moon's orbit makes with the ecliptic is 5° 18′. The intersection from which the moon ascends northward from the ecliptic, is called the moon's ascending node; and the opposite intersection, from which the moon descends southward from the ecliptic, is called the moon's descending node. These nodes move backward in the ecliptic $19\frac{1}{3}$ degrees every year, from the consequent toward the antecedent signs; and therefore they go quite round the ecliptic, in 18 years, 223 days, and 5 hours.

From the time of the sun's being in conjunction with either of the moon's nodes, to the time of his being in conjunction with the other, is about $173\frac{1}{2}$ days, at a mean rate; within which number of days the eclipses must always happen, in different times of the year.

The days of these conjunctions are shewn in the following table, from A. D. 1781 to 1800, N. S.

Mean conjunctions of the Sun and Nodes.

Years	Ascend. Node. Mon.	D.	Descend. Node. Mon.	D.	Years	Ascend. Node. Mon.	D.	Descend. Node. Mon.	D.
1781	April	26	Oct.	16	1791	Oct.	3	April	13
1782	April	8	Sept.	28	1792	Sept.	14	March	25
1783	March	20	Sept.	10	1793	Aug.	27	March	7
1784	March	1	Aug.	22	1794	Aug.	8	Feb.	16
1785	Feb.	10	Aug.	1	1795	July	21	Jan.	28
1786	Jan.	22	July	15	1796	July	2	Jan.	9
1787	Jan.	4	June	27				Dec.	22
	Dec.	16			1797	June	14	Dec.	4
1788	Nov.	27	June	9	1798	May	26	Nov.	15
1789	Nov.	8	May	20	1799	May	8	Oct.	28
1790	Oct.	21	May	2	1800	April	19	Oct.	9

When the moon changes within 18 days before or after the day of the sun's being in conjunction with either of her nodes, the sun will be eclipsed. And when the moon is full within 12 days before or after the day of the sun's conjunction with either of the nodes, the moon will be eclipsed. At greater distances of the sun from the nodes, there can be no eclipses of these luminaries.

As the table contained in page 493, shews the days on which the mean changes of the moon happen, and the moon is always full on the 15th day before or after the change; and the above table shews the days on which the sun is in conjunction with the moon's nodes; we may easily find, by these tables, on what days

days of any given year, from A. D. 1781 to 1800, the sun and moon must be eclipsed.

EXAMPLE. In the year 1781, the sun is in conjunction with the moon's ascending node, on the 26th of April; and with the descending node, on the 16th of October.

Now, I find by the table (page 493) that in the year 1781, the changes of the moon are on Jan. 24, Feb. 23, March 24, April 23, May 22, June 21, July 20, Aug. 19, Sept. 17, Oct. 17, Nov. 16, Dec. 15; and consequently, as the change on April 23 is within 18 days of April 26, when the sun is in conjunction with the ascending node, the sun must be eclipsed at the time of that change. And as the change on October the 17th is within 18 days of October the 16th, when the sun is in conjunction with the descending node of the moon's orbit, the sun must be eclipsed at the time of that change also. But as all the other changes of the moon, in that year, are more than 18 days from the times of the conjunctions of the sun and nodes, there can be no more than the two above eclipses of the sun in the year 1781.

By adding 15 days to all the changes of the moon in the same year, we find the days of all the full moons to be Jan. 1, Feb. 8, March 10, April 8, May 8, June 6, July 6, Aug. 4, Sept. 3, Oct. 2, Nov. 1, and Dec. 1.

But of all these full moons, there is none that happens within 12 days of the conjunctions of the sun and nodes; therefore there can be no eclipse of the moon in 1781.

And thus we have a very plain and easy method for finding how many eclipses there must be of the sun and moon in any given year, and the days on which they must fall, according to the mean times of new and full moons, from A. D. 1781 to A. D. 1800.

The period of eclipses, according to the learned Dr. Halley, is 18 years, 10 days, 7 hours, 43 minutes; in leap-year, 11 days, 7 hours, 43 minutes.

EXAMPLE. In 1779, the sun is 3° 15′ eclipsed, June 14th, 30 minutes past 7 in the morning; I desire to know when this eclipse returns?

		D.	H.	M.	
1779	June	14	7	30	m.
18		10	7	43	m.
1797	June	25	3	13	a.

This is the best general rule that can be given to continue eclipses from one year to another.

How to calculate the true times and places of eclipses, particular and general, I recommend the curious reader to Mr. Ferguson's or Mr. Leadbetter's Astronomy, where he will find satisfaction to his desire.

The Copernician, or Solar System.

This system of the world, as described on plate 2, is not a late invention, but was known and taught by the wise Samian Pythagoras,

goras, and others among the ancients; which, in after-times, was lost; 'till, in the 15th century, it was again revived by the famous Polish philosopher, Nicholas Copernicus, who was born at Thorn, in the year 1473. In this he was followed by the greatest mathematicians and philosophers that have since lived, as *Kepler*, *Galileo*, *Descartes*, *Gessendus*, and Sir *Isaac Newton*; who have established this system on such an everlasting foundation of mathematical and physical demonstration, that the gates of ignorance shall never prevail against it.

The most famous of the antiquated systems are two; viz. one taught by *Ptolemy*, the *Egyptian*, astronomer, said to have lived 138 years before Christ; the other by the noble Dane, *Tycho Brahe*, born in Schonen, A. D. 1546.

The *Ptolemean system* supposed the earth immoveably fixed in the center of the world; about which moved seven planets, viz. the Moon, Mercury, Venus, the Sun, Mars, Jupiter, and Saturn; above these is placed the firmament of the fixed stars, then the two crystalline spheres; all which were included in, and received motion from, the primum mobile; which constantly revolved about the earth in 24 hours from east to west.

The *Tychonian system* succeeded the *Ptolemean*, but was never so universal. This supposed the earth in the center of the world, or firmament of fixed stars; as also of the two luminaries, the Moon and Sun: but then he supposes the Sun the center of the planetary motions, viz. of Mercury, Venus, Mars, Jupiter, and Saturn; these, with the Sun, all revolved about the earth in the space of a year, to account for the annual motion; and the earth he made to revolve about the axes every 24 hours, from west to east. This hypothesis being part false, was embraced by few, and soon gave way to the only true and rational solar system, restored by *Copernicus* as aforesaid.

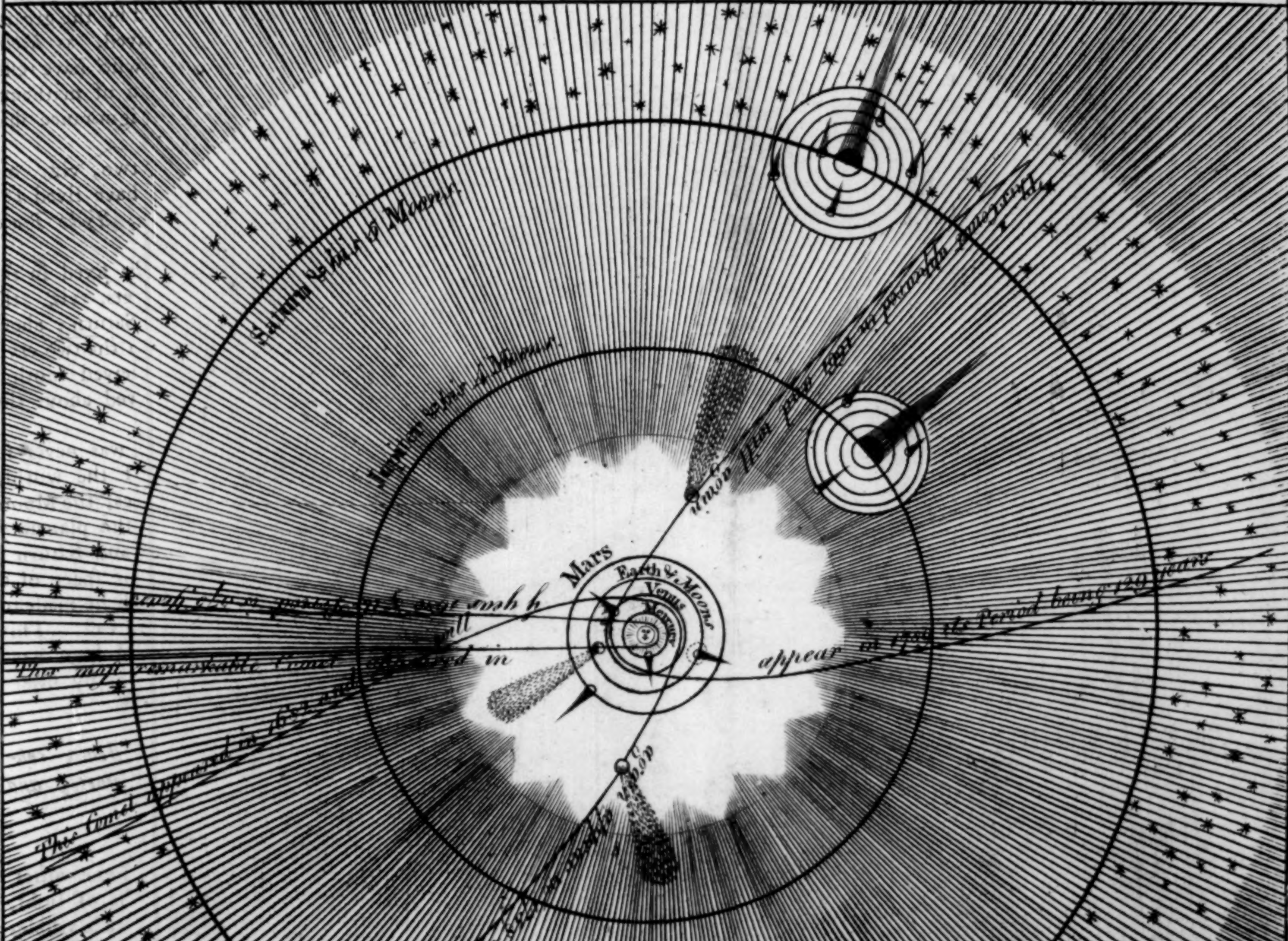
A general View of the Universe, or the Copernician System; containing the Doctrine of the Sun, Moon, Planets, Comets, and fixed Stars; and first,

Of the Sun.

The sun is a huge body of light or fire, whence all the other planets receive their light; and by whose rays and beams of light, the whole system of beings about us is illuminated and made visible,

The diameter of the sun is computed at 822,148 miles, and its bulk, or solid content, at 290,971,000,000,000,000 miles, which is about 1,000,000, or a million times greater than the globe of our earth; and Sir Isaac Newton saith, the sun's light and heat, at the distance of Mercury, is seven times as great as the greatest with us; therefore our water there would be always boiling hot.

The sun is placed in the midst of an immense space, wherein six opaque spherical bodies revolve about him as their center; and we have no certain knowledge that the sun hath any real and proper motion at all, though he seemeth indeed to us to move from east to west, every day; but that is one of the fallacies of sight, not-



Pl. 2. The Copernican *or* Solar System. *Page 496*

notwithſtanding the literal meaning of ſcripture, and the evidence of all our ſenſes to the contrary. No one needs be convinced of the fallacy of ſight, who hath ever been in a ſhip under ſail. The ſacred ſcriptures are certainly a juſt hiſtory of facts and divine revelation; but I am apt to believe, the natural ſciences, eſpecially philoſophy and aſtronomy, as they are now truly taught and underſtood, were entirely unknown to the writers of thoſe books*.

Of the Moon.

The moon is allowed to be a large, dark, opake, ſpherical body, like to our earth in matter and form, borrows her light from the ſun, by reflection, and conveys it to us in the ſun's abſence.

The moon's diameter is 2175 miles; her circumference, therefore, muſt be 6829 miles; whence the ſuperficies of the moon will contain 1415440 ſquare miles, and her ſolid content will be 5386333000 cubic miles, and her diſtance from the earth about 238920 miles; whence the diameter of the moon's orbit will be 477840 miles, and circumference thereof 1500418 miles, which is the journey the moon performs every revolution; and ſhe moves round her own axes, and round the earth, in the ſame ſpace of time; for ſhe revolves about her axes in 27 days, 7 hours, and 43 minutes, at a mean rate; and alſo round the earth in the ſame time; and it is very evident it cannot be otherwiſe, becauſe, at the ſame time, we always ſee the ſame face, or ſide of the moon; and this could not happen, unleſs a proper motion about her axes turned every day juſt ſo much of the moon's body to the earth, as her periodical motions turn from it. Her orbit is eliptical; but ſo irregular is her motion, that ſhe is, by this means, perpetually changing both her place and figure; which proceeds from the excentricity of her orb, and the obliquity, crookedneſs, or irregularity of her axes, in her diurnal motion in the ecliptic; for if the moon was to move regularly in the ecliptic, we ſhould have an eclipſe of the ſun every new moon.

There are ſome parts of the moon's face look dark, and others light; the bright parts are the more eminent parts of land, which reflect the light of the ſun, as hills, mountains, iſlands, &c. and the dark parts of the moon are thought to be ſeas, lakes, rivers, fens, &c. by ſome; and by others, they are ſaid to be ſhaded vallies, caverns, &c. but they repreſent both water and ſhaded places; for neither of theſe reflecting light, muſt appear dark.

The reaſon of the moon's riſing and ſetting an hour later every night than it did the night before, is owing to its diurnal motion of about 13° 10′ from weſt to eaſt round her orbit; which, together with the motion of the earth round her orbit, makes very near an hour's difference of riſing and ſetting, one night with another; but it is not ſo every night, being ſometimes as much

* For a phyſical demonſtration of the earth's motion about the ſun, ſee Kill's Aſtronomical Lectures, pages 34, 35.

again as at others, according to the different times of the year, and the different parts of the ecliptic, the earth and moon may then happen to be in; as is evident from the different aspects of the moon towards the autumn, vulgarly called the harvest moon; which does not set the next night after full, but rises about the same time for several nights together.

In an oblique sphere, all great circles intersecting the equinoctial will, in the revolution of the sphere, intersect the horizon with different angles, at every different part thereof. Thus, with respect to the ecliptic, when the beginning of Libra is orient, or rising in the east, it then makes the greatest angle with the horizon; when Capricorn is orient, the angle is mean; and when Aries is orient, the angle is least of all; therefore, when the moon is full in the beginning of Libra, one day's motion depresses her farthest below the horizon, and least when in the beginning of Aries; consequently, the difference of her rising each day at the vernal equinox will be greatest, and least of all in the autumnal equinox.

The revolution of the moon through the zodiac is called a lunation, and 12 of these lunations or revolutions is a lunar year; which takes up the space of 354 days, 8 hours, 48′ 38″. The difference between this and the solar year, which contains 365d. 5h. 48′ 57″ is almost 11 days, which chronologers call the epact. See sect. 78, page 489.

And because the moon's motion about her axes is performed in the same time as about the earth, the lunarians have their natural days equal to their months.

Of the Planets.

The word planet is derived from the Greek, and in English signifies a wandering star, of which there are two sorts; the first are called planets, the other moons, or satellites.

The primary planets are Mercury, Venus, the Earth, Mars, Jupiter, and Saturn; all of which revolve about the Sun as the center of our system, from west to east, in different distances, as represented in the solar system. The form of their motion is elliptical, more or less, and not perfectly circular; and they all move round the sun in such a manner, as to describe equal areas of space in equal times.

The planets all move about the sun, as I said before, from west to east; and yet they sometimes appear to move the contrary way, from east to west; and sometimes not to move at all, for some time; and thus they are said to be direct, stationary, or retrograde.

Besides the six primary planets above-mentioned, there are ten other lesser planets, called moons or satellites: the Earth hath one moon, Jupiter hath four moons, and Saturn five. The times in which they severally revolve about their primaries, and their distance in semi-diameters of the bodies of Jupiter and Saturn, are as follow:

In Jupiter.

	D.	H.	M.			
1 satellite	1	18	27	Distance from Jupiter's center.	$5\frac{6}{10}$	Semi-diameter of Jupiter.
2 ——	3	13	13		9	
3 ——	7	3	42		$14\frac{3}{10}$	
4 ——	16	16	32		$25\frac{29}{100}$	

In Saturn.

	D.	H.	M.			
1 satellite	1	21	18	Distance from Saturn's center.	$1\frac{9}{10}$	Semi-diameter of Saturn's ring.
2 ——	2	17	41		$2\frac{2}{3}$	
3 ——	4	12	25		$3\frac{2}{5}$	
4 ——	15	22	41		8	
5 ——	79	22	4		$23\frac{3}{10}$	

These moons, and their affections, were all discovered by means of the telescope; and before its use, were unknown to the ancients.

The biggest of Saturn's satellites was discovered with a twelve feet telescope, in the year 1665, by the famous Christopher Hugens; the other four were all discovered by Mr. Cassini, by the help of extraordinary glasses.

All Jupiter's satellites were discovered by Galileo, on Jan. 7, 1610; and from that time, no more than four could ever be seen.

The different dimensions, revolutions, densities, quantities of matter, light, heat, &c. of the six primary planets, will be seen to the best advantage, in one general view, in the following table,

A TABLE of the Six Primary PLANETS, with their affections, &c.

	SATURN.	JUPITER	MARS.	EARTH.	VENUS.	MERCURY.
Their diameters in English miles	67870	81155	4444	7964	7906	2460
Circumference of their bodies	213112	254908	13960	25020	24823	7724
Superfices in square miles	14468430000	20688000000	62032000	199250205	196238000	1900804
Solidity or magnitude in cub. miles	163637700000000	281042300000000	45966600000	264466789070	258445900000	7793273000
Mean distance from the sun in miles	777000000	424000000	123000000	81000000	59000000	32000000
Diameter of their orbits in miles	1554000000	848000000	246000000	162000000	118000000	64000000
Circumference of orbits in miles	4881891000	2662280000	773686000	508939200	370636000	201024000
Periodical times	10759d. 6h. 36m.	433d. 12h. 20m.	686d. 23h. 27m.	365d. 6h. 9m.	224d. 16h. 49m.	87d. 23h. 16m.
Daily revolution round their axes	— — —	9 9 56	1 0 40	0 23 56	0 23 0	— — —
Daily mean motions in the ecliptic	0° 2′ 0″	0° 4′ 59″	0° 31′ 27″	0° 59′ 8″	1° 36′ 8″	4° 5′ 32″
Inclination of orbits to the ecliptic	2 30 0	1 21 0	1 52 0	0 0 0	3 24 0	6 54 0
Excentricities of their orbits	54700	25050	141000	1490	517	7970
Propor. of weight on their surfaces	529	943	—	435	—	—
Proportion of their bulks	621350	1064500	170	1000	985	30
Densities of their masses	67	$94\frac{1}{2}$	—	400	—	—
Quantity of matter in each	33	92	—	$\frac{59}{100}$	—	—
Proportion of light and heat	$1\frac{1}{10}$	$3\frac{1}{12}$	43	100	200	700
Moons, or satellites	5	4	—	1	—	—
Propor. of diam. to the sun, 1000	137	181	6	12	12	4
Proportional mean distances	953800	520110	152369	100000	72333	38710

Besides the moons, these planets are accompanied with, Saturn is known to be encompassed with a ring; which surprizing phænomenon was discovered about 132 years since. It is said, the inner border of the ring, from the body of Saturn, is equal to the breadth of the ring itself; each is computed to be, at least, 21,000 miles; though others make the interval between the ring and Saturn's body to be 210,265, and the breadth of the ring to be 29,200 miles; its thickness is unknown, being too small for observation: it hath a variety of aspects, sometimes appearing a large ellipsis, then a smaller; sometimes only a straight line, and sometimes not visible at all. These are the most remarkable particulars of this prodigy of nature, known by astronomers; as to the matter, of which it doth consist, it is not known by any.

Of Comets.

Comets, or blazing stars, according to Sir *Isaac Newton**, are solid, compact, fixed, and durable substances; and are a kind of planets, which move about the sun in stated periods of time, and shine by the light of the sun-beams reflected from them; and that the orbits are very eccentric and elliptical, but some more and some less; so consequently their periods are longer or shorter. The forms of three remarkable cometary orbs are described in the solar system before-going

Dr. *Halley* has determined the longest axis of the orbit of that comet, which appeared in 1680, and whose period is 575 years, to be 1382975 parts, of which the mean distance of the earth from the sun is 10000; therefore, supposing this mean distance to be 81000000 English miles, then the length of that comet's orb will be above eleven thousand and two hundred millions of English miles.

And Sir *Isaac Newton* has computed the heat of the aforesaid comet, when nearest the sun, to be 2000 times hotter than red-hot iron; and it is computed, that a ball of iron, as big as the globe of our earth, would, if red-hot, require 50,000 years to grow cold in; and the bodies of comets being so much greater than our earth, can never be cold at their greatest distance: and the learned Dr. *Halley* has compiled a set of tables, whereby the places in the zodiac, of above 20 comets, may be determined for any given time.

These comets were formerly taken to be the forerunners of plagues, famine, and war; and some have conjectured they were appointed to demolish planetary worlds, and to supply materials again for building them anew; others, that they are so many hells to punish the damned with perpetual vicissitudes of intolerable heat and cold; but all is uncertain †.

The most fatal accident that can happen from a shock of the earth and a comet, may be the breaking them both in pieces, which would prove their total destruction; yet the force of gravity might new mould them into one or more new planets; but whether this earth has undergone such a catastrophe or not, is

* See Principia Philos. page 508.

† The latter is Mr. Whiston's opinion. See his Astronom. Princip. of Religion.

doubt-

doubtful. However, we are certain, it has undergone very great changes; as the putrified fiſhes, &c. that are often found at a great diſtance from the ſea, and even at the tops of very high mountains, are inconteſtable proofs of ſome ſuch event.

Of the fixed Stars.

Fixed ſtars are ſo called, in oppoſition to the planets, or moving ſtars, becauſe they always keep the ſame place in the heaven, and do not ſeem to move for ſeveral ages together; yet they have an apparent motion, occaſioned by a certain contrary motion of the earth, ariſing from the ſpheroidical figure thereof.

This motion of the fixed ſtars does not exceed 50″ of a degree in a year, or one degree in 70 years; therefore, to complete one revolution of a circle, is required 25,920 years; after which time the ſtars all return again to their former places.

The diſtance of the fixed ſtars is but imperfectly known; however, the famous Hugens has computed the brighteſt, and of courſe the neareſt of all the fixed ſtars, viz. *Syrius*, to be, in appearance, 27,664 times leſs than the ſun; and ſince their diſtances are greater, as their magnitudes are leſſer, therefore this ſtar muſt be at the rate of above two millions of millions of miles; which is ſo great, that a cannon ball would ſpend almoſt 700,000 years in paſſing through it; and it is probable, that all fixed ſtars are equally diſtant from each other, in proportion to the diſtance of the neareſt of them from our ſun.

The number of viſible fixed ſtars, whoſe places have been rectified by aſtronomers, are theſe:

Hypparchus, 140 years before Chriſt, had a catalogue of ſtars, containing — —	1022
Pliny — — —	1600
Ptolemy, 135 years after Chriſt —	1026
1437, Uligh Beighi — — —	1017
1572, Tycho Brahe — — —	777
1620, John Kepler — — —	1163
1630 Dr. John Bayerus — —	1725
Prince of Heſſe — — —	400
1635 Brachius — — —	1672
1651 John Ricciolus — — —	1468
1670 John Hevetius — — —	1888
1676 Dr. Edmund Halley — — —	373
1690 John Flamſtead — — —	3000

Aſtronomers have divided the ſtars into ſix ſeveral ſizes or magnitudes, of which the greateſt or brighteſt of them are called ſtars of the firſt magnitude, as *Aclurus*, *Regulus*, *Sirius*, &c. and the next to them in brightneſs, are called the ſtars of the ſecond magnitude; next to them in brightneſs, are ſtars of the third magnitude, &c. till we come to ſtars of the ſixth magnitude, which comprehend the ſmalleſt ſtars that can be diſcerned with the naked eye; and in the above catalogues, the moſt complete contains only 3000 ſtars, though aſſiſted by the beſt glaſſes; but the moſt that can be diſcovered

covered by the naked eye, in the most serene night, are not above three or four hundred; and Dr. *Keill*, in his Astronom. Lect. vi. page 51, 52, 53, 54. says, of the 3000 stars in Mr. *Flamstead's* catalogue, it is seldom that a very good eye can reckon more than one hundred together; and the famous Mr. *Flamstead* himself asserts, that the naked eye cannot discover above 384 stars in the serenest night, in both the hemispheres.

The Galaxy, via lectea, or milky way, is a broad white tract, encompassing the whole heavens, and extending itself in the sign of *Capricorn*, from the equinoctial to the tropic of *Cancer*, with a double path, and the rest of it is a single one. Some of the ancients, as *Aristotle*, imagined that this path consisted only of a certain exhalation hanging in the air; but by the observations with the telescope, made in this age, it has been discovered to consist of an innumerable quantity of fixed stars, different in situation and magnitude; from the confused mixture of whose light, its white colour is supposed to be occasioned.

The fixed stars are known from the planets by their scintillation, or sparkling; for the planets have no such vibration, twinkling, or glimmering of light; but, generally, all the fixed stars, more or less, and at some times more than others. The cause of their scintillation is variously discoursed of, both by philosophers and astronomers. *Aristotle*, among the ancients, assigns the cause thereof to their remoteness from our sight, by which they are weakly, and as it were by a trembling weariness reached. Some, again, will have the cause of this twinkling to proceed from refraction; others assign the cause to arise from the unequal superfices of the fluctuating air or medium; as stones in the bottom of a river, by the rapid motion of the water, seem to have a kind of tremulous motion: but *Gasseneus*, more probably, conceives this scintillation of the fixed stars to proceed from that native and primogenial light they are indued with, like that of the sun's sparkling, casting forth such quick darted rays, as our weaker light cannot behold without that trembling passion.

GEOGRA-

GEOGRAPHY.

PART VII.

SECTION LXXX.

THE word Geography comes from the Greek; and, in a proper ſenſe, ſignifies nothing more than a deſcription of ſuch parts of the ſurface of the earth as are really land; the other parts, which deſcribes the water, being called Hydrology. For the globe of our earth, having its external ſurface partly land and partly water, has been from thence always denominated the terraqueous globe, which is the foundation of the two above-mentioned ſciences, Geography and Hydrology.

The figure of the earth has been long well known to be globular, or ſpherical: it was originally ſuppoſed flat, or a plane; but this was too great an error for any perſon to continue in long; becauſe if a perſon walks directly north or ſouth, it will cauſe the ſtars to have a greater or leſſer elevation above the horizon; but no alteration, in that reſpect, would happen to them, in walking on a plane, though the diſtance be ever ſo great. This, therefore, afforded an evident proof, that the ſurface of the earth was of a curvilineal form; and becauſe walking over equal ſpaces occaſioned an equal difference in the meridian altitude of the ſtars, it was a proof that the curve ſurface was of the ſpherical kind; and therefore, the body of the globe was in the form of a globe or ſphere.

And this was the general opinion, till the beginning of the laſt century; when experiments on pendulums, the nature of gravity, a centrifugal force in revolving bodies, and ſome other phyſical principles, came to be underſtood, there was great reaſon to ſuſpect, that the figure of the earth could not poſſibly be that of a globe, but that of a ſpheroid as above-mentioned.

Theſe diſcoveries excited a great deſire among the learned, to be ſatisfied (experimentally) of the true figure of the earth; which they eaſily knew could not be done, without actually meaſuring a degree on the ſurface of the earth, in ſeveral different parts of it; and the more remote from each other, the better. At length, by the munificence of Kings, and great propenſity of philoſophers and mathematicians, the arduous undertaking was attempted, proſecuted, and finiſhed, with ſucceſs.

Norwood, and others, make the circumference of the globe 25020 miles; the diameter in the equator is 7964 miles, at the poles 7930 miles: but to form a general idea of theſe things, we may, without much error, look upon the earth as a globe or ſphere; and ſo the dimenſions of its ſurface, computed in ſquare miles, 60 to a degree, will be expreſſed in the following table; whereby you may ſee, at one view, the ſuperficial content of the whole, and its ſeveral parts.

The superficial CONTENT of the whole GLOBE, in square miles, 60 to a degree.

	Square Miles.
The globe	148510627
Seas & unknown parts	117843821
The habitable world	30666806
Europe	2749349
Asia	10275487
Africa	8506208
North-America	3699087
South-America	5454675
Persian empire under Darius	1650000
Roman empire in its utmost height	1610000
Russian	3303485
Chinese	1749000
Great Mogul	1116000
Turkish	960057
Present Persia	800000
Islands.	
Borneo	228000
Madaga	168000
Sumatra	129000
Japan	118000
Great-Britain	72926
Celebes	68400
Manilla	58500
Iceland	46000
Terra del Fuego	42075
Mindinao	39200
Cuba	38400
Javan	38250
Hispaniola	36000
Newfoundland	35500
Ceylon	27730
Ireland	27457
Formosa	17000
Arion	11900
Gilolo	10400
Sicily	9400
Timor	7800
Sardinia	6600
Cyprus	6300
Jamaica	6000

Islands.	Square Miles.
Flores	6000
Ceram	5400
Briton	4000
Socatora	3600
Candia	3220
Porto Rico	3200
Corsica	2520
Zealand	1935
Majorca	1400
St. Jago	1400
Negropont	1300
Teneriff	1272
Gotland	1000
Madeira	950
St. Michael	920
Skye	900
Lewis	880
Funen	768
Yvica	625
Minorca	520
Rhodes	480
Cephalonia	420
Amboyna	400
Orkney Pomona	324
Scio	300
Martinico	260
Lemnos	220
Corfu	194
Providence	168
Man	160
Bornholms	160
Wight	150
Malta	150
Barbadoes	140
Zant	120
Antigua	100
St. Christopher	80
St. Helena	80
Guernsey	50
Jersey	43
Bermudas	40
Rhode	36

GEOGRAPHICAL DEFINITIONS.

The situation of places upon the earth is determined by their latitude and longitude.

1. The latitude of any place upon the earth is its nearest distance, either north or south, from the equator; and if the place be in the northern hemisphere, it is called north latitude; if in the southern hemisphere, it is called south latitude, and is measured by an arch of

the meridian passing through the zenith of the said place, and intercepted betwixt it and the equator: and all places that lie on the same side, and at the same distance from the equator, are said to be in the same parallel of latitude; the parallels of latitude in geography, are the same with the parallels of declination in astronomy.

COROLLARIES. 1. No place can have above 90 degrees of latitude, either north or south.

2. Those places that lie under the equinoctial have no latitude, it being from thence that the calculation of latitude is counted; and those places that lie under the poles have the greatest latitude, those points being at the greatest distance from the equator, or equinoctial line.

3. The latitude of any place is always equal to the elevation of the pole in the same place, above the horizon; and is therefore often expressed by the pole's height, or elevation of the pole: the reason of which is, because from the equator to the pole there is always the distance of 90 degrees, and from the zenith to the horizon the same number of degrees, each of these including the distance from the zenith to the pole. That distance, therefore, being taken away from both, will leave the distance from the zenith to the equator (which is the latitude) equal to the distance from the pole to the horizon.

4. The elevation of the equator in any place, is always equal to the complement of the latitude of the same place.

5. A ship sailing directly towards the equator, lessens her latitude, or depresses the pole just so much as her distance sailed; and sailing directly from the equator augments her latitude, or raises the pole just so much as her distance sailed.

2. DIFFERENCE of LATITUDE, is the nearest distance betwixt any two parallels of latitude, shewing how far the one is to the northward or southward of the other, which can never exceed 180 degrees; and when the two places are in the same hemisphere, or on the same side of the equator, the lesser latitude subtracted from the greater, and when they are on different sides of the equator, the two latitudes added gives the difference of latitude.

3. The longitude of any place upon the earth, is an arch of the equator, contained betwixt the meridian of the given place, and some fixed or known meridian; or it is equal to the angle formed by the two meridians; which properly can never exceed 180 degrees, though sometimes the longitude is counted easterly quite round the globe.

Since the meridians are all moveable, and not one that can be fixed in the heavens, the longitudes of places cannot so well be fixed from any one meridian; but every geographer is at his liberty to make which he pleases his first meridian, from whence to calculate the longitudes of other places: hence it is, that the geographers of different nations, reckon their longitudes from different meridians, commonly chusing the meridian passing through the metropolis of their own country for their first; thus the English geographers generally make the meridian of London to be their first; the French, that of Paris; and the Dutch, that of Amsterdam, &c. And mariners generally

reckon their longitude from the last known land they saw. This arbitrary way of reckoning the longitude from different places, makes it necessary, whenever we express the longitude of any place, that the place from whence it is counted be also expressed.

4. ZONES are large tracts of the surface of the earth, distinguished by the tropicks and polar circles, being five in number, viz. one torrid, two temperate, and two frigid.

The torrid, or burning zone, is all the space comprehended between the two tropicks: the ancients imagined this tract of the earth to be uninhabitable, because of the excessive heat, it being so near the sun. All the inhabitants of the torrid zone have the sun in their zenith, or exactly over their heads, twice in every year; excepting those who live exactly under the two tropicks, where the sun comes to their zenith only once in every year.

The two temperate zones lie on either side of the globe between the tropicks and the polar circles.

The two frigid zones are those spaces upon the globe that are included within the two polar circles.

The inhabitants of the earth are also distinguished by the diversity of their shadows; those who live in the torrid zone are called Amphiscians, because their noon-shadow is cast different ways, according as the sun is to the northward or southward of their zenith; but when the sun is in their zenith, they are called Ascians.

The inhabitants of the temperate zones are called Heteroscians, because their noon-shadow is always cast the same way: but those who live under the tropicks are called Ascians-Heteroscians. Those who live in the frigid zones are called Periscians, because sometimes their shadow is cast round about them.

The inhabitants of the earth are also distinguished into three sorts, in respect to their situation one to another; and these are called, the Perioeci, Antoeci, and Antipodes.

5. The Perioeci are those who live under opposite points of the same parallel of latitude; they have the seasons of the year at the same time, and their days and nights always of the same length with one another; but the one's noon is the other's midnight; and when the sun is in the equinoctial, he rises with the one, when he sets with the other; those who live under the poles have no Perioeci.

6. The Antoeci live under the same meridian, and in the same latitude, but on different sides of the equator; their seasons of the year are contrary, and the days of the one are equal to the nights of the other; but the hour of the day and night is the same with both; and when the sun is in the equinoctial, he rises and sets to both exactly at the same time. Those who live under the same equator have no Antoeci.

7. The Antipodes are those who live diametrically opposite to one another, standing, as it were, exactly feet to feet; their days and nights, summer and winter, are at direct contrary times.

The surface of the earth is by some distinguished into climates.

8. A climate is a tract of the surface of the earth, included between two such parallels of latitude, that the length of the longest day in the one, exceeds that in the other, by half an hour.

 The

The whole ſurface of the earth is conſidered as being divided into 60 climates, viz. from the equator to each of the polar circles, 24, ariſing from the difference of half an hour in the length of their longeſt days; and from the polar circles to the poles themſelves are ſix, ariſing from the difference of an entire month; the ſun being ſeen, in the firſt of theſe, a whole month without ſetting; in the ſecond, two, and in the third, three months, &c. Theſe climates continually decreaſe in breadth, the further they are from the equator. How they are framed, viz. the parallel of latitude in which they end (that being likewiſe the breadth of the next) with the reſpective breadth of each of them, is ſhewed in the following table.

A TABLE of the CLIMATES.

Climates between the Equator and the Polar Circles.

Climates.	Longeſt day.	Latitude. D.	Latitude. M.	Breadth D.	Breadth M.	Climates.	Longeſt day.	Latitude. D.	Latitude. M.	Breadth D.	Breadth M.
1	12½	8	25	8	25	13	18½	59	58	1	29
2	13	16	25	8	00	14	19	61	18	1	20
3	13½	23	50	7	25	15	19½	62	25	1	07
4	14	30	20	6	30	16	20	63	22	0	57
5	14½	36	28	6	08	17	20½	64	06	0	44
6	15	41	22	4	54	18	21	64	49	0	43
7	15½	45	29	4	07	19	21½	65	21	0	32
8	16	49	01	3	32	20	22	65	47	0	26
9	16½	51	58	2	57	21	22½	66	06	0	19
10	17	54	27	2	29	22	23	66	20	0	14
11	17½	56	37	2	10	23	23½	66	28	0	08
12	18	58	29	1	51	24	24	66	31	0	03

Climates between the Polar Circles and the Poles.

Length of days. Months.	Latitude. D.	Latitude. M.	Length of days. Months.	Latitude. D.	Latitude. M.
1	67	21	4	78	30
2	69	48	5	84	05
3	37	37	6	90	00

Of the coſmical, achronical, and heliacal, riſing and ſetting of the Stars.

A ſtar is ſaid to riſe or ſet coſmically, when it riſes or ſets at ſun-riſing; and when it riſes or ſets at ſun-ſetting, it is ſaid to riſe or ſet achronically. A ſtar riſes heliocally, when firſt it becomes viſible, after it has been ſo near the ſun, as to be hid by the ſplendor of his rays; and a ſtar is ſaid to ſet heliacally, when it is firſt immerſed, or hid by the ſun's rays.

The fixed ſtars, and the three ſuperior planets, Mars, Jupiter, and Saturn, riſe heliacally in the morning; but the moon riſes heliacally in the evening; becauſe the ſun is ſwifter than the ſuperior planets, and ſlower than the moon.

Of the Surface of the Earth, considered as it is composed, of Land and Water.

The earth consists naturally of two parts, land and water; and therefore it is called the terraqueous globe: each of these elements are subdivided into various forms and parts, which accordingly are distinguished by different names.

I. *Of the Land.*

The Land is distinguished into Continents, Islands, Peninsulas, Isthmusses, Promontories, Mountains, or Coasts.

9. A Continent is a large tract of land, comprehending several countries, not separated by the sea; such as Europe, Asia, Africa, and America; which four are the principal divisions of the earth.

10. An Island is a portion of the earth, entirely surrounded with water, such as Great-Britain and Ireland; also a small part of dry land, in the midst of a river, is called an island.

11. A Peninsula is a part of land almost environed with water, save one narrow neck of land adjoining it to the Continent; such as Denmark joining to Germany; also Africa is properly a large peninsula, joining to Asia.

12. An Isthmus is a narrow neck of land, joining a peninsula to the continent; as the isthmus of Suez, which joins Africa to Asia; that of Panama, joining North and South America, &c.

13. A Promontory is a high part of land, stretching out into the sea; and is often called a Cape or Headland; such is the Cape of Good Hope, in the south of Africa; the Lizard Point, &c. A Mountain is a high part of land, in the midst of a country, overtopping the adjacent parts.

14. A Coast, or Shore, is that part of land which borders upon the sea, whether it be an island, or a continent; and that part of the land, which is far distant from the sea, is called the inland country. These are the usual distinctions of the land.

II. *Of Water.*

The Water is distinguished into Oceans, Seas, Lakes, Gulfs, Straits, and Rivers.

15. The Ocean, or Main Sea, is a vast spreading collection of water, not divided by lands running between; such as the Atlantic, or Western Ocean, between Europe and America; the Pacific Ocean or South Sea, &c.

Note. Those parts of the Ocean, which border upon the land, are called by various names, according to those of the adjacent countries; as the British Sea, the Irish Sea, the French and Spanish Sea.

16. A Lake is a collection of standing water, surrounded by land, and having no communication by sea; as the Lake of Geneva, the Lake of Aral, and the Caspian Sea, which is properly a lake.

17. A Gulf is a part of the sea, almost encompassed with land, or, that which runs up a great way into the land; as the Gulf of Venice, &c. but if it be very large, it is rather called an Inland-Sea; as the Baltic Sea, the Mediterranean Sea, the Red Sea, or the Arabian

bian Gulf, &c. And a small part of the sea, thus environed with land, is usually called a Bay. If it be but a very small part, or as it were, a small arm of the sea, that runs but a few miles between the land, it is called a Creek or Haven.

18. A Strait is a narrow passage lying between two shores, whereby two seas are joined together; as the Straits of Dover, between the British Channel and the German Sea; the Straits of Gibraltar, between the Atlantic and the Mediterranean Sea. These are all the necessary terms used in Geography.

The names of the several countries, seas, and all the principal divisions of the earth, the reader will find expressed upon the terrestrial globes.

To give a proper account of the produce of each country, the genius of the people, their political institutions, &c. is, properly, a subject of itself.

A Description of the Globes.

A globe is a sperical, or round body, whereon those circles, that are fixed, are drawn; those that are moveable, are supplied by the brass meridian, the wooden horizon, and the quadrant of altitude.

The appurtenances of a globe are, 1st. the axes, represented by a wire run through both poles. 2. A brass circle, representing the first meridian, wherein the globe turns on its axes. 3. A wooden frame, representing the horizon, on which the course of the sun is inscribed; and within which, the brazen meridian turns, by means of the notches. 4. The horary circle; it is fixed in the brazen meridian, in such a manner, as to make the pole its center. 5. The quadrant of altitude; which is a thin brass plate, screwed to the brazen meridian, and graduated with 90 degrees, answering to one-fourth part of the equator. 6. The semi-circle of position; this is a thin narrow plate of brass, exactly answering to one-half of the equator, containing 180 degrees. 7. The compass; it is a round circle, like a wheel, with 32 points issuing from its center; one of which is a flower-de-luce, and points due north; it usually stands on the pedestal of the horizon.

The things above described are common to both globes; but there are some others, which are peculiar, or proper to one sort of globe. The two colures, and the circles of latitude, from the ecliptic, belong only to the celestial globe; also the ecliptic itself does properly belong only to this globe, though it is drawn on the terrestrial, for the sake of those that might not have the other by them. The equinoctial, on the celestial globe, is always numbered into 360 degrees, beginning at the equinoctial point ♈; but on the terrestrial, it is arbitrary, where these numbers commence, according to the meridian of what place you intend for your first; and the degrees may be counted, either quite round to 360, or both ways, till they meet in the opposite part of the meridian, at 180.

The globe is of great use to explain geography: is very easy and pleasant to learners, and will explain a great number of problems; some of which are the following

The USE of the TERRESTRIAL GLOBE.

PROBLEM I. *To find the latitude and longitude of any given place upon the globe.*

Turn the globe round its axes, till the given place lie exactly under the brazen meridian; then that degree upon the meridian which is directly over it is the latitude; likewise that degree upon the equator which is cut by the brazen meridian is the longitude required from the first meridian upon the globe.

EXAMPLE. What is the latitude and longitude of Mexico, Pekin in China, and Cape Horn?

	Latitude.	Longitude.
Mexico - -	20° N.	102° W.
Peking - -	39° 45 N.	111° E.
Cape Horn -	57 S.	80 W.

Prob. 2. *The latitude and longitude being known, to rectify the globes fit for use.*

Raise the pole to the given latitude, as suppose London; then fix the quadrant of altitude in the zenith, and by the compass on the pedestal set the globe so that the brazen meridian may stand due north and south, according to the needle, and then it is done.

EXAMPLE. By the preceding problem I find the latitude of London to be 51½ degrees north latitude; therefore I count 51½ degrees from the pole downwards, and turn the meridian through the notches of the horizon till those 51½ degrees come exactly to the uppermost edge of the north point in the horizon; and then is the meridian rectified to the latitude of London.

2. Next rectify the quadrant of altitude, by screwing the edge of the nut that is even with the graduated edge of the thin plate, to 51½ degrees of the brazen meridian counted from the equinoctial, which is the zenith of London; and thus is your globe rectified for the solution of such questions as are to be wrought thereby in that latitude.

Prob. 3. *The latitude and longitude being given, to find the place.*

Seek for the given longitude in the equator, and bring that point to the meridian; then count from the equator on the meridian, the degree of latitude given towards the arctic or antarctick pole, according as the latitude is northerly or southerly; and under that degree of latitude lies the place required.

EXAMPLE. What is the name of that place, whose latitude is 18° N. and longitude 76½ W.?

Answer, Jamaica.

Prob. 4. *To find the difference of latitude between any two given places.*

Bring each of the places proposed successively to the meridian, and observe where they intersect it; then the number of degrees upon the meridian, contained between the two intersections, will be the difference of latitude required; or, if the places proposed are on the same side of the equator, having first found their latitudes, subtract the lesser from the greater; but if they are on the contrary

trary sides of the equator, add them together; and the difference in the first case, and the sum in the latter, will be the difference of latitude required.

EXAMPLE. What is the difference of latitude between London and Rome; also between Paris and Cape Bona?

The difference of latitude between London and Rome, is 9° 45″; and between Paris and Cape Bona, is 83°.

Prob. 5. *To find the difference of longitude between any two given places.*

Bring each of the places successively to the meridian, and see where the meridian cuts the equator each time; the number of degrees contained between those two points, if it be less than 180 degrees, otherwise the remainder to 360 degrees, will be the difference of longitude required.

EXAMPLE. What is the difference of longitude between Rome and Constantinople?

By working as above, you will find the difference of longitude to be 19° (which are reduced into miles, by multiplying the degrees by 60, and allowing for every minute, one mile) makes 1140 miles for their distance.

Prob. 6. *The day of the month being given, to find the sun's place in the ecliptic, and his declination.*

First, to find the sun's place, look for the day of the month, given in the kalendar of months, upon the horizon; and against it, you will find that sign and degree of the ecliptic, which the sun is in. The sun's place being thus found, look for the same in the ecliptic line, which is drawn upon the globe, and bring that point to the meridian; then that degree of the meridian, which is over the sun's place, is the declination required; which is either north or south, according as the sun is in the northern or southern signs: thus,

	Sun's place.			Declination.	
		Deg.	Min.	Deg.	Min.
April 12	♉	3	00	12	32 N.
July 20	♌	7	51	18	20 N.

Prob. 7. *To find the angle of position of places; or, the angle formed by the meridian of one place, and a great circle passing through both the places.*

First, rectify the globe, for the latitude and zenith of one of the given places; then bring the said place to the meridian, and turn the quadrant of altitude about, until the fiducial edge thereof cuts the other place; and the number of degrees upon the horizon, contained between the said edge and the meridian, will be the angle of position sought.

Thus, the angle of position at the Lizard, between the meridian of the Lizard, and the great circle passing from thence to Barbadoes, is 69 degrees south-westerly; but the angle of position between the same places, at Barbadoes, is but 38 degrees, north-easterly.

 SCHO-

SCHOLIUM.—The angle of poſition between two places, is a different thing from what is meant by the bearings of places; the bearings of two places is determined by a ſort of ſpiral line, called a rhumb line, paſſing between them in ſuch a manner, as to make the ſame or equal angles, with all the meridians through which it paſſeth. But the angle of poſition is the very ſame thing with what we call the azimuth in aſtronomy; both being formed by the meridian, and a great circle, paſſing through the zenith of a given place, and a given point, either in the heavens, then called the azimuth, or upon the earth, then called the angle of poſition.

From hence may be diſcovered the error of that geographical paradox, viz. if a place, A, bears from another, B, due weſt, B ſhall not bear from A due eaſt; for if it be admitted, that the eaſt and weſt lines make the ſame angles with all the meridians through which they paſs, it will follow, that theſe lines are the parallels of latitude: for the path deſcribed in travelling from eaſt to weſt, is the continuation of the ſurface of a cone, whoſe ſides are the radii of the ſphere, and baſe the parallel of latitude of the traveller; and it is evident, that all the meridians cut the ſaid ſurface at right (and therefore at equal) angles; whence it follows, that the rhumbs of eaſt and weſt are the parallels of latitude; though the caſe may ſeem different, when we draw inclining lines (like meridians) upon paper, without carrying our idea any further.

Prob. 8. *To find the Antœci, Periœci, and Antipodes, to any given place.*

Bring the given place to the meridian, and having found its latitude, count the ſame number of degrees on the meridian, from the equator towards the contrary pole, and that will give the place of the Antœci. The globe being ſtill in the ſame poſition, ſet the hour index to 12 at noon; then turn the globe about, till the index points to the lower 12; the place which then lies under the meridian, having the ſame latitude with the given place, is the Periœci required. As the globe now ſtands, the Antipodes of the given place are under the ſame point of the meridian that its Antœci ſtood before.

EXAMPLE. Required the Antœci, Periœci, and Antipodes, to London, whoſe latitude is 51° 30′ South.

That place which lies under the ſame meridian, and in the latitude of 51° 30′ ſouth, is the Antœci. That which lies in the ſame parallel with London, and 180° of longitude from it, is the Periœci, and the Antipodes is that place whoſe longitude from London is 180°, and latitude 51° 30′ ſouth.

Prob. 9. *The hour of the day at one place, being given to find the correſponding hour, or what o'clock it is at that time in any other place.*

The difference of time between two places, is the ſame with their difference of longitude; wherefore, having found their difference of longitude, reduce it into time, by allowing one hour for every 15 degrees, &c. and if the place, where the hour is required, lies eaſterly from the place where the hour is given, add the difference

of longitude, reduced into time, to the hour given, and the sum will be the hour required; and if the place lies westerly, subtract the difference of longitude, reduced into time, the remainder will be the hour required. Or,

Bring the place, where the hour is given, to the meridian, and set the hour index to the given hour, then turn the globe about, until the place, where the hour is required comes to the meridian, and the hour index will point out the hour at the said place.

Thus, when it is noon at London, it is

	H.	M.	
At Rome —	0	52	P. M.
Constantinople	2	07	P. M.
Vera-Cruz —	5	30	A. M.
Pekin, in China	7	50	P. M.

Prob. 10. *The day of the month being given, to find those parts on the globe where the sun will be vertical, or in the zenith that day.*

Having found the sun's place in the ecliptic, bring the same to the meridian, and note the degree over it: then turning the globe round, all places that pass under that degree, will have the sun vertical that day.

Prob. 11. *A place being given in the torrid zone, to find those two days in which the sun shall be vertical to the same.*

Place the town in question (suppose Goa, which is in the 16th degree of north latitude) under the brass meridian, and observe what degree of the ecliptic will pass under this latitude, when the globe is turned about, and you will find two; the 13th degree of Taurus, and the 17th degree of Leo; when the sun, therefore, shall be in the 13th degree of Taurus, on the 30th of May, or in the 17th degree of Leo, on the 10th of August, it will be perpendicular to Goa.

Prob. 12. *To find where the sun is vertical at any given time assigned, or the day of the month, and the hour, at any place, being given, to find in what place the sun is vertical at that very time.*

Having found the sun's declination, and brought the place (suppose London) to the meridian, set the index to the given hour, and turn the globe about, until the index points to 12 at noon; which being done, that place upon the globe, which stands under the point of the sun's declination upon the meridian, has the sun that moment in the zenith.

Prob. 13. *The day, and hour of the day, at any one place being given, to find all those places upon the earth where the sun is then rising, setting, culminating (or on the meridian); also when it is day, night, twilight, dark night, and midnight; where the twilight then begins, and where it ends; the height of the sun, in any part of the illuminated hemisphere, also his depression in the obscure hemisphere.*

Having found the place where the sun is vertical at the given hour, rectify the globe for the latitude, and bring the said place to the meridian.

Then

Then all those places, that are in the western semi-circle of the horizon, have the sun rising at that time.

Those in the eastern semi-circle, have it setting.

To those who live under the upper semi-circle of the meridian, it is 12 o'clock at noon. And,

Those who live under the lower semi-circle of the meridian have it at midnight.

All those places that are above the horizon, have the sun above them, just so much as the places themselves are distant from the horizon, which height may be known by fixing the quadrant of altitude in the zenith, and laying it over any particular place.

In all those places, that are 18 degrees below the western side of the horizon, the twilight begins in the morning; and in all those places, that are 18 degrees below the eastern side of the horizon, the twilight ends, and the total darkness begins.

The twilight is in all those places, whose depression below the horizon does not exceed 18 degrees. And,

All those places, that are lower than 18 degrees, have dark night.

Prob. 14. *The day of the month being given; to shew, at one view, the length of day and night in all places upon the earth at that time; and to explain, how the vicissitudes of day and night are really made by the motion of the earth round her axes in 24 hours; the sun standing still.*

The sun always illuminates one half of the globe, or that hemisphere which is next towards him, while the other remains in darkness; and if we elevate the globe, according to the sun's place in the ecliptic (by prob. 13.) it is evident, that the sun (he being at an immense distance from the earth) illuminates all that hemisphere, which is above the horizon; the wooden horizon itself will be the circle terminating light and darkness; and all those places, that are below it, are wholly deprived of the solar light.

The globe standing in this position, those arches of the parallels of latitude, which stand above the horizon, are the diurnal arches, or the length of the day in all those latitudes at that time of the year, and the remaining part of those parallels, which are below the horizon, are the nocturnal arches, or the length of the night in those places. The length of the diurnal arches may be found, by counting how many hours are contained between the two meridians, cutting any parallel of latitude in the eastern and western parts of the horizon.

In those places that are in the western semi-circle of the horizon, the sun appears rising; for, the sun standing still in the vertex (or above the brass meridian) appears easterly, and 90 degrees distant from all those places that are in the western semi-circle of the horizon; and therefore, in those places he is then rising.—Now, if we pitch upon any particular place upon the globe, and bring it to the meridian, and then bring the hour index to 12 at noon, and afterwards turn the globe about, until the aforesaid place be brought to the western side of the horizon; the index will then shew the time of sun-rising in that place. Supposing the hour circle numbered the contrary way, then turning the globe gradually

dually about from west to east, and minding the hour index, we shall see the progress made in the day every hour, in all latitudes upon the globe, by the real motion of the earth round its axes; until, by their continual approach to the brass meridian (over which the sun stands still all the while) they at last have noon-day, and the sun appears at the highest; and then by degrees, as they move easterly, the sun seems to decline westward, until, as the places successively arrive in the eastern part of the horizon, the sun appears to set in the western; for the places that are in the horizon, are 90 degrees distant from the sun.

We may observe, that all places upon the earth, that differ in latitude, have their days of different length (except when the sun is in the equinoctial) being longer or shorter, in proportion to what part of the parallels stand above the horizon: those that are in the same latitude, have their days of the same length: but have them commence sooner or later, according as the places differ in longitude.

Prob. 15. *The latitude of any place, not exceeding 66 ½ degrees, and the day of the month being given, to find the time of sun-rising and setting, and the length of the day and night.*

Having rectified the globe according to the latitude, bring the sun's place to the meridian, and put the hour index to 12 at noon; then bring the sun's place to the eastern part of the horizon, and the index will shew the time when the sun rises; again, turn the globe until the sun's place be brought to the western side of the horizon, and the index will shew the time of sun-setting.

The hour of sun-setting, doubled, gives the length of the day; and the hour of sun-rising, doubled, gives the length of the night.

EXAMPLE. Let it be required to find when the sun rises and sets at London on the 20th of April?

Rectify the globe for the latitude of London, and having found the sun's place, corresponding to April the 20th, viz. ♉ 10¾ degrees, bring ♉ 10¾ degrees to the meridian, and set the index to 12 at noon; then turn the globe about, till ♉ 10¾ degrees be brought to the eastern part of the horizon, and you will find the index point to 4¼ hours, the time of sun-rising: again, bring the sun's place to the western part of the horizon, and the index will point to 7¾ hours, which is the time of sun-setting.

Prob. 16. *The latitude, the sun's place, and his altitude, being given; to find the hour of the day, and the sun's azimuth from the meridian.*

Having rectified the globe for the latitude, the zenith, and the sun's place, turn the globe and the quadrant of altitude, so that the sun's place may cut the given degree of altitude; then the index will shew the hour, and the quadrant will cut the azimuth in the horizon.

Thus, if at London, on the 10th of August, the sun's altitude be 36 degrees in the forenoon, the hour of the day will be 9, and the sun's azimuth about 58 degrees from the south part of the meridian.

Prob.

Prob. 17. *The latitude, the sun's place, and his azimuth, being given, to find his altitude, and the hour.*

Rectify the globe for the latitude, the zenith, and the sun's place; then put the quadrant of altitude to the sun's azimuth in the horizon, and turn the globe till the sun's place meets the edge of the quadrant; then the said edge will shew the altitude, and the index point to the hour. Thus,

On May the 10th, at London, when the sun is due east, his altitude will be about 24 degrees, and the hour 7 in the morning; and when his azimuth is 60 degrees south-westerly, the altitude will be about $44\frac{1}{2}$ degrees, and the hour about $2\frac{3}{4}$ in the afternoon.

Prob. 18. *The latitude, the sun's altitude, and his azimuth being given; to find his place in the ecliptic, and the hour.*

Rectify the globe for the latitude and zenith, and set the edge of the quadrant to the given azimuth, then turning the globe about, that point of the ecliptic, which cuts the altitude, will be the sun's place. Keep the quadrant of altitude in the same position, and having brought the sun's place to the meridian, and the hour index to 12 at noon, turn the globe about till the sun's place cuts the quadrant of altitude, and then the index will point the hour of the day.

Prob. 19. *The declination and meridian altitude of the sun, or of any star, being given, to find the latitude of the place.*

Mark the point of declination upon the meridian, according as it is either north or south from the equator, then slide the meridian up or down in the notches, till the point of declination be so far distant from the horizon, as is the given meridian altitude; that elevation of the pole will be the latitude. Thus,

If the sun's, or any star's meridian altitude, be 50 degrees south, and its declination $11\frac{1}{2}$ degrees north, the latitude will be $51\frac{1}{2}$ degrees north.

Prob. 20. *The day and hour of a lunar eclipse being known, to find all those places upon the globe, in which the same will be visible.*

Find, by prob. 12. where the sun is vertical at the given hour, and bring that point to the zenith, then the eclipse will be visible in all those places that are under the horizon, or if you bring the Antipodes to the place where the sun is vertical into the zenith, you will have the places where the eclipse will be visible above the horizon.

Note. Because lunar eclipses continue sometimes for a long while together, they may be seen in more places than one hemisphere of the earth; for, by the earth's motion round its axes, during the time of the eclipse, the moon will rise in several places, after the eclipse began.

When an eclipse of the sun is central, if you bring the place where the sun is vertical at that time into the zenith, some part of the eclipse will be visible in most places within the upper hemisphere; but, by reason of the short duration of solar eclipses;

and

and the latitude which the moon has at that time (though but small) there is no certainty in determining the places where those eclipses will be visible by the globe, but recourse must be had to calculations.

The USE of the Celestial Globe.

Prob. 1. *To find the right ascension and declination of the sun, or any fixed star.*

Bring the sun's place, in the ecliptic, to the meridian, then that degree of the equator, which is cut by the meridian, will be the sun's right ascension; and that degree of the meridian, which is exactly over the sun's place, is the sun's declination.

After the same manner, bring the place of any fixed star to the meridian, and you will find its right ascension in the equinoctial, and declination on the meridian. Thus, the right ascension and declination of the sun,

		Right Asc. Degrees.	Declination. Degrees.
Is on	January 20	314	$17\frac{3}{4}$ S.
	March 25	14	6 N.
	July 10	$120\frac{1}{4}$	$20\frac{1}{4}$ N.
	November 15	242	21 S.

	Right Asc. Degrees.	Declination. Degrees.
Aldebaran —	65	$16\frac{3}{4}$ N.
Spica Virginis	$197\frac{3}{4}$	$9\frac{3}{4}$ S.
Capella —	74	$45\frac{2}{3}$ N.
Syrus, or the dog-star	$98\frac{1}{4}$	$16\frac{1}{3}$ S.

Note. The insensible change in the longitude, right ascension, and declination of the fixed stars, made by their slow motion, parallel to the ecliptic (being but 1 degree in 72 years) is not worth observation in this place. The right ascension and declination of the sun varies every day.

Prob. 2. *The right ascension and declination of any star being given, to find the star upon the globe.*

Bring the given degree of right ascension on the equator to the meridian, then look under the degree of declination on the meridian, and you will find the star at the meridian, under the given degree of declination.

Thus, suppose I wanted to find Aldebaran, whose right ascension is 65°, and his declination 16° 45′ N. I first bring 65° of the equinoctial to the meridian; and looking under 16° 45′ N. declination on the meridian, I find Aldebaran: proceed in the same manner for any other star.

Prob. 3. *To find the longitude and latitude of a given star.*

Having brought the solstitial colure to the meridian, fix the quadrant of altitude over the proper pole of the ecliptic, whether it be north or south; then turn the quadrant over the given star, and the

the arch contained between the star and the ecliptic, will be the latitude, and the degree cut on the ecliptic will be the star's longitude.

Thus you will find the latitude of Arcturus to be 31° N. and the longitude 200° from ♈, or 20° from ♎. Also the latitude of Fomalhaunt, in the southern Fish, 21° S. and longitude $299\frac{1}{2}$ degrees, or ♒ 29° 30′.

The distance between two stars, or the number of degrees contained between them, may be found, by laying the quadrant of altitude over each of them, and counting the number of degrees intercepted, after the same manner, as we found the distance between two places on the terrestrial globe.

Prob. 4. *To find the rising and setting of the stars; and the point of the compass any star rises or sets upon in any latitude, and on any day of the year*

Rectify the globe, and bring the sun's place to the meridian, then turn the globe till the given star comes to the eastern verge of the horizon, and the index will point to the time of rising, and the horizon will shew the point it rises upon. Turn it to the west, and the index will point to the time of setting, and the horizon will shew you the point it sets upon.

Proceed thus, and you will find that Aldebaran, on Nov. 5, at London, rises a little past six in the evening, and sets about nine in the morning. The point he rises upon is E. N. E. and the point he sets upon is W. N. W. Also Regal in Orian, the same night, rises a little before nine, and sets about half past seven in the morning. The points of rising are W. by S. and setting E. by S.

Note. The stars rise and set every day on the same point of the compass, though at contrary hours.

Prob. 5. *To find the time, viz. how many hours any star continues above the horizon, from its rising to its setting, in any latitude.*

Rectify the globe, then bring the star to the eastern verge, and note the time of rising; then turn the globe to the western side, and the number of hours that passed through the dial-plate, tells you the continuance of that star above the horizon. Thus,

I find Aldebaran, at London, continues up from the time of his rising on any day (for example, take Dec. 25.) about 15 hours, and Regil about ten hours and a half.

At Port Royal, he continues up only about twelve hours and three quarters.

Prob. 6. *The latitude, day of the month, and height of any star given, to tell the time or hour of the night.*

Rectify the globe for the latitude, &c. then fix the quadrant in the zenith, and move the globe and the quadrant together, till the star cuts the quadrant in the given height, and the index will point to the hour.

Thus, on January the 21st (at London) in the evening, I observed Aldebaran E. S. E. to be about 40° high; I demand the

time

time of this observation? Answ. A little past five in the evening. Again, on Dec. 25, in the evening, I observed Sirius to be about 28° 30′ high; I demand the hour? Answ. About ten at night; and Aldebaran is under the meridian at the same time.

Prob. 7. *The latitude of the place, and the day of the month being given, to know where to find any star, or tell the name of any star, at pleasure.*

Rectify the globe for the day, and turn it till the index points to the given hour: then, by a quadrant, take the height of the required star, or, for want of this (in a common way of guessing) observe well what part of the heavens it is in, viz. whether E. N. E. S. W. or the like; as also its height, as near as you can guess. This being done, set the globe in due order, for the day and hour, and you will find the same star on the globe; and, by applying the quadrant, you will find the exact point of the compass, and the real height the star then was; which, though not, perhaps, near to what you guessed it at, yet, if it be any noted star, you may assure yourself it was right; as there is no other star of note near it about that height, and upon the same point.

Thus, on Dec. 25, at eight at night, I observed a bright star (as near as I can guess) on the south-east point, and about 48° high; I would know what star it is? Answ. Aldebaran.

I rectify the globe, and turn the index to the hour, and then turn the quadrant to the given point of the compass, and looking about 48° high on the quadrant, I find Aldebaran to be the nearest bright star by the quadrant, on that point and height; therefore, I conclude it is Aldebaran.

After the same manner, the globe being rectified, you may distinguish those stars that are to the southward of you, and be soon acquainted with all the stars that are visible in our hemisphere.

SCHOLIUM.—The globe being rectified to the latitude of any place, if you turn it round its axes, all those stars that do not go below the horizon, during a whole revolution of the globe, never set in that place; and those that do not come above the horizon, never rise.

Prob. 8. *To find the time of the achronical rising and setting of any star.*

Bring the sun's place, for the given day, to the western side of the horizon, and all those stars that are on, or near the eastern side of the horizon, rise achronically; and those on the western verge of the horizon, set achronically.

Thus I find, on Dec. the 6th, that Aldebaran rises achronically, but it sets achronically on May the 21st. Also Sirius rises achronically on February 4th, and sets achronically on May 14th.

Prob. 9. *To find the cosmical rising and setting of the stars, in any latitude.*

Rectify the globe for the latitude, and bring the sun's place to the eastern side of the horizon, for the given day; then all those stars, cut by the eastern verge of the horizon, rise cosmically.

The globe still remaining in the same position, look at the western verge, or edge of the horizon; and all those stars cut by it, or that are very near it, set on that day cosmically.

Thus I find, that Arcturus, and two small stars in Hercules's thigh, rise cosmically Sept. 25. Also two stars in Eridanus, Essengue in Lyra, &c. set cosmically.

Again, for the cosmical setting. Turn the globe, till the star comes to the western side of the horizon, and observe the degree of the ecliptic; then cut by the eastern side of the horizon, for that will answer to the day of the cosmical setting.

Thus Arcturus sets cosmically, June 22. Also Aldebaran sets cosmically, December 20.

Prob. 10. *To tell the heliacal rising or setting of the Stars.*

Rectify the globe, and bring the given star to the eastern verge of the horizon; then fix the globe, and turn the quadrant to the western side, till 12° degrees of the quadrant touches the ecliptic; this done, note the degree of the ecliptic that is cut by 12° of the quadrant, on the western side (for then will the real place of the sun be depressed 12° on the eastern side) and that degree, sought in the calendar, gives the heliacal rising. The same is to be observed with the quadrant on the eastern side for the heliacal setting. Thus you will find, Aldebaran rises heliacally, July 4; sets heliacally, May 5; and Sirius, the dog-star, rises heliacally, about Aug. 26.

Prob. 11. *The sun's declination and hour, when he is due east, given; to find the latitude, viz. the elevation of the pole.*

Rectify the globe to the same latitude as the given number of degrees of declination, and fix the quadrant in the zenith; then convert the hours, that the sun is due E. before or after six o'clock, into degrees; and count the same number of degrees on the horizon, from the east point, southward, and bring the quadrant to that degree of the horizon; so shall the degree on the quadrant, that is cut by the equator, be the complement of latitude; which, taken from 90°, gives the latitude itself, or height of the pole.

EXAMPLE. Sailing May 21, I made an observation, that the sun was due east about seven minutes past seven in the morning, and his declination 20° N. I demand what latitude I was in?

Proceed as above directed, and you will find the latitude to be $51\frac{1}{2}$° nearly.

Prob. 12. *Having the sun's azimuth at six o'clock, and declination; to find the latitude.*

As many degrees as are contained in the azimuth given, so much elevate the pole, and fix the quadrant in the zenith, and bring ♈ to the meridian; this done, count on the quadrant upwards, the complement of the sun's declination, to ninety, and bring that degree to the equator; then the degree of the horizon, cut by the quadrant, shall be the complement of latitude, counted from the south point, or else from the north, as it may happen; and the remainder,

remainder to 90, is the latitude required; or otherwise, the degrees counted from the other two cardinal points, either E. or W. as it may happen, will give the latitude.

Thus, I find the sun's azimuth, at 6 o'clock, to be 12° 15', and his declination 20° 10'; What is the latitude? Work as taught above, and you will have the answer 38½° complement; that is, 51½° latitude required.

Prob. 13. *The sun's altitude E. and his declination given; to prove the elevation of the pole.*

Elevate the pole to the complement of the sun's altitude at E. and fix the quadrant in the zenith, and bring ♈ to the meridian; then number, on the quadrant of altitude, the degree of declination; and bringing the same to the equator, observe what degree the quadrant cuts the equator in; for its complement to 90° is the height of the pole.

EXAMPLE. The sun's declination is 20° 10' N. his altitude at east (at London) is nearly 26°; I would know, whether the supposed latitude (51½°) agrees with the operation?

First, I subtract 26° from 90°, and there remains 64° complement of altitude, and I elevate the pole accordingly; then I bring ♈ to the meridian, and cause 20° 10' on the quadrant to cut the equator, and find it nearly 38½° the complement of latitude required; which, subtracted from 90°, gives 51½°, the real latitude of the place.

Prob. 14. *To find the place of any planet upon the globe; and so, by that means, to find its place in the heavens. Also to find at what hour any planet will rise or set, or be on the meridian, at any day in the year.*

You must first seek in an ephemerides, for the place of the planet proposed on that day; then mark that point of the ecliptic, either with chalk, or by sticking on a little black patch; and then, for that night, you may perform any problem, as before by a fixed star.

Prob. 15. *To find all that space upon the earth, where an eclipse of one of the satellites of Jupiter will be visible.*

Having found that place upon the earth, in which the sun is vertical at the time of the eclipse, by problem 12, on the terrestrial globe, elevate the globe according to the latitude of the said place, then bring the place to the meridian, and set the hour index to 12 at noon. If Jupiter be in consequence of the sun, draw a line with black lead, or the like, along the eastern side of the horizon; which line will pass over all those places where the sun is setting at that time; then count the difference between the right ascension of the sun and that of Jupiter, and turn the globe westward until the hour index points to this difference, keeping the globe from turning round its axes, and elevate the meridian according to the declination of Jupiter. The globe being in this position, draw a line along the eastern side of the horizon, the space between this line and the line before drawn, will comprehend all those places of

of the earth where Jupiter will be visible, from the setting of the sun to the setting of Jupiter.

But if Jupiter be in antecedence of the sun (i. e. rises before him) having brought the place where the sun is vertical to the zenith, and put the hour index to 12 at noon, draw a line on the western side of the horizon, elevating the globe according to the declination of Jupiter, and turn it about eastwards until the index points to so many hours distant from noon, as is the difference of right ascension of the sun and Jupiter. The globe being in this position, draw a line along the western side of the horizon; then the space contained between this line and the other line last drawn, will comprehend all those places upon the earth where the eclipse is visible, between the rising of the sun and Jupiter.

ALGEBRA.

PART VIII.

SECTION LXXXI.

ALGEBRA is a science which teaches, in a general manner, the relations and comparisons of abstract quantities; by means whereof, such questions are resolved whose solutions would be sought in vain from common arithmetic.

There are two kinds of Algebra, Numeral and Specious, or Literal.

Numeral Algebra is that wherein all the given quantities are represented by numbers, and the unknown quantity by a letter or other symbol, as was used by the ancients.

Specious or Literal Algebra, is that wherein all the quantities, as well known as unknown, are expressed by letters of the alphabet; the given ones, for distinction sake, being usually denoted by the initial letters, a, b, c, d, &c. and the unknown or required ones by the final letters, u, w, x, y, &c. There are, likewise, certain signs and characters made use of to shew the relation and dependence of quantities one upon another, which are the foundation of this celebrated science. See the mathematical abbreviations facing page 1.

When the reader has a clear understanding of what the signs and characters used in algebra are intended to express, it will be necessary to inform him, that when any quantity is to be taken more than once, the number is to be prefixed, which shews how many times it is to be taken; thus, $5a$ denotes that the quantity a is to be taken five times, and $3ab$ stands for three times ab, or the quantity which arises by multiplying ab by 3; also $8\sqrt{a^2+b^2}$ signifies that $\sqrt{a^2+b^2}$ is to be taken eight times, and so of others.

The numbers thus prefixed are called coefficients, and that quantity which stands without a coefficient is always understood to have an unit prefixed, or to be taken once, and no more; thus, a is the same as $1a$.

Thoſe quantities are ſaid to be alike, that are expreſſed by the ſame letters under the ſame powers, or which differ only in their coefficients; thus $3bc$, $8bc$, $10bc$, are like quantities; and the ſame is to be underſtood of the radicals, $2\sqrt{\frac{b+c}{a}}$ and $8\sqrt{\frac{b+c}{a}}$. But unlike quantities are thoſe which are expreſſed by different letters, or by the ſame letters under different powers; thus $2ab$, $3abc$, $8ab^2$, and $5ba^2$, are all unlike.

When a quantity is expreſſed by a ſingle letter, or by ſeveral ſingle letters joined together in multiplication, without any ſign between them, as a, $3a$ or $4ab$, it is called a ſimple quantity.

But when theſe are connected by the ſigns $+$ or $-$, as $b+dc$, $ab-d$, $dy+az$, they are called compound quantities, whereof the ſimple quantities, b, d, ab, d, dy and az, are called the terms or members.

The letters by which any ſimple quantity is expreſſed may be ranged according to any order at pleaſure, and yet the ſignification continue the ſame; thus ab may be wrote ba, for ab denotes the product of a by b, and ba the product of b by a; for it is well known, that when two numbers are to be multiplied together, it matters not which of them is made the multiplicand, nor which the multiplier; the product either way comes out the ſame. In like manner it will appear, that abc, acb, bac, bca, cab, and cba, all expreſs the ſame thing (as will be demonſtrated in its proper place) but it is ſometimes convenient, in long operations, to place the ſeveral letters according to the order which they ſtand in the alphabet.

Likewiſe the ſeveral members, or terms, of which any quantity is compoſed, may be diſpoſed according to any order at pleaſure, and yet the ſignification be no ways altered thereby; thus $a-2ab+5a^2b$ may be wrote $a+5a^2b\,2ab$, or $-2ab+a+5a^2b$, &c. for all theſe repreſent the ſame thing; that is, the quantity which remains, when, from the ſum of a and $5a^2b$, the quantity $2ab$ is deducted.

When any calculation is to be made, it is done either by Addition, Subtraction, Multiplication, or Diviſion of quantities, which four fundamental rules I ſhall now proceed to explain.

ADDITION.

Addition, in Algebra, is performed by connecting the quantities by their proper ſigns, and joining into one ſum, ſuch as can be united; for the more ready performing of which, obſerve the following rules.

RULE I. If the quantities are alike, and have all the ſame ſigns, add the coefficient of thoſe terms together, and to their ſum join the letters common to each term, prefixing the common ſign.

EXAMPLES.

	E. 1	E. 2	E. 3.
To	$3a$	$5ab$	$18xyz$
Add	$4a$	$3ab$	$12xyz$
Sum	$7a$	$8ab$	$30xyz$

In

In example 1, the coefficients are 4 and 3, which, added together, make 7; to which joining a, the quantity, it is $7a$; and no sign being prefixed to either $3a$ or $4a$, the affirmative sign is understood as prefixed to both; hence $7a$ or $+7a$ is the sum required.

If there are two or more quantities connected by the signs + or −, and are like two or more quantities connected by the signs + or −, they are added as in the former examples, taking due care that the quantities, which compose their sum, are connected with their proper signs, according to rule 1.

	E. 4.	E. 5.
To -	$4a+7b$	$21ab+2cd$
Add -	$6a+2b$	$3ab+3cd$
Sum - -	$10a+9b$	$24ab+5cd$

In example 4, there is $4a+7b$, to be added to $6a+2b$; the quantities being disposed as in the example, it follows from the former examples, that $6a$ being added to $4a$, makes $10a$, and $2b$ added to $7b$ makes $9b$; and as $7b$ and $2b$ have both the affirmative sign, to $10a$ connect $9b$ with the sign +; hence $10a+9b$ is the sum required.

E. 6. To - - $2\sqrt{ab}+7\sqrt{bc}$
Add - - $3\sqrt{ab}+2\sqrt{bc}$
$6\sqrt{ab}+9\sqrt{bc}$

Sum will be $11\sqrt{ab}+18\sqrt{bc}$

In example 6, $6\sqrt{ab}$; $3\sqrt{ab}$; $2\sqrt{ab}$, added together, make $11\sqrt{ab}$; and $9\sqrt{bc}$; $2\sqrt{bc}$; $7\sqrt{bc}$, added together, make $18\sqrt{bc}$; then connecting them with the affirmative sign +, we have $11\sqrt{ab}+18\sqrt{bc}$, the sum required.

	E. 7.	E. 8.
To - -	$3yb+7a$	$-14y+d$
Add -	$2yb+a$	$-y+2d$
Sum - -	$5yb+8a$	$-15y+3d$

In example 7, when you come to $+7a$ to $+a$, there being no coefficient prefixed to a, unity or 1 in such cases is always the coefficient; and by what has been already taught, $+7a$ being added to $+a$, the sum is $+8a$, as in the example.

In example 8, when $2d$ is added to d, the sum is $3d$, for the same reason.

RULE 2. When in the quantities to be added, there are like terms, whereof some are affirmative, and others negative, add together the affirmative terms (if there be more than one) and do the same by the negative ones; then take the difference of the two sums (not regarding

regarding the signs) by subtracting the coefficient of the lesser from that of the greater, and joining the letters common to each; to which difference prefix the sign of the greater.

It is of no signification whether the quantity that has the greatest coefficient stands above or below.

	E. 1.	E. 2.	E. 3.
To	$6a$	$18b$	$36ad$
Add	$-3a$	-12	$-8ad$
Sum	$3a$	$6b$	$28ad$

In example 1, the coefficient 3, subtracted from 6, leaves 3, to which joining a, it is $3a$, and the sign of 6, the greatest coefficient, is affirmative; therefore $3a$, or $+3a$ is the sum required.

	E. 4.	E. 5.	E. 6.
To	$-18e$	$8ac$	$5ax$
Add	$6e$	$-14ac$	$-ax$
Sum	$-12e$	$-6ac$	$4ax$

Example 4, the coefficient 6, subtracted from 18, leaves 12, to which joining e, it is $12e$; but the sign of 18, the greatest coefficient, being —, prefix that sign to $12e$, then is $-12e$ the sum required.

E. 7.

$$\begin{array}{rrrr}
12abc & -16abd & +25acd & -72bcd \\
16abc & +12abd & +20acd & -18bcd \\
-13abc & +26abd & -15acd & +12bcd \\
32abc & -18abd & -10acd & +16bcd \\
\hline
\text{Sum } 47abc & +4abd & +20acd & -62bcd
\end{array}$$

In example 7 the coefficients 32, 16, 12, which have the affirmative sign being added together, make 60, then subtracting 13, the coefficient which hath the negative sign, leaves 47, to which joining abc, it is $47abc$: proceed thus through all the quantities as the rule directs, and you will find the sum to be $47abc + 4abd + 20acd - 62bcd$, as in the example.

E. 8.

$$\begin{array}{l}
\frac{5a}{b} - \frac{3cc}{a} + 7\sqrt{\frac{bc}{a}} - 9\sqrt{\frac{ab+cc}{a}} \\
\frac{8a}{b} + \frac{7cc}{a} - 12\sqrt{\frac{bc}{a}} + 6\sqrt{\frac{ab+cc}{a}} \\
\hline
\text{Sum } \frac{13a}{b} + \frac{4cc}{a} - 5\sqrt{\frac{bc}{a}} - 3\sqrt{\frac{ab+cc}{a}}
\end{array}$$

In example 8, and all others, where fractional and radical quantities are concerned, every such quantity, exclusive of its coefficient, is to be treated in all respects like a simple quantity expressed by a single letter.

RULE 3. When, in the quantities to be added, there are terms without others like to them, write them down one after the other, with the same coefficients and signs they have in the example.

The

The quantities may be set in any order; that is, any quantity may be set first, in the middle, or last, as it is not material how they are ranged, so as they are but connected with their proper signs.

	E. 1.	E. 2.
To	$3a$	$a+d$
Add	$4d$	$2x$
Sum	$3a+4d$	$a+d+2x$

E. 1. The quantities or letters being unlike, I place down $3a$, and because $4d$ has the sign $+$, therefore after the $3a$ put $+4d$, so is $3a+4d$ the sum required.

E. 2. Having put down a, after that put $+d$, and after that $+2x$, so is $a+d+2x$ the sum required.

	E. 3.	E. 4.
To	$4a-14m$	$aa+bb$
Add	$3x+5z$	$z-7y$
Sum	$4a-14m+3x+5z$	$aa+bb+z-7y$

To	$8b+6y-a$
Add	$3e-4c+2x$
Sum	$8b+6y-a+3e-4c+2x$

Note. In rule 3, where the quantities are unlike, it is plain that such quantities cannot be united into one, or otherwise added, than by their signs; thus, for example, let a be supposed to represent a crown, and b a shilling, then the sum of a and b can be neither $2a$ nor $2b$, that is, neither two crowns nor two shillings, but one crown $+$ one shilling, or $a+b$.

Here follows a few examples, wherein all the three foregoing rules are promiscuously used.

	E. 1.	E. 2.
To	$3a-7d+z$	$-4a+7m-21x$
Add	$2a+9d$	$11a-12m+2y$
Sum	$5a+2d+z$	$7a-5m-21x+2y$

E. 1. $2a$ added to $3a$ makes $5a$, and $-7d$ added to $9d$ makes $2d$, by rule 2; and there being no quantity like z, that must be placed by itself, by rule 3; and connecting these quantities with their proper signs, we have $5a+2d+z$, the sum required.

$$\text{Add}\left\{\begin{array}{llll} 5\sqrt{ax} & -\;8\sqrt{aa-xx} & +\;12\sqrt{aa+4xx} \\ 8\sqrt{ax} & +\;15\sqrt{aa-xx} & -\;8\sqrt{aa+4xx} \\ 6\sqrt{ax} & -\;7\sqrt{aa-xx} & +\;10\sqrt{aa+4xx} \end{array}\right.$$

$$\text{Sum}\quad 19\sqrt{ax} \quad \ast\ast \quad +\;14\sqrt{aa+12xx}$$

E. 3. The coefficients 6, 8, and 5, being added together, make 19; then joining $\sqrt{ax}$, it makes $19\sqrt{ax}$; and -7 and -8 added to 15, the sum is 0; and -8 added to $12+10$, the sum is 14; and

and prefixing the ſign of the two greateſt coefficients, and $\sqrt{aa}$ being joined, we have $+14\sqrt{aa}$, likewiſe $4+4+4=12$, to which joining the quantity xx, and connecting the quantities with their proper ſigns, we have $19\sqrt{ax}+14\sqrt{aa+12xx}$, the ſum required.

E. 4.	To	$-14m+30+8a$
	Add	$-8a-22+16m$
	Sum	$2m+8$

E. 4. $-14m$ added to $16m$, the ſum is $2m$, and -22 added to 30, the ſum is 8, and $8a$ added to $-8a$, the ſum is 0; hence $2m+8$ is the ſum required.

Note. In this example, the ſame quantities are not ſet under one another, that the learner may ſee it is not material how they are placed, if the quantities are alike, for they muſt be added the ſame as if they ſtood one under the other.

That this rule may be the better underſtood by the learner, I ſhall give one more example, wherein let us ſuppoſe a to denote a pound ſterling, b a ſhilling, and c a penny, thus,

E. 5.			£. s. d.
To	$7a-9b+5c$	=	$7-9+5$
Add	$3a+5b-9c$	=	$3+5-9$
Sum	$10a-4b-4c$	=	$10-4-4$

By this example may be underſtood, why addition is changed into ſubtraction, when the ſigns differ, and the ſum of the greater quantity is prefixed to the remainder: for in the ſum of 10*l.* nine ſhillings are wanting; therefore, if 5*s.* are added, the defect is leſſened, and brought to 4*s.*; and becauſe there are not 5 whole ſhillings, but 5*s.* — 9*d.* to be added, the ſum 10*l.* — 4*s.* exceeds the truth by 9*d.* which are therefore to be ſubtracted.—Now in the upper number, to which the lower is to be added, there are 5*d.* theſe may be ſubtracted, and the other 4*d.* in the lower number, ſet down as wanting, or negative quantities; and this was the way the rule was firſt diſcovered.

SUBTRACTION.

Subtraction in algebra is performed by the following general

RULE. Change all the ſigns of thoſe quantities which are to be ſubtracted, or conceive them to be changed; then add theſe quantities to the others, according to the ſeveral rules of addition, and you will have the difference or remainder required.

	E. 1.	E. 2.	E. 3.
From	$6a$	$-8b$	$12ab$
Take	$3a$	$-2b$	$-6ab$
Remains	$3a$	$-6b$	$18ab$

In example 1. there is $3a$, having the ſign $+$, to be ſubtracted, which being made, or ſuppoſed to be made $-$, then by the general rule

rule $6a$ is to be added to $-3a$, the sum of which is $3a$ by rule 2 of addition, and is the remainder required.

	E. 4.	E. 5.	E. 6.	E. 7.
From	$5ac$	$-ab$	$-7ad$	$5xy$
Take	$-ac$	$-5ab$	$+ad$	xy
Remains	$6ac$	$4ab$	$-8ad$	$4xy$
Proof	$5ac$	$-ab$	$-7ad$	$5xy$

From the four preceding examples, it may be easily perceived, that subtraction in algebra is proved as in common arithmetic, by adding the remainder to the quantity which is subtracted.

	E. 8.	E. 9.
From	$-5zy - 2am$	$14a - 5b$
Take	$3zy + 4am$	$-3a - 5b$
Remains	$-8zy - 6am$	$17a \quad 0$

If the quantities to be subtracted are unlike those from which the subtraction is to be made, set down these with the same signs and coefficients they have in the example; after which, place the quantities to be subtracted with their coefficients, but change their signs.

	E. 10.	E. 11.
From	$3ac$	$5x^3 + 8xz$
Take	bd	$8x^2 - x$
Remains	$3ac - bd$	$5x^3 + 8xz - 8x^2 + x$

In E. 10. having put down $3ac$, after which put $-bd$, the quantity to be subtracted being $+bd$, and $3ac - bd$ is the remainder required.

MULTIPLICATION.

In Multiplication there is one general rule for the signs, viz. when the signs of the factors are alike (that is, both + or both −) the sign of the product is +; but when the signs of the factors are unlike, the sign of the product is −. This general rule will resolve itself into four particular cases, which I shall illustrate separately in simple quantities.

CASE 1. When any positive quantity, as $+a$, is multiplied by a positive quantity $+b$, the meaning is, that $+a$ is to be taken so many times as there are units in b, and the product is evidently b times a or ba.

EXAMPLES.

Multiply	$+a$	$3a$	$5bx$	$9dc$
By	$+b$	$6b$	7	$8x$
Product	$+ba$	$18ab$	$35bx$	$72xdc$

In example 1, having joined the letters ba, and each of them having the affirmative sign, therefore, by the rule, ba or $+ba$ is the product required: and so of others.

CASE 2. When $-a$ is multiplied by b, then $-a$ is to be taken as often as there are units in b, and the product must be b times $-a$, or $-ba$.

EXAMPLES.

Multiply	$-a$	$-2a$	$-bx$	$-9dc$
By	b	$4b$	7	$3z$
Products	$-ba$	$-8ab$	$-7bx$	$-27dcz$

In example 1, case 2, the product of a by b is ba, and as the sign of a is $-$, and that of b is $+$, therefore to ba prefix the sign $-$, so is $-ba$ the product required.

CASE 3. As multiplication by a positive number implies a repeated addition, so multiplication by a negative implies a repeated subtraction; and therefore, when a or $+a$ is to be multiplied by $-b$, it means only, that $+a$ is to be subtracted as often as there are units in b, and therefore the product being negative, must also be $-ba$; see the following

EXAMPLES.

Multiply	$+a$	$3a$	$6bc$	$9acd$
By	$-b$	$-4b$	-8	$-4x$
Product	$-ba$	$-12ab$	$-48bc$	$36acdx$

CASE 4. When $-a$ is to be multiplied by $-b$, then $-a$ is to be subtracted as often as there are units in b; but to subtract $-a$ is equivalent to adding $+a$; therefore this case is the same in effect as in case 1, and the product is evidently $+ba$, or ba.

EXAMPLES.

Multiply	$-a$	$-4a$	$-6by$	$-8xy$
By	$-b$	$-3b$	-9	$-5a$
Product	ba	$12ab$	$54by$	$40axy$

A compound quantity is multiplied by a simple one, by multiplying every term of the multiplicand by the multiplier.

EXAMPLES.

Multiply	$a+d$	$-mx-y$	$a-b+c$
By	x	$-v$	$-b$
Sum	$xa+xd$	$vmx+vy$	$-ab+bb-bc$

If there are coefficients, or numbers prefixed to the quantity, then multiply the numbers as in common arithmetic, and to their products join the products of the quantities found by the last example.

Multiply	$3a-b+2c$
By	$8b$
Product	$24ab-8bb+16bc$

Multiplply	$2a^2-4ab+3ac-2bc+3b^2-2c^2$
By	$2abc$
Product	$4a^3bc-8a^2b^2c+6a^2bc^2-4ab^2c^2+6ab^3c-4abc^3$

If any algebraic quantities are to be multiplied by a pure number, this number is to be multiplied into every one of the coefficients of the

the other quantities, in all respects as before, and to each particular product set or join that quantity whose coefficient was multiplied.

EXAMPLES.

Multiply	$3a + 4b$	$2x - 6c$
By	6	8
Product	$18a + 24b$	$16x - 48c$

Compound quantities are multiplied into one another, by multiplying every term of the multiplicand by each term of the multiplier, successively, and connecting the several products thus arising, with the signs of the multiplicand, if the multiplying term be affirmative, but with contrary signs, if negative.

EXAMPLES.

Multiply	$3a + 2x$	$a^3 + a^2b + ab^2 + b^3$
By	$4a + 3x$	$a - b$
	$12aa + 8ax$	$a^4 + a^3b + a^2b^2 + ab^3$
	$9ax + 6xx$	$- a^3b - a^2b^2 - ab^3 - b^4$
Product	$12aa + 17ax + 6xx$	a^4 * * * $- b^4$

In the above example, by striking out all the terms that destroy each other, the product becomes $a^4 - b^4$.

Note. If the sign of any proposed term of the multiplier, in any case whatever, be affirmative, it is easy to conceive, that the required product will be greater than it would be if there was no such term, by the product of that term into the whole multiplicand; and therefore it is, that this product is to be added or wrote down with its proper signs. But if, on the contrary, the sign of the term by which you multiply be negative, then, as the required product must be less than it would be, if there were no such term, by the product of that term into the whole multiplicand, this product, it is manifest, ought to be subtracted or wrote down with contrary signs.

Hence is derived the common rule, that like signs produce +, and unlike signs —.

For, first, if the signs of both the quantities or terms to be multiplied are affirmative, it is plain, that the sign of the product must likewise be affirmative.

Secondly, if the signs of both quantities are negative, that of the product will be affirmative, because contrary to that of the multiplicand, as proved above.

Thirdly, if the sign of the multiplicand be affirmative, and that of the multiplier negative, the sign of the product will be negative, because the same with that of the multiplicand.

Lastly, if the sign of the multiplicand be negative, and that of the multiplier affirmative, the sign of the product will be negative, because the same with that of the multiplicand.

And these are all the cases that can possibly happen with regard to the variation of signs.

Examples for the learner's exercise.

Multiply $a^3 - 3a^2b + 3ab^2 - b^3$
By $a^2 - 2ab + b^2$

$$a^5 - 3a^4b + 3a^3b^2 - a^2b^3$$
$$- 2a^4b + 6a^3b^2 - 6a^2b^3 + 2ab^4$$
$$+ a^3b^2 - 3a^2b^3 + 3ab^4 - b^5$$

Product $a^5 - 5a^4b + 10a^3b^2 - 10a^2b^3 + 5ab^4 - b^5$

Multiply $xx + xy + yy$
By $xx - xy + yy$

$$x^4 + x^3y + x^2y^2$$
$$- x^3y - x^2y^2 - xy^3$$
$$x^2y^2 + xy^3 + y^4$$

Product $x^4 \quad * + x^2y^2 \quad * + y^4$

DIVISION.

In division of algebraic quantities, the rule for the signs is the same as in multiplication, viz. if the signs of the divisor and dividend are alike, the sign of the quotient must be $+$; but if they are unlike, the sign of the quotient must be $-$.

This is a general rule for all operations in division, which are only the reverse of multiplication, and therefore will be easy to understand, when illustrated by examples.

EXAMPLES.

Divide acd $\qquad -mad$
By ac $\qquad -md$

Quotient d $\qquad a$

In the first example, because ac is in the dividend and divisor, reject it, and put down d for the quotient.

The truth of these operations in division may be proved like those in arithmetic; for the quotient and divisor being multiplied, the product will be the dividend, if the work is true; thus, in the second example, by multiplying a the quotient into $-md$ the divisor, the product is mda, or adm, or mad, to which must be prefixed the sign $-$, because the signs of md and a are unlike; hence the product with its sign is $-mad$, the given dividend.

E. 3. $-a) + ab(-b$ $\qquad$ E. 4. $2ab)2ab(1$

E. 5. $2a)6ab(3b$
$6ab$
$\cdots$

E. 6. $a)aa + ab(a + b$
aa
$+ ab$
ab
$\cdot\cdot$

$$\text{E. 7.}\quad 4a+b)\,4aa-7ab-2bb\,(a-2b$$
$$4aa+ab$$
$$8ab-2bb$$
$$8ab-2bb$$
$$\cdot \qquad \cdot$$

The truth of these examples are proved as in common arithmetic.

$$\text{E. 8.}\quad a+x)\,a^3+5a^2x+5ax^2+x^3\,(a^2+4ax+x^2$$
$$a^3+a^2x$$
$$4a^2x+5ax^2$$
$$4a^2x+4ax^2$$
$$ax^2+x^3$$
$$ax^2+x^3$$
$$* \qquad *$$

To work the above example, say, How often is a contained in a^3; the answer is a^2, which I write down in the quotient, and multiply the whole divisor $a+x$ thereby, and there arises a^3+a^2x, which, subtracted from the two first terms of the dividend, leaves $4a^2x$; to this remainder I bring down $+5ax^2$, the next term of the dividend, and then seek again how many times a is contained in $4a^2x$; the answer is $4ax$, which I also put down in the quotient, and by it multiply the whole divisor, and there arises $4a^2x+4ax^2$, which subtracted from $4a^2+5ax^2$, leaves ax^2, to which I bring down x^3, the last term of the dividend, and seek how many times a is contained in ax^2, which I find to be x^2, which I also put in the quotient, and by it multiply the whole divisor, and then, having subtracted the product from ax^2+x^3, I find there is nothing remains.

$$\text{E. 9.}\qquad (a^3-3a^2x+39x^2-x^3$$
$$a^2-2ax+x^2)\,a^5-5a^4x+10a^3x^2-10a^2x^3+5ax^4-x^5$$
$$a^5-2a^4x+a^3x^2$$
$$-3a^4x+9a^3x^2-10a^2x^3$$
$$-3a^4x+6a^3x^2-3a^2x^3$$
$$3a^3x^2-7a^2x^3+5ax^4$$
$$3a^3x^2-6a^2x^3+3ax^4$$
$$-a^2x^3+2ax^4-x^5$$
$$-a^2x^3+2ax^4-x^5$$
$$* \qquad * \qquad *$$

E. 10.

E. 10.

$$3a - 6)\ 6a^4 - 96\ (2a^3 + 4a^2 + 8a + 16, \text{ \&c. ad infinitum, by adding the power of } a.$$

$$\begin{array}{l} 6a^4 - 12a^3 \\ \hline 12a^3 - 96 \\ 12a^3 - 24a^2 \\ \hline 24a^2 - 96 \\ 24a^2 - 48a \\ \hline 48a - 96 \\ 48a - 96 \\ \hline * \end{array}$$

E. 11.

$$4x - 5a)\ 48x^3 - 76ax^2 - 64a^2x + 105a^3\ (12x^2 - 4ax - 21a^2$$

$$\begin{array}{l} 48x^3 - 60ax^2 \\ \hline 16ax^2 - 64a^2x \\ 16ax^2 + 20a^2x \\ \hline -84a^2x + 105a^3 \\ -84a^2x + 105a^3 \\ \hline * \end{array}$$

It may be proper to obſerve to the learner, that the work will not always terminate without a remainder, in which caſe this method is of little uſe. In all ſuch caſes it will be moſt proper to expreſs the quotient in the manner of a fraction, by writing the diviſor under the dividend, with a line between them, as in the following examples.

	E. 1.	E. 2.	E. 3.
Divide	$8my$	$3a + 4b$	$14db - 5xz$
By	z	$6m$	$6y$
Quotient	$\frac{8my}{z}$	$\frac{3a + 4b}{6m}$	$\frac{14db - 5xz}{6y}$

INVOLUTION.

Involution is the raiſing of powers from any propoſed root, and is therefore performed by multiplication; for the given quantity being multiplied by itſelf, will be the ſquare of that quantity, and that product being multiplied by the given quantity, will be the cube of that quantity; and ſo on as in common arithmetic.

EXAMPLES.

Required to find the cube of a ... $2a$

$$\begin{array}{lcll} & & a & 2a \\ & & a & 2a \\ \hline \text{the ſquare of } a & = & aa & 4aa \\ & & a & 2a \\ \hline \text{the cube of } a & = & aaa & 8aaa \end{array}$$

In

In like manner, any other single quantity may be raised to any required power.

If there are two or more quantities connected by the signs $+$ or $-$, to be raised to any given power, it is still performed by common multiplication.

Two quantities, when connected by the sign $+$, are commonly called a binomial; and by the sign $-$, a residual. Let it be required to raise the binomial, $a+b$ to the sixth power.

$$\begin{array}{l}
a+b \\
a+b \\
\hline
aa+ab \\
\quad\quad ab+bb \\
\hline
aa+2ab+bb, \text{ the square, or 2d power.} \\
a+b \\
\hline
a^3+2a^2b+ab^2 \\
\quad\quad a^2b+2ab^2+b^3 \\
\hline
a^3+3a^2b+3ab^2+b^3, \text{ the cube, or 3d power.} \\
a+b \\
\hline
a^4+3a^3b+3a^2b^2+ab^3 \\
\quad\quad a^3b+3a^2b^2+3ab^3+b^4 \\
\hline
a^4+4a^3b+6a^2b^2+4ab^3+b^4, \text{ the 4th power.} \\
a+b \\
\hline
a^5+4a^4b+6a^3b^2+4a^2b^3+ab^4 \\
\quad\quad a^4b+4a^3b^2+6a^2b^3+4ab^4+b^5 \\
\hline
a^5+5a^4b+10a^3b^2+10a^2b^3+5ab^2+b^5, \text{ the 5th power} \\
a+b \\
\hline
a^6+5a^5b+10a^4b^2+10a^3b^3+5a^2b^4+ab^5 \\
\quad\quad a^5b+5a^4b^2+10a^3b^3+10a\,b^4+5ab^5+b^6 \\
\hline
a^6+6a^5b+15a^4b^2+20a^3b^3+15a^2b^4+6ab^5+b^6, \text{ the 6th power.}
\end{array}$$

Let it be required to involve, or raise the residual, $a-b$ to the sixth power.

$$\begin{array}{l}
a-b \\
a-b \\
\hline
aa-ab \\
\quad -ab+b^2 \\
\hline
a^2-2ab+b^2, \text{ the 2d power.} \\
a-b \\
\hline
\end{array}$$

a^3-2

$$a^3 - 2a^2b + ab^2$$
$$\quad - a^2b + 2ab^2 - b^3$$

$a^3 - 3a^2b + 3ab^2 - b^3$, the 3d power.
$a - b$

$$a^4 - 3a^3b + 3a^2b^2 - ab^3$$
$$\quad - a^3b + 3a^2b^2 - 3ab^3 + b^4$$

$a^4 - 4a^3b + 6a^2b^2 - 4ab^3 + b^4$, the 4th power.
$a - b$

$$a^5 - 4a^4b + 6a^3b^2 - 4a^2b^3 + ab^4$$
$$\quad - a^4b + 4a^3b^2 - 6a^2b^3 + 4ab^4 - b^5$$

$a^5 - 5a^4b + 10a^3b^2 - 10a^2b^3 + 5ab^4 - b^5$, the 5th power.
$a - b$

$$a^6 - 5a^5b + 10a^4b^2 - 10a^3b^3 + 5a^2b^4 - ab^5$$
$$\quad - a^5b + 5a^4b^2 - 10a^3b^3 + 10a^2b^4 - 5ab^5 + b^6$$

$a^6 - 6a^5b + 15a^4b^2 - 20a^3b^3 + 15a^2b^4 - 6ab^5 + b^6$, the 6th power.

There is a theorem, given by Sir Isaac Newton, whereby any power of a binomial, $a + b$, or residual $a - b$, may be expressed in simple terms, without the trouble of those tedious multiplications required in the preceding operations; which is thus:

Let n denote any number at pleasure; then the n^{th} power of $a + b$ will be $a^n + na^{n-1}b + \frac{n \cdot \overline{n-1}}{1 \cdot 2} \cdot a^{n-2}b^2 + \frac{n \cdot \overline{n-1} \cdot \overline{n-2}}{1 \cdot 2 \cdot 3} \cdot a^{n-3}b^3 + \frac{n \cdot \overline{n-1} \cdot \overline{n-2} \cdot \overline{n-3}}{1 \cdot 2 \cdot 3 \cdot 4} \cdot a^{n-4}b^4 + \frac{n \cdot \overline{n-1} \cdot \overline{n-2} \cdot \overline{n-3} \cdot \overline{n-4}}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5} \cdot a^{n-5}b^5$, &c. And the n^{th} power of $a - b$ will be expressed in the same manner, only the signs of the second, fourth, sixth, &c. terms, where the odd powers of b are involved, must be negative.

EXAMPLE.

Let it be required to raise $a + b$ to the sixth power?

Here n, the index of the proposed power being 6, the first term, a^n, of the general expression is equal to a^6; the second $na^{n-1}b = 6a^5b$; the third $\frac{n \cdot \overline{n-1}}{1 \cdot 2} \cdot a^{n-2}b = 15a^4b^2$; the fourth $\frac{n \cdot \overline{n-1} \cdot \overline{n-2}}{1 \cdot 2 \cdot 3} \cdot a^{n-3}b = 20a^3b^3$; the fifth $\frac{n \cdot \overline{n-1} \cdot \overline{n-2} \cdot \overline{n-3}}{1 \cdot 2 \cdot 3 \cdot 4} \cdot a^{n-4}b = 15a^2b^4$; the sixth $\frac{n \cdot \overline{n-1} \cdot \overline{n-2} \cdot \overline{n-3} \cdot \overline{n-4}}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5} \cdot a^{n-5}b = 6ab^5$; the seventh $\frac{n \cdot \overline{n-1} \cdot \overline{n-2} \cdot \overline{n-3} \cdot \overline{n-4} \cdot \overline{n-5}}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6} \cdot a^{n-6}b = b^6$; the eighth $\frac{n \cdot \overline{n-1} \cdot \overline{n-2} \cdot \overline{n-3} \cdot \overline{n-4} \cdot \overline{n-5} \cdot \overline{n-6}}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \cdot 7} \cdot a^{n-7}b = b^7$, &c. = nothing. Therefore the 6th power of $a + b = a^6 + 6a^5b + 15a^4b^2 + 20a^3b^3 + 15a^2b^4 + 6ab^5 + b^6$, the same as before determined by multiplication.

From

From the preceding operations it may be obſerved, that the terms of the powers are made up of certain letters multiplied into the unciæ, or coefficients, and theſe coefficients increaſe, till the indices of the two letters a and b become equal, or change values, and then return or decreaſe again, according to the ſame order.

Likewiſe, that the products may be found by making two geometrical progreſſions, the one beginning at the deſired power of the firſt part of the root, and ending at an unite; and the other beginning at an unite, and ending at the power of the other part of the root: to find the 6th power of $a + b$, write the powers,

$$\text{Thus}\left\{\begin{array}{ccccccc} a^6 & a^5 & a^4 & a^3 & a^2 & a & 1 \\ 1 & b & b^2 & b^3 & b^4 & b^5 & b^6 \end{array}\right.$$

Then $a^6 + a^5 b + a^4 b^2 + a^3 b^3 + a^2 b^4 + a b^5 + b^6$, will be the terms in the 6th power of $a + b$, by multiplying the powers above by thoſe below; and to find their unciæ or coefficients, that of the firſt term is always an unit, and that of the ſecond is the exponent of the firſt; and of the third is the exponent of a in the ſecond term, multiplied by the affixed unciæ, and divided by $2 = 15$; and of the third is the exponent of a in the third term, multiplied by the prefixed unciæ 15, and divided by 3, and ſo of the fourth, &c.

EXAMPLE.

Let it be required to complete all the terms of the aforeſaid ſeveral powers, viz. $a^6 + a^5 b + a^4 b^2 + a^3 b^3 + a^2 b^4 + a b^5 + b^6$.

1. The index of a^6, the firſt term, will be the unciæ of the ſecond term. Thus $a^6 + 6 a^5 b$.

2. Then half the ſecond term's index into its unciæ, viz. $\frac{6 \times 5}{2} = 15$, will be the third term's unciæ.

Thus $a^6 + 6 a^5 b + 15 a^4 b^2$ will be the three firſt terms.

Third, again $\frac{15 \times 4}{3} = 20$, the unciæ of the fourth term.

Then it will be $a^6 + 6 a^5 b + 15 a^4 b^2 + 20 a^3 b^3$.

Fourth, and $\frac{20 \times 3}{4} = 15$, the unciæ of the fifth term.

Then $a^6 + 6 a^5 b + 15 a^4 b^2 + 20 a^3 b^3 + 15 a^2 b^4$, &c. until all the terms are compleated with their reſpective unciæ's, and then they will ſtand thus:

$$a^6 + 6 a^5 b + 15 a^4 b^2 + 20 a^3 b^3 + 15 a^2 b^4 + 6 a b^5 + b^6.$$

If the work of the preceding examples be well underſtood, it will be found very eaſy to raiſe any power from a binomial or reſidual root, to what height you pleaſe, without the trouble of a continued involution, or without the help of a table of powers, as propoſed by ſeveral authors.

EXAMPLE.

Let it be required to raise $a+b$ to its cube or third power, both in numbers and species, where $a=10$, and $b=2$?

$$\begin{array}{llll}
a+b & = 10+2 & & \\
a+b & = 10+2 & & \\
\hline
a^2+ab & = 100+20 & & 12 \\
\quad ab+b^2 & = \quad 20+4 & & 12 \\
\hline
a^2+2ab+b^2 & = 100+40+4 & = 144 = \text{squa.} & \\
a+b & = 10+2 & & \\
\hline
a^3+2a^2b+ab^2 & = 1000+400+40 & & 144 \\
\quad a^2b+2ab^2+b^3 & = \quad 200+80+8 & & 12 \\
\hline
a^3+3a^2b+3ab^2+b^3 & = 1000+600+120+8 & = 1728 = \text{cube.} &
\end{array}$$

EVOLUTION.

Evolution is the extraction of roots, and therefore opposite to involution, or raising of powers, and is performed by converse operations, viz. by the division of the indices.

To extract the root of any simple quantity, consider how many times the letter is repeated, or how high the power of it is; and if the given power have no numbers prefixed to it, and its index can be divided by the index of the root required, the quotient will be the index of the root sought.

Thus the square root of a^6, by dividing the exponent by 2, is found to be a^3, and the cube root of a^6 is a^2; also the biquadratic root of $\overline{a+y}|^8$, will be $\overline{a+y}|^2$; likewise the cube root of $\overline{xx+yy}|^{\frac{1}{2}}$ will be $\overline{xx+yy}|^{\frac{1}{6}}$.

If the given powers have coefficients, then you must extract their respective roots, as in vulgar arithmetic.

Thus the square root of $81\,a^4$, is $9\,a^2$; and the square root of $1296\,a^8\,b^8$, is $36\,a^4\,b^4$; and so of others.

If the quantity given be a fraction, its root will be extracted, by extracting the root of each particular factor.

Thus the square root of $\frac{a^2b^2}{c^2}$ will be $\frac{ab}{c}$ and that of

$\frac{81 \times aa \times \overline{aa+xx}|^4}{16 \times \overline{a-x}|^2}$ will be $\frac{9a \times \overline{aa+xx}|^2}{4 \times a-x}$; likewise, the square root of $\overline{aa+xx}|^{\frac{1}{2}}$ will be $\overline{aa+xx}|^{\frac{1}{4}}$; its cube root $\overline{aa+xx}|^{\frac{1}{6}}$; and its biquadratic root $\overline{aa+xx}|^{\frac{1}{8}}$; and so of others.

If we cannot extract the square root of both the numerator and denominator, the given quantity is a surd, and must have the sign of the root required prefixed to it.

Thus, suppose it was required to extract the square root of $xx+2xn-nn$; the answer will be $\sqrt{xx+2xn-nn}$, the square root required.

In

In the above example there are three quantities, and two different letters, x and n; two of these three quantities, viz. xx and nn, are pure powers of x and n; but both these powers have not the sign $+$, for it is $-nn$, therefore, I conclude, that the given quantity is a surd quantity, whose square root cannot be extracted any otherwise than by prefixing the sign $\surd$ to it as above, which expresses the square root.

Evolution of compound quantities is performed by the following

RULE. Place the several terms, whereof the given quantity is composed, in order, according to the dimensions of some letter therein, as shall be judged most proper; then let the root of the first term be found, and placed in the quotient, which term being subtracted, let the first term of the remainder be brought down, and divided by twice the first term of the quotient, or by three times its square, or four times its cube, &c. according as the root to be extracted is a square, cubic, or biquadratic one, &c. and let the quantity thence arising be also wrote down in the quotient, and the whole raised to the second, third, or fourth, &c. power, according to the aforesaid cases, respectively, and subtracted from the given quantity, and if any thing remains, let the operation be repeated, by always dividing the first term of the remainder by the same divisor, found as above.

EXAMPLES.

It is required to extract the square root of $x^2+2xy+y^2$?

$$\text{Thus, } x^2+2xy+y^2\ (x+y, \text{ the root required.}$$
$$2x)\ 2xy$$
$$x^2+2xy+y^2, \text{ second power of } x+y.$$

$$*$$

The square root of compound quantities may be extracted according to the common method of extracting the square root in numbers.—See the last example.

$$\text{Thus, } x^2+2xy+y^2\ (x+y, \text{ root as before}$$
$$x^2$$
$$2x+y)\quad 2xy+y^2$$
$$2xy+y^2$$

$$*$$

It is required to extract the square root of $a^4-2a^3x+3a^2x^2-2ax^3+x^4$?

$$a^4-2a^3x+3a^2x^2-2ax^3+x^4\ (a^2-ax+x^2, \text{ root.}$$
$$2a^2)\ -2a^3x$$
$$a^4-2a^3x+a^2x^2, \text{ second power of } a^2-ax.$$

$$2a^2)\ 2a^2x^2, \text{ first term of the remainder.}$$
$$a^4-2a^3x+3a^2x^2-2ax^3+x^4, \text{ square of } a^2-ax+x^2.$$

$$*\quad *\quad *$$

Or

Or thus, as in common arithmetic.

$$a^4 - 2a^3x + 3a^2x^2 - 2ax^3 + x^4\ (a^2 - ax + x^2, \text{ root (as before.}$$

$$a^4$$

$$2a^2 - ax)\quad -2a^3x + 3a^2x^2$$

$$-2a^3x + a^2x^2$$

$$2a^2 - 2ax + x^2)\quad 2a^2x^2 + 2ax^3 + x^4$$

$$2a^2x^2 + 2ax^3 + x^4$$

$$*\qquad *$$

It is required to extract the square root of $a^4 + 4a^3x + 6a^2x^2 + 4ax^3 + x^4$?

$$a^4 + 4a^3x + 6a^2x^2 + 4ax^3 + x^4\ (a^2 + 2ax + x^2, \text{ root.}$$

$$2a^2)\ 4a^3x$$

$$a^4 + 4a^3x + 4a^2x^2$$

$$2a^2)\quad 2a^2x^2$$

$$a^4 + 4a^3x + 6a^2x^2 + 4ax^3 + x^4$$

$$*\qquad *\qquad *$$

Or thus, as in common arithmetic,

$$a^4 + 4a^3x + 6a^2x^2 + 4ax^3 + x^4\ (a^2 + 2ax + x^2, \text{ (root as before.}$$

$$a^4$$

$$2a^2 + 2ax)\quad 4a^3x + 6a^2x^2$$

$$4a^3x + 4a^2x^2$$

$$2a^2 + 4ax + x^2)\quad 2a^2x^2 + 4ax^3 + x^4$$

$$2a^2x^2 + 4ax^3 + x^4$$

$$*\qquad *$$

It is required to extract the cube root of $a^3 - 6a^2x + 12ax^2 - 8x^3$.

$$a^3 - 6a^2x + 12ax^2 - 8x^3\ (a - 2x, \text{ root.}$$

$$3a^2)\ -6a^2x$$

$$a^3 - 6a^2x + 12ax^2 - 8x^3$$

$$*\qquad *\qquad *$$

To prove the above example.

$a - 2x$, supposed root.

$$a - 2x$$

$$a^2 - 2ax$$

$$-2ax + 4x^2$$

$a^2 - 4ax + 4x^2$, square, or second power.

$$a - 2x$$

$$a^3 - 4a^2x + 4ax^2$$

$$-2a^2x + 8ax^2 - 8x^3$$

Proof $a^3 - 6a^2x + 12ax^2 - 8x^3$, Product.

Same as the given quantity, which proves the cube root to be as above.

Required

Required to extract the biquadratic root of

$$16x^4 - 96x^3y + 216x^2y^2 - 216xy^3 + 81y^4 \quad (2x - 3y.$$

$$32x^3) - 96x^3y$$

$$16x^4 - 96x^3y + 216x^2y^2 - 216xy^3 + 81y^4$$

$$* \qquad * \qquad * \qquad *$$

And in the same manner the root may be determined in any other case, where it is possible to be done; but after all, if there is a remainder, the root is to be expressed in the manner of a surd.

FRACTIONS.

The reduction of fractional quantities is of use in changing an expression to the most simple form it is capable of.

To change different fractions into one denomination, retaining the same value.

RULE. Multiply all the denominators into each other for a new denominator, and each numerator into all the denominators except its own, for a new numerator.

EXAMPLES.

Reduce $\frac{a}{b}$, $\frac{b}{c}$ and $\frac{c}{d}$ to a common denominator.

$$\text{Thus}\left\{\begin{array}{l} a \times c \times d = acd \\ b \times b \times d = bbd \\ c \times b \times c = bcc \end{array}\right\}\text{Numerators.}$$

And $b \times c \times d = bcd$ common denominator. Therefore $\frac{a}{b}$ $\frac{b}{c}$ $\frac{c}{d}$ becomes $\frac{acd}{bcd}$ $\frac{bbd}{bcd}$ $\frac{bcc}{bcd}$ the fractions required; and so of others.

When the denominators have a common divisor, instead of multiplying the terms of each fraction by the denominator of the other, you only multiply by that part which arises by dividing by the common divisor.

E. 2. Reduce $\frac{b^2}{ad}$ and $\frac{ab}{cd}$ to a common denominator.

$$\left.\begin{array}{ll} \text{Thus} & bb \times c = bbc \\ \text{And} & ab \times a = aab \end{array}\right\}\text{Numerators.}$$

Also $a \times c \times d = acd$ common denominator.

So $\frac{b^2}{ad}$ $\frac{ab}{cd}$ become $\frac{bbc}{acd}$ $\frac{aab}{acd}$ the fractions required.

To reduce fractional quantities into their lowest terms.

RULE. Divide both the numerator and denominator by the greatest common divisor.

Thus, $\frac{ab}{bc}$, by dividing by b, is $\frac{a}{c}$, and $\frac{2abc}{abb}$, by dividing by ab, is $\frac{2c}{b}$; also, $a + \frac{bdc}{bc} = a + d$.

In ſuch ſingle fractions as theſe, the common diviſors (if there be any) are eaſily diſcovered, but the compound diviſors, whereby a fraction can be reduced to lower terms, are not ſo eaſily diſcovered, for which reaſon I have laid down the following

RULE. Divide the numerator by the denominator until nothing remains, when that can be done; or find their common meaſure by dividing the denominator by the numerator, and the numerator by the remainder, and ſo on, as in vulgar fractions.

EXAMPLES.

Reduce $\frac{a^2 c - a^2 d}{cd - d^2}$ to its loweſt terms.

Thus $cd - d^2\ )\ a^2 c - a^2 d\ (\frac{aa}{d}$ the fraction required.

$$\begin{array}{l} a^2 c - a^2 d \\ \hline * \quad * \end{array}$$

Let it be required to reduce $\frac{a^3 - ab^2}{a^2 + 2ab + b^2}$ to its loweſt terms.

$$\begin{array}{l} a^2 + 2ab + b^2\ )\ a^3 - 0 \quad - ab^2\ (\ a \\ \qquad\qquad a^3 + 2a^2 b + ab^2 \\ \hline \end{array}$$

Remainder $- 2a^2 b - 2ab^2\ )\ a^2 + 2ab + b^2 (-\frac{1}{2b} - \frac{1}{2a}$

$$\begin{array}{l} a^2 + ab \\ \hline ab + b^2 \\ ab + b^2 \\ \hline * \quad * \end{array}$$

Hence it appears, that $-2a^2 b - 2ab^2$ is the common meaſure, by which $a^3 - 2ab^2$ being divided,

viz. $- 2a^2 b - 2ab^2\ )\ a^3 - 0 - ab^2\ (-\frac{a}{2b} + \frac{1}{2}$

$$\begin{array}{l} a^3 + a^2 b \\ \hline - a^2 b - ab^2 \\ - a^2 b - ab^2 \\ \hline * \quad * \end{array}$$

Then $-\frac{a}{2b} + \frac{1}{2} = -\frac{a+b}{2b}$, the new numerator, and $\frac{1}{2b} - \frac{1}{2a} = \frac{-a-b}{2ba}$, the new denominator.

Let both be multiplied with $2ba$, and we ſhall have

$\frac{-a^2 + ab \text{ numerator.}}{-a - b \text{ denominator.}}$ Or change the ſigns of all the quantities, it will be $\frac{a^2 - ab}{a+b}$ the new fraction required, that is, $\frac{a^2 - ab}{a+b} = \frac{a^3 - ab^2}{a^2 + 2ab + b^2}$.

Beſides

Besides these, there are other sorts of reductions, which some authors have treated of under the head of fractions; but as they are of little use in the solution of problems, I shall pass them by, and proceed to addition.

ADDITION and SUBTRACTION.

RULE. Add or subtract their numerators as occasion requires, and to their sum or difference subscribe the common denominator, as in vulgar fractions.

EXAMPLES in ADDITION.

Add $\frac{a}{b}$ to $\frac{c}{d}$, first reduce them to a common denominator, and they will be $\frac{ad}{bd}+\frac{bc}{bd}=\frac{ad+bc}{bd}$, the sum required.

Add $\frac{a}{b}+\frac{c}{d}+\frac{d}{e}$ into one sum. First, these reduced to a common denominator will be $\frac{ade}{bde}+\frac{bce}{bde}+\frac{ddb}{bde}=\frac{ade+bce+ddb}{bde}$ the sum required.

EXAMPLES in SUBTRACTION.

From $\frac{bb+aa}{c}$ take $\frac{bb}{c}$. Thus, $\frac{bb+aa}{c}-\frac{bb}{c}=\frac{aa}{c}$, the difference.

From $\frac{a}{2}$ take $\frac{a}{3}$. Thus reduced, $\frac{3a}{6}$ and $\frac{2a}{6}$, then $\frac{3a}{6}-\frac{2a}{6}=\frac{a}{6}$, the difference required.

From $\frac{b^2+a^2}{c}$ take $\frac{bb}{c}$. Thus, $\frac{b^2+a^2}{c}-\frac{b^2}{c}=\frac{a^2}{c}$, the difference.

From $\frac{2b}{d+a}$ take $\frac{a+b-d}{d+a}$. Thus, $\frac{2b}{d+a}-\frac{a+b-d}{d+a}=\frac{a-b+d}{d+a}$, the difference.

MULTIPLICATION.

First, prepare the quantities as directed in vulgar fractions, then multiply the numerators together for a new numerator; and the denominators together, for a new denominator.

EXAMPLES.

EXAMPLES.

Multiply $\frac{a}{b}$ by $\frac{c}{d}$. Thus $\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$, the product.

And $\frac{2ab}{3c} \times \frac{5ad}{3f} = \frac{10a^2bd}{9cf}$, the product.

Likewise $\frac{2ac + b - 25c}{c} \times \frac{3b + 4c}{1} =$

$$\left.\frac{6bac + 3bb - 75bc + 8acc + 4bc - 100cc}{c}\right\}\text{the product.}$$

DIVISION.

One fraction is divided by another, by multiplying the denominator of the divisor into the numerator of the dividend, for a new numerator; and the numerator of the divisor into the denominator of the divided, for a new denominator.

EXAMPLES.

Divide $\frac{a}{b}$ by $\frac{c}{d}$. Thus $\frac{a}{b} \div$ by $\frac{c}{d} = \frac{ad}{bc}$, the quotient.

And $\frac{5ax}{3c} \div \frac{6bc}{7d} = \frac{35adx}{18bcc}$, the quotient.

Also $\frac{6a^2b}{5x} \div \frac{5ab^2}{3x} = \frac{18a^2bx}{25ab^2x}$, the quotient.

In cases like the last, where the two numerators, or the denominators, have factors common to both, the conclusion will become more neat by first casting off such common factors.

Thus, casting away ab out of the two numerators, and x out of both denominators, we have $\frac{6a}{5}$ to be divided by $\frac{5b}{3}$ whereof the quotient is $\frac{18a}{25b}$; and so of others.

To divide any quantity by a compound divisor, let the division be made the common way continually, till it appear the terms in the quotient will come out infinitely, remembering the rules of changing the signs when multiplying, subtracting, or dividing.

Let it be required to divide b by $a + c$?

$a + c)\ b\ \left(\frac{b}{a} - \frac{bc}{aa} + \frac{bc^2}{a^3} - \frac{bc^3}{a^4}\right.$, &c. infinitely.

$$b + \frac{bc}{a}$$

$$\overline{\quad - \frac{bc}{a}\quad}$$

$$- \frac{bc}{a} - \frac{bc^2}{a^2}$$

$$\overline{\qquad\qquad}$$

$$+\frac{bc^2}{a^2}$$

$$+\frac{bc^2}{a^2}+\frac{bc^3}{a^3}$$

$$-\frac{bc^3}{a^3}, \text{ \&c. infinitely.}$$

In this example, b divided by a, the quotient is $\frac{b}{a}$, the product of $\frac{b}{a}$ into $a+c$ is $\frac{ab}{a}+\frac{bc}{a}=b+\frac{bc}{a}$, which being taken from the dividend b, leaves $-\frac{bc}{a}$; again, if $-\frac{bc}{a}$ be divided by a, the quotient will be $-\frac{bc}{aa}$, then the product of $a+c$ into $-\frac{bc}{aa}$ is $-\frac{abc}{aa}-\frac{bcc}{aa}$ or $-\frac{bc}{a}-\frac{bc^2}{a^2}$, which being taken from the dividend $-\frac{bc}{a}$, leaves $+\frac{bc^3}{a^2}$. Thus it appears how the division is to be continued.

SURDS.

Surds are such numbers as cannot be exactly expressed in figures, and as they arise in the solution of algebraic questions, I shall explain to the young algebraist so much of them only as is necessary to the present design.

ADDITION of SURD QUANTITIES.

CASE. 1. When the quantities under the radical signs are alike, add the rational quantities, or those which are without the radical signs together, by the rules of addition, and to this join the surd quantities, and this will be the sum required.

If there be no rational quantities without the radical sign, then unity, or 1, is always supposed to be the rational quantity.

EXAMPLES.

To	$\sqrt{ab}$	$2\sqrt{xy}$	$5y\sqrt{dm+z}$
Add	$\sqrt{ab}$	$\sqrt{xy}$	$y\sqrt{dm+z}$
Sum	$2\sqrt{ab}$	$3\sqrt{xy}$	$6y\sqrt{dm+z}$

In example 1, there being no rational quantities, therefore unity or 1 is the rational quantity to each. Now 1 added to 1 makes 2, to which joining the surd $\sqrt{ab}$, we have $2\sqrt{ab}$, the sum required.

To	$-15m\sqrt{da-zy}$	$16ab\sqrt{14+x}$
Add	$7m\sqrt{da-zy}$	$-12ab\sqrt{14+x}$
Sum	$-8m\sqrt{da-zy}$	$4ab\sqrt{14+x}$

 CASE 2.

CASE 2. When the letters under the radical signs are different, place them down one after the other, with the same signs they have in the question.

EXAMPLES.

To	$\sqrt{a}$	$a\sqrt{bx+y}$
Add	$\sqrt{b}$	$a\sqrt{z}$
Sum	$\sqrt{a}+\sqrt{b}$	$a\sqrt{bx+y}:+a\sqrt{z}$

To	$-5y\sqrt{bx-y}$
Add	$7y\sqrt{zm+a}$
Sum	$-5y\sqrt{bx-y}:+7y\sqrt{zm+a}$

SUBTRACTION.

CASE 1. When the letters under the radical signs are alike, subtract the rational quantities from the rational quantities, and to the difference join the surd quantities, which will be the remainder required.

EXAMPLES.

From	$5\sqrt{ad}$	$6n\sqrt{nz}$	$14y\sqrt{d+z}$
Take	$3\sqrt{ad}$	$3n\sqrt{nz}$	$3y\sqrt{d+z}$
Remains	$2\sqrt{ad}$	$3n\sqrt{nz}$	$11y\sqrt{d+z}$

From	$12y\sqrt{d-n}$	$-5a\sqrt{x+y}$
Subtract	$-3y\sqrt{d-n}$	$3a\sqrt{x+y}$
Remains	$15y\sqrt{d-n}$	$-8a\sqrt{x+y}$

The truth of these operations are proved as in subtraction of common numbers.

CASE 2. When the letters under the radical signs are different, set them down one after the other, but care must be taken to change the signs of those quantities that are to be subtracted.

EXAMPLES.

From	$2a\sqrt{en}$	$5y\sqrt{a}$
Subtract	$y\sqrt{rv}$	$-3\sqrt{b}$
Remains	$2a\sqrt{en}-y\sqrt{rv}$	$5y\sqrt{a}+3\sqrt{b}$

From	$18\sqrt{da}$
Subtract	$7\sqrt{b+c}$
Remains	$18\sqrt{da}:-7\sqrt{b-c}$

These operations are proved in the same manner as in the last case, by adding the remainder to the quantity that was subtracted.

MULTIPLI-

MULTIPLICATION.

CASE 1. When there are no rational quantities joined to the surd, multiply the surd quantities together, and to their product prefix the radical signs.

EXAMPLES.

Multiply	$\sqrt{a}$	$\sqrt{xy}$	$\sqrt{a+b}$	$\sqrt{ax-ay}$
By	$\sqrt{b}$	$\sqrt{d}$	$\sqrt{x}$	$\sqrt{n}$
Product	$\sqrt{ab}$	$\sqrt{xyd}$	$\sqrt{ax+xb}$	$\sqrt{axn-ayn}$

CASE 2. If rational quantities be joined to the surds, then multiply the rational into the rational, and the surd into the surd, and join the products together.

EXAMPLES.

Multiply	$a\sqrt{x}$	$8a\sqrt{3x}$	$n\sqrt{a+y}$	$\sqrt{n+b}$
By	$b\sqrt{y}$	$3b\sqrt{2y}$	$a\sqrt{b}$	$a\sqrt{z}$
Product	$ab\sqrt{xy}$	$24ab\sqrt{6xy}$	$na\sqrt{ab+by}$	$a\sqrt{zn+zb}$

DIVISION.

CASE 1. When there are no rational quantities joined with the surd quantities, reject all those quantities in the dividend and divisor that are alike, and set down the remainder, to which prefix the radical signs, and this will be the quotient sought.

EXAMPLES.

Divide	$\sqrt{abx}$	$\sqrt{bxd}$	$\sqrt{ypa}$
By	$\sqrt{x}$	$\sqrt{bd}$	$\sqrt{yp}$
Quotient	$\sqrt{ab}$	$\sqrt{x}$	$\sqrt{a}$

In example 1, because x is in both dividend and divisor, reject it, and put down ab with the sign $\sqrt{}$ before it, and $\sqrt{ab}$ is the quotient required; and so of others.

Divide	$\sqrt{bn+ba}$	$\sqrt{mz-mp}$
By	$\sqrt{b}$	$\sqrt{m}$
Quotient	$\sqrt{n+a}$	$\sqrt{z-p}$

CASE 2. When there are rational quantities joined with the surds, divide the rational quantities by the rational quantities, and to their quotient join the quotient of the surds found by the last case.

EXAMPLES.

Divide	$ay\sqrt{mn}$	$ba\sqrt{ayn}$	$bx\sqrt{anp}$	$4an\sqrt{dy+dn}$
By	$a\sqrt{m}$	$a\sqrt{ay}$	$x\sqrt{an}$	$2a\sqrt{d}$
Quotient	$y\sqrt{n}$	$b\sqrt{n}$	$b\sqrt{p}$	$2n\sqrt{y+n}$

The truth of these operations are proved by multiplying the quotient by the divisor, for if that produces the dividend, the work is true, otherwise it is erroneous.

EQUATIONS.

An equation is, when two equal quantities, differently expressed, are compared together by means of the sign $=$ placed between them.

Thus $6 - 2 = 4$ is an equation expressing the equality of the quantities $6 - 2$ and 4: and $x = a + b$ is an equation, shewing that the quantity represented by x is equal to the sum of the two quantities represented by a and b.

Equations are the means whereby we come at such conclusions as answer the conditions of a problem, wherein, from the quantities given, the unknown ones are determined, and this is called the reduction of equations.

REDUCTION of SINGLE EQUATIONS.

Single equations are such as contain only one unknown quantity, which must be so ordered by addition, subtraction, multiplication, division, &c. of equal quantities, that a just equality between the two parts thereof may be still preserved, and that there may result, at last, an equation wherein the unknown quantity stands alone on one side, and all the known ones on the other; the best manner of doing which will be obtained by the following rules.

RULE 1. Any term of an equation may be transposed to the contrary side, if its sign be changed.

EXAMPLE 1. Thus, $x + 8 = 18$, then will $x = 18 - 8 = 10$.

In this equation $x + 8 = 18$, which by transposition becomes $x = 18 - 8 = 10$, by only subtracting the number 8 from both sides.

E. 2. If $x + 19 = 107$, what is the value of x?

By transposition the above equation is changed into this, $x = 107 - 19$, therefore $107 - 19 = 88$ the value of x.

E 3. Given $x - 107 = 19$, required the value of x?

By transposition of -107 to the other side of the equation, and changing the sign, the equation stands thus, $x = 19 + 107$, therefore $19 + 107 = 126$, the value of x.

E. 4. If $20 - 3x - 8 = 60 - 7x$, what is the value of x?

By transposing $7x$, we shall have $-3x + 7x = 60 - 20 + 8$, or $4x = 48$, therefore $x = \frac{48}{4} = 12$ the value of x.

For $\overline{20 - 12} \times 3 - 8 = 60 - \overline{12 \times 7} = -24$, proof.

RULE 2. If there is any quantity by which all the terms of the equation are multiplied, let them all be divided by that quantity; but, if all of them be divided by any quantity, let the common divisor be cast away.

E. 1. Suppose $ax = ab$, then by the rule $x = b$; also $10x = 60$, reduced $x = 6$; and by the latter part of the rule $\frac{x}{a} = \frac{b}{a}$ is reduced to $x = b$.

E. 2. Required the value of x, when $36 - \frac{4x}{9} = 8$.

By

By multiplying both sides by 9, we have $324 - 4x = 72$, therefore $4x = 324 - 72 = 252$, consequently $x = \frac{252}{4} = 63$.

E. 3. If $6x^2 - 20x = 16x + 2x^2$, what is the value of x?

By dividing by $2x$ we have $3x - 10 = 8 + x$, and by transposition $3x - x = 8 + 10$, that is $2x = 18$, therefore $x = \frac{18}{2} = 9$ answer.

RULE 3. If there are reduceable fractions, let the whole equation be multiplied by the product of all their denominators, or, which is the same, let the numerator of every term in the equation be multiplied by all the denominators, except its own, supposing such terms (if any there be) that stand without a denominator, to have an unit subscribed.

E. 1. If $x + \frac{x}{2} + \frac{x}{4} = 10$, what is x equal to?

By multiplying the equation by 8, the product of the two denominators 2 and 4, we have $8x + 4x + 2x = 80$, or $14x = 80$; therefore $x = \frac{80}{14} = 5.728$.

E. 2. Let $\frac{x}{5} + \frac{x}{3} = x - 7$, required the value of x?

This reduced will become $\frac{3x + 5x}{15} = \frac{8x}{15} = x - 7$, consequently $8x = 15x - 105$, whence $7x = 105$, therefore $x = \frac{105}{7} = 15$ the answer.

RULE 4. If in your equation there is an irreducable surd, wherein the unknown quantity enters, let all the other terms be transposed to the contrary side (by rule 1) and then, if both sides be involved to the power denominated by the surd, an equation will arise free from radical quantities, unless there happens to be more surds than one, in which case the operation is to be repeated.

Thus, $\sqrt{x} + 6 = 10$, by transposition becomes $\sqrt{x} = 10 - 6 = 4$, which, by squaring both sides, gives $x = 16$.

So likewise $\sqrt{aa + xx} - c = x$ becomes $\sqrt{aa + xx} = c + x$, which, squared, gives $aa + xx = cc + 2cx + xx$, or $aa - c = 2cx$, per rule 1.

E. 1. If $\sqrt{\frac{5x}{3}} + 12 = 17$, what is x?

By transposition becomes $\sqrt{\frac{5x}{3}} = 17 - 12 = 5$, and $\sqrt{5x} = 5 \times 3 = 15$; then by involving 15 to the power denominated by the surd, we have $5x = 225$, therefore $x = \frac{225}{5} = 45$.

E. 2. What

E. 2. What is the value of x when $\sqrt{12+x} = 2 + \sqrt{x}$?

By squaring both sides, we have $12 + x = 4 + 4\sqrt{x} + x$, and by transposition $4\sqrt{x} = 8$, consequently $x = 4$.

E. 3. Suppose $\sqrt{4x+16} = 12$, query the value of x?

By squaring both sides we have $4x + 16 = 144$, and by transposition $4x = 144 - 16 = 128$, therefore $x = \frac{128}{4} = 32$.

E. 4. If $\sqrt{x} + 6 = 10$, what is the value of x?

By transposition becomes $\sqrt{x} = 10 - 6 = 4$, and by squaring both sides we have $x = 16$.

E. 5. Suppose $\sqrt{ax+b^2} - c = d$, what is the value of x?

Then $\sqrt{ax+b^2} = d + c$, and by squaring we have $ax + b^2 = d^2 + 2dc + c^2$, whence $x = \frac{d^2 + 2dc + c^2 - b^2}{a}$.

RULE 5. Having, by the preceding rules, cleared your equations of fractional and radical quantities, and so ordered it by transposition, that all the terms wherein the known quantities are found may stand on the same side thereof, let the whole be divided by the coefficients, or the sum of the coefficients of the highest power of the said unknown quantity.

E. 1. If $12x = 48$, what is x equal to?

By dividing the whole by the coefficient of x, we have $x = \frac{48}{12} = 4$.

E. 2. Suppose $5x = 79$, what is x equal to?

To disengage x, you must take away the 5, and place it under the 79, thus $x = \frac{79}{5} = 15.8$.

E. 3. What is the value of x, when $5x - 16 = 3x + 12$?

By transposition becomes $5x - 3x = 12 + 16$, whence $x = \frac{28}{2} = 14$.

To reduce two or more equations to a single one.

RULE. Let the given quantities or equations be multiplied or divided by such numbers or quantities, whether known or unknown, that the term which involves the highest power of the unknown quantity to be exterminated, may be the same in each equation, and then, by adding or subtracting the equations, as occasion shall require, that term shall vanish, and a new equation emerge, wherein the number of dimensions (if not the number of unknown quantities) will be diminished.

E. 1. Given $\left\{\begin{array}{l} 5x + 8y = 106 \\ 4x - 5y = 5 \end{array}\right\}$ Query the value of x and y?

Here, by multiplying the first equation by 4, and the second by 5, in order that the coefficients of x may be the same in both, we have

$$20x + 32y = 424$$
$$20x - 25y = 25$$

By

By subtracting the latter from the former, we have $57y = 399$: hence $y = \frac{399}{57} = 7$. And so by the first equation $x = \frac{5 + 5 \times 7}{4} = \frac{40}{4} = 10$.

E. 2. Given $\left\{ \begin{array}{l} 5x - 3y = 90 \\ 2x + 5y = 100 \end{array} \right\}$ Query the value of x and y?

Here, by multiplying the first equation by 2, and the second by 5, in order that the coefficients of x may be the same in both, there arises

$$10x - 6y = 180$$
$$10x + 25y = 800$$

By subtracting the former from the latter, we have $31y = 620$; hence $y = \frac{620}{31} = 20$. And so by the first equation $x = \frac{90 + 3y}{5} = \frac{90 + 60}{5} = 30$.

E. 3. Given $\left\{ \begin{array}{l} \frac{x}{2} + \frac{y}{3} = 16 \\ \frac{x}{5} - \frac{y}{9} = 2 \end{array} \right\}$ Query x and y.

Here the equations cleared of fractions will be

$$3x + 2y = 96$$
$$9x - 5y = 90$$

Now if, from the triple of the former, the latter be subtracted, we have $6y + 5y = 288 - 90$, that is, $11y = 198$; hence $y = \frac{198}{11} = 18$; and $x = \frac{96 - 2y}{3} = \frac{96 - 36}{3} = 20$.

E. 4. Let $\left\{ \begin{array}{l} x + y = 13 \\ x + z = 14 \\ y + z = 15 \end{array} \right\}$ Query x, y, and z.

Here, by subtracting the first equation from the second, in order to exterminate x, we have $z - y = 1$, to which the third equation being added, y will likewise be exterminated, there coming out $2z = 16$, or $z = 8$; whence $y = z - 1 = 7$, and $x = 13 - y = 6$.

E. 5. Given $\left\{ \begin{array}{l} x + 100 = y + z \\ y + 100 = 2x + 2z \\ z + 100 = 3x + 3y \end{array} \right\}$ Query x, y, and z.

To the double of the first, let the second equation be added, so shall the x's, on the contrary sides, destroy each other, and we shall have $300 + y = 2y + 4z$, or $y + 4z = 300$.

Moreover, to the triple of the first let the third equation be added, whence will be had $z + 400 = 6y + 3z$, or $2z + 6y = 400$.

Now if, from the double of this last equation (viz. $4z + 12y = 800$) the former (viz. $y + 4z = 300$) be subtracted, there will come out $11y = 500$; therefore $y = \frac{500}{11} = 45\frac{5}{11}$, and $z = \frac{300}{4} = 75$

$-\frac{y}{4} = 75 - 11\frac{4}{11} = 63\frac{7}{11}$, and $x = y + z - 100 = 109\frac{1}{11} - 100 = 9\frac{1}{11}$.

A collection of questions producing simple equations.

When a question is proposed to be solved algebraically, its true design and signification ought to be perfectly understood, so that it may be abstracted from all ambiguous and unnecessary phrases, and the conditions thereof exhibited in the clearest light possible, this being done, and the several quantities therein concerned being denoted by proper signs, let the true sense and meaning of the question be translated from the English into algebra, and the conditions thus expressed in algebraic terms will, if it be properly limited, give as many equations as is necessary to its solution; but these things will be best understood by examples.

QUESTION 1. What two numbers are those, whose difference is 14, and sum 48?

Let - - -	$x =$ the lesser number.
Then the greater number will be	$x + 14$
Which by addition gives	$2x + 14 = 48$ per question.
Therefore by transposition	$2x = 48 - 14 = 34$
And - - -	$x = \frac{34}{2} = 17$ the lesser number.
Consequently - -	$17 + 14 = 31$ the greater number.
For - - -	$31 - 17 = 14$
And - - -	$31 + 17 = 48$ proof.

Q. 2. Four men, A, B, C, and D, built a ship, which cost 5214l. whereof B paid twice as much as A, C paid as much as A and B, and D paid as much as C and B; what did each pay;

Suppose A paid	x pounds
Then B paid	$2x$
C - -	$3x$
And D -	$5x$

Whence the whole sum paid is	$11x = 5214$l. by question,	
Therefore - -	$x = \frac{5214}{11} = 474$l. A's	share.
Consequently - -	$2x = 948$l. B's	
	$3x = 1422$l. C's	
And - - -	$5x = 2370$l. D's	
	Proof £. 5214	

Q. 3. A borrowed of B as much money as A had, and spent 6d. to treat him; after which, meeting with C, A borrowed of him twice as much money as he had left, and treated him with 12d.; lastly, A borrowed of D three times as much money as he had left, and spent on him 18d. after which he had 30d. left; what had he at first?

Suppose he had x pence at first

Then he borrowed x pence of B,

And after spending 6d. had $2x - 6$ left,

Then

Then he borrowed - $4x - 12$ of C,
And after ſpending 12*d*. had $6x - 30$ left;
Then he borrowed - $18x - 90$ of D,
And after ſpending 18*d*. had $24x - 138$ left:
But - $24x - 138 = 30$ by the queſtion.
Therefore, - $24x = 168$
And - - - $\frac{168}{24} = 7d.$ the anſwer.

Q. 4. A charitable lady, relieving four poor perſons, gave among them 6s. 8d. to the ſecond ſhe gave twice, to the third thrice, and to the fourth four times as much as to the firſt; what did ſhe give to each?

Suppoſe ſhe gave - x pence to the firſt,
Then ſhe gave - $2x$ pence to the ſecond,
——— $3x$ —— to the third,
And ——— $4x$ —— to the fourth;

And ſhe gave in all $10x$ ($= 6s.\ 8d.$) $= 80$, by the queſtion.
Therefore —— $x = \frac{80}{10} = 8d.$ the ſum the firſt had.
For $8d. + 16d. + 24d. + 32d. = 80d.$ the proof.

Q. 5. In a lump of mixed metal, weighing 29lb. there was 2lb. of ſilver more than of gold, 4lb. of copper more than of ſilver, and 3lb. of braſs more than of copper; how many pounds were there of each?

Suppoſe there were - x *lb.* of gold,
Then there were - $x + 2$ of ſilver,
And —— —— $x + 6$ of copper,
Alſo —— —— $x + 9$ of braſs:

But their ſum is - $4x + 17 = 29$ per queſtion.
Therefore by ſubtraction, $4x = 29 - 17 = 12$;
Conſequently $x = \frac{12}{4} = 3$ *lb.* of gold, the anſwer.
For $3 + 5 + 9 + 12 = 29$ *lb.* Proof.

Q. 6. Being to buy a ſuit of clothes for each of my ſix children, I propoſe to lay out four times as much on the eldeſt as I do on the youngeſt, and to beſtow twelve ſhillings a ſuit leſs on each than on the next elder; what will each ſuit coſt?

Suppoſe the youngeſt's ſuit coſt x ſhillings,
Then the ſecond's will coſt - $x + 12$,
the third's —— $x + 24$,
the fourth's —— $x + 36$,
the fifth's —— $x + 48$,
And the eldeſt's —— $x + 60$,
But —— —— $x + 60 = x\,4$ per queſtion.
Therefore —— $3x = 60$,
Conſequently $x = \frac{60}{3} = 20s.$ what the youngeſt's ſuit coſt.
For $20 + 12 + 12 + 12 + 12 + 12 = 80$, and $\frac{80}{4} = 20$. Proof.

Q. 7. The paving of a square, at 2*s.* a yard, cost as much as the inclosing it at 5*s.* a yard; the side of that square is required?

Let - - x = side of the square,
Then $4x$ = yards of inclosure,
And xx = yards of pavement,
Whence $20x = (4x \times 5 =)$ price of inclosing,
And $2xx = (xx \times 2 =)$ price of paving;
But - $2xx = 20x$ by the question
$\therefore$ - $xx = 10x$ } by division, the answer.
And - $x = 10$ }
For $10 \times 4 \times 5 = 100 \times 2 = 200$, Proof.

Q. 8. A general disposing his army into a square battle, finds he has 284 men more than a perfect square, but increasing the side by one man, he will want 25 men; how many had he?

Let - - - - x = side of the first square,
Then - - $xx + 284$ = army,
And $\overline{x+1} \times \overline{x+1} - 25$ = army,

Hence $xx + 2x - 24 = xx + 284$;
Then - - $2x = 308$;
$\therefore$ - - $x = 154$;
Consequently he had $(154 \times 154 + 284 =)$ 24000 men, the answer.

Q. 9. A person being asked how old he was, answered, that the product of $\frac{1}{20}$ of the years he had lived, being multiplied by $\frac{5}{8}$ of the same, would be his age; what was it?

Suppose his age was x years,

Then - - $\frac{x}{20} \times \frac{5x}{8} = x$ per question,

That is $\left(\frac{5xx}{20 \times 8} =\right) \frac{xx}{4 \times 8} = x$,

But by multiplication $xx = 32x$,
$\therefore$ by division - - $x = 32$, his age.

Q. 10. A man dying, left his wife big with child, ordering by will, that if the child proved a daughter, then his wife should have $\frac{2}{3}$ and the child $\frac{1}{3}$ of his estate, but if it was a son, then he should have $\frac{2}{3}$ and the mother $\frac{1}{3}$ thereof; now it happened that the mother was delivered of a son and daughter; how must the estate (which was 6300*l.*) be divided among them?

Suppose the daughter's share was x £.
Then the mother's would be - $2x$,
And the son's - - $4x$.

For then { the son's share is to the mother's, } as $\frac{2}{3}$ to $\frac{1}{3}$:
{ the mother's to the daughter's }

But the whole estate $6300 = 7x$ per question.

$\therefore \left(\frac{6300}{7} =\right) 900 = x$, the daughter's share.

Q. 11.

Q. 11. Four persons, A, B, C, D, spent twenty shillings in company together, whereof A proposed to pay $\frac{1}{3}$, B $\frac{1}{4}$, C $\frac{1}{5}$, and D $\frac{1}{6}$ part, but when the money came to be collected, they found it was not sufficient to answer the intended purpose; how much must each person contribute to make up the whole reckoning, supposing their several shares to be still to each other in the proportion above specified?

Suppose A was to pay x shillings.

Then - $(\frac{1}{3} : \frac{1}{4} :: x :)\ \frac{3x}{4} =$ the sum B must pay,

And - $(\frac{1}{3} : \frac{1}{5} :: x :)\ \frac{3x}{5} =$ the sum C must pay,

And - $(\frac{1}{3} : \frac{1}{6} :: x :)\ \frac{x}{2} =$ the sum D must pay;

But - $x + \frac{x3}{4} + \frac{3x}{5} + \frac{x}{2} = 20$ per question.

Or $20x + 15x + 12x + 10x = (20 \times 20 =)\ 400$,

That is - - $57x = 400$

Therefore - $x = \left(\frac{400}{57} =\right) 7s.\ \frac{1}{57}$, the sum A pays;

B must pay $\left(\frac{400}{57} \times \frac{3}{4} = \frac{300}{57} =\right) 5\frac{15}{57}$;

C must pay $\left(\frac{400}{57} \times \frac{3}{5} = \frac{80 \times 3}{57} = \frac{240}{57} =\right) 4\frac{12}{57}$;

D must pay $\left(\frac{400}{57} \times \frac{1}{2} = \frac{200}{57} =\right) 3\frac{29}{57}$;

For $7\frac{1}{57} + 5\frac{15}{57} + 4\frac{12}{57} + 3\frac{29}{57} = 20s.$ Proof.

QUADRATIC EQUATIONS.

A simple quadratic equation is that which involves the square of the unknown quantity only.

An adfected quadratic equation is that which involves the square of the unknown quantity, together with the product that arises by multiplying it by some known quantity.

Of these equations there are three forms, viz.

1. $x^2 + ax = b$
2. $x^2 - ax = b$
3. $x^2 - ax = -b$

To find the value of x in each of these equations, observe the following rules:

1. Transpose all the terms that involve the unknown quantity to one side of the equation, and the known terms to the other side, and let them be ranged according to their dimensions.

2. When the square of the unknown quantity has any coefficient prefixed to it, let all the rest of the terms be divided by that coefficient.

3. Add the square of half the coefficient of the second term to both sides of the equation, and that side which involves the unknown quantity will then be a complete square.

4. Extract from both sides of the equation, and the value of the unknown quantity will be determined.

EXAMPLE 1. Suppose $x^2 + 4x = 140$, what is the value of x?

First, $x^2 + 4x + 4 = 140 + 4 = 144$, by completing the square,

Then $\sqrt{x^2 + 4x + 4} = \sqrt{144}$ by evolution,

$\therefore x = 12 - 2 = 10$.

E. 2. Suppose $x^2 - 6x + 8 = 80$, what is the value of x?

First, $x^2 - 6x = 80 - 8 = 72$ by transposition,

Then $x^2 - 6x + 9 = 72 + 9 = 81$ by completing the square,

And $x - 3 = \sqrt{81} = 9$ by extracting the root,

$\therefore x = 9 + 3 = 12$.

E. 3. Suppose $2x^2 + 8x - 20 = 70$, what is x equal to?

First, $2x^2 + 8x = 70 + 20 = 90$ by transposition

Then $x^2 + 4x = 45$ by dividing by 2,

And $x^2 + 4x + 4 = 49$ by completing the square,

Whence $x + 2 = \sqrt{49} = 7$ by extracting the root,

Consequently $x = 7 - 2 = 5$.

E. 4. Suppose $3x^2 - 3x + 6 = 5\frac{1}{3}$, query x?

Here $x^2 - x + 2 = 1\frac{7}{9}$ by dividing by 3,

And $x^2 - x = 1\frac{7}{9} - 2$ by transposition,

Also $x^2 - x + \frac{1}{4} = 1\frac{7}{9} - 2 + \frac{1}{4} = \frac{1}{36}$ by completing the square,

And $x - \frac{1}{2} = \sqrt{\frac{1}{36}} = \frac{1}{6}$ by extracting the root,

$\therefore x = \frac{1}{6} + \frac{1}{2} = \frac{2}{3}$.

E. 5. Suppose $ax^2 + bx = c$, what is x equal to?

First, $x^2 + \frac{b}{a}x = \frac{c}{a}$ by division,

Then $x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} = \frac{c}{a} + \frac{b^2}{4a^2}$ by completing the square,

And $x + \frac{b}{2a} = \sqrt{\frac{c}{a} + \frac{b^2}{4a^2}} = \sqrt{\frac{4ac + b^2}{4a^2}}$ by evolution,

$\therefore x = \pm\sqrt{\frac{4ac + b^2}{4a^2}} - \frac{b}{2a}$.

Questions producing quadratic equations.

Q. 1. What two numbers are those, whose difference is 8, product 240?

Let $x =$ the lesser number,

Then will $x + 8 =$ the greater,

And $x^2 + 8x = 240$ by the question,

Whence $x^2 + 8x + 16 = 240 + 16 = 256$ by completing the square,

Also $x + 4 = \sqrt{256} = 16$ by extracting the root,

$\therefore x = 16 - 4 = 12$ the lesser number,

And $x + 8 = 20$ the greater.

Q. 2. It is required to divide 100 into two such parts, that if they be multiplied together, the product shall be 2100.

Let the greater part above $50 = x$,

Then will $50 + x =$ the greater part,

And $50 - x =$ the lesser,

$\therefore \overline{50 + x} \times \overline{50 - x}$, or $2500 - x^2 = 2100$ by the question,

Whence

Whence $x^2 = 400$, and $x = \sqrt{400} = 20$ by evolution,

$\therefore 50 + x = 70$ the greater part,

And $50 - x = 30$ the lesser part,

For $70 \times 30 = 2100$ proof.

Q. 3. What number is that, whose third part added to its fourth part will be 21?

Let $x =$ the given number,

Then $\frac{x}{3} + \frac{x}{4} = 21$ by the question,

$\therefore 7x = 252$, consequently $x = 36$ the number required,

For $\frac{36}{3} + \frac{36}{4} = 12 + 9 = 21$ proof.

Q. 4. Sold a piece of cloth for 24l. and gained as much per cent. as the cloth cost me; what was the price of the cloth?

Let $x =$ pounds the cloth cost,

Then $24 - x =$ the whole gain,

But $100 : x :: x : \overline{24 - x}$ per question,

Or $x^2 = 100 \times \overline{24 - x} = 2400 - 100x$,

That is, $x^2 + 100x = 2400$,

Whence $x^2 + 100x + 2500 = 2400 + 2500 = 4900$ by completing the square,

And $x + 50 = \sqrt{4900} = 70$ by evolution,

Consequently $x = 70 - 50 = 20$ the price of the cloth.

Q. 5. The difference of the means (a) and the difference of the extremes (b) of four numbers in continued geometrical proportion being given, to find the numbers.

Let the sum of the means be $= x$,

Then the greater of them will be $= \frac{x+a}{2}$,

And the lesser $\frac{x-a}{2}$, whence by the nature of proportionals it will be $\frac{x+a}{2} : \frac{x-a}{2} :: \frac{x-a}{2} : \frac{\overline{x-a}|^2}{2x+2a}$ the lesser extreme, and $\frac{x-a}{2} : \frac{x+a}{2} :: \frac{x+a}{2} : \frac{\overline{x+a}|^2}{2x-2a}$ the greater extreme; therefore by the question we have $\frac{\overline{x+a}|^2}{2x-2a} - \frac{\overline{x-a}|^2}{2x+2a} = b$, consequently $\overline{x+a}|^3 - \overline{x-a}|^3 = 2b \times \overline{x-a} \times \overline{x+a}$, that is, $6x^2a + 2a^3 = 2b \times \overline{x^2 - a^2}$, whence $x^2 = \frac{ba^2 + a^3}{b - 3a}$, and consequently $x = a\sqrt{\frac{b+a}{b-3a}}$.

MISCELLANEOUS QUESTIONS.

1. Suppose a hole perforated through the earth, in a right line perpendicular to its surface, in latitude 51°, and that a stone be let fall into

into it, so as to go on in its descent without interruption; where would it stop, and how many English miles would it run through, supposing it not to go beyond the place of its final rest*, and supposing the earth to be an oblate spheroid, whose diameter at the equator is 7974, and the length of its axes 7940 English miles?

Put $2t = 7974$, $2c = 7940$, $m =$ the sine, and n the cosine of 51° the given latitude, radius $= 1$, then will $\sqrt{t^2 m^2 + c^2 n^2} + \dfrac{c t^2 n^2 - c^3 n^2}{t\sqrt{t^2 m^2 + c^2 n^2}} = 3976{,}88$ miles, the space run through by the stone, where it rests at the distance $\dfrac{n m t^2 - n m c^2}{\sqrt{t^2 m^2 + c^2 n^2}} = 16{,}4$ miles from the center of the earth. Q. E. I.

2. From the Old Cross in Birmingham, two men they set out,
Resolving to travel the whole world about;
The one, he went easterly steering his way,
The other went north, as some people do say;
The first travell'd $7\frac{455}{21533}$ miles every day,
The other $11\frac{143}{4307}$, but now, I pray,
How many times round the world must they go,
And how many miles † will each travel also,
And how many days must they be to obtain,
To meet at the Cross aforesaid again:

Put $7\frac{455}{21533} = a$, $11\frac{143}{4307} = b$, r for the circumference of the earth $= 21600$ miles, x and y (which must be whole numbers) for the respective number of times that each man must travel round the earth before they meet at the same point from which they set out; then $rx =$ the miles travelled by the first man, $ry =$ those travelled by the second man. Hence $\frac{rx}{a}$ or $\frac{ry}{b} =$ the common time of both their travelling $\therefore \frac{rx}{a} = \frac{ry}{b}$, or $\frac{x}{a} = \frac{y}{b}$, that is, $bx = ay$, in numbers $6512184\,y = 10233432\,x$, consequently $y : x :: 10233432 : 6512184$ $= \frac{6512184}{10233432} = \frac{7}{11}$ in its lowest terms, whose numerator shews the number of times travelled round the earth by the first man, and the denominator those of the second man.

4. A ball, descending by the force of gravity from the top of a tower, was observed to fall half the way in the last second of time; required the tower's height, and the whole time of descent?

Let $t =$ the whole time of descent, so will $t - 1 =$ the time of descent through the first half of the tower's height, and therefore (the spaces described being always as the squares of the times) we have $tt : \overline{t-1}|^2 :: 2 : 1$, whence $tt - 2t + 1 = \frac{1}{2}tt$, from which $t = 2 + \sqrt{2} = 3{,}414$, and the tower's height $= 187{,}48$ feet.

* Some people think a stone thus descending by the force of gravity would go beyond the center, and returning again, continue for some time moving backward and forward, like the swinging of a pendulum, till it rests in its proper place.

† Reckoning 360 degrees, each 60 English miles, according to vulgar computation.

APPENDIX.

APPENDIX:

CONTAINING

1. A COURSE of BOOK-KEEPING, according to the method of Single Entry; with a description of the Books, and directions for using them.

2. BOOK-KEEPING by Double Entry, according to the Italian method; with various Forms of Acquittances, Promissory Notes, and Bills of Exchange; very useful either for young Book-keepers entering into business, or for Masters to teach in their schools.

BOOK-KEEPING, BY SINGLE ENTRY.

IN book-keeping by single entry, two books are indispensibly necessary, viz. Day Book and Ledger; the forms of which may be sufficiently known by inspection.

In the Day Book every person is written down Debtor to the things he receives from you on trust, and Creditor by those which you receive from him.—In the margin of the Day Book are written the pages where the accounts stand in the Ledger. Instead of these marginal figures, some make only a dash with the pen, to shew that the account has been posted, that is, entered in the ledger; but it is better to use the figures, for they shew not only that the account has been posted, but likewise where to find it in the Ledger, without looking in the alphabet. I have entered in the Day Book what is received, as well as what is delivered, which is very necessary in teaching; for the learner ought to make out his Ledger from his Day Book.

There are several other books kept by most merchants, as the Cash Book; the Book of House Expences, the Invoice Book, the Bill Book, &c. &c.

DIRECTIONS

DIRECTIONS FOR THE READER.

Your books being ruled in the proper form, copy into your Day Book one month's accounts; then calculate them upon your slate, to find if they be rightly cast up. Next, rule your slate in the form of the Ledger, and upon it post the accounts that were copied in the Day Book, with their dates prefixed, observing to put on the Debtor side of each person's account, those accounts to which he is debtor in the Day Book, and on the Creditor side, those by which he is Creditor, and if any accounts consist but of one article, you are to express it particularly with its money in the columns; but if of several, write *to* or *by* sundries, placing the sum of the amounts of all the articles in the columns. After the accounts are properly placed, transcribe them into your Ledger, leaving a proper space under each person's name, to receive more accounts.

Then under the proper letters in the alphabet, enter those names with the pages where they stand in the Ledger; and lastly, write the Ledger pages to the several accounts in the Day Book.

Do the same with the next month's accounts, and so on till the whole be finished: You must not enter any person's name down again, which has been entered before, till the space first assigned to it shall be filled with articles, and then the account must be transferred to a new place; and at the end of the old Ledger, draw out a balance account, placing your Debts on one side and your Credits on the other.

THE DAY BOOK.

		£	s.	d.
	January 1, 1782.			
1	*Mr. John, Holland of York, Dr.*			
	To 7 yards of fine broad cloth, at 18s 6d ——	6	9	6
	— 20 —— superfine ditto, - 19s 8d ——	19	13	4
	—— 1. ——	26	2	10
1	*George Birch, Esq. of Bath, Dr.*			
	To 6 gallons palm sack, at 8s 6d per gal. ——	2	11	0
	— 9 —— port, red, - 5 8 —— ——	2	11	0
	— 9 —— claret, —— 8 9 —— ——	3	18	9
	—— 4. ——	9	0	9
1	*Mrs. Sarah Moore, - Dr.*			
	To 2 *lb.* green tea, - at 18s 0d ——	1	16	0
	— 2½ congou, —— 9 6 ——	1	3	9
	— ½ stone of sugar, - 5 0 per stone ——	0	2	6
	— A lump of sugar, weight 20*lb.* at 10*d* per pound	0	16	8
	—— 9. ——	3	18	11
2	*Sir Joseph Johnson, Dr.*			
	To a silver punch bowl, wt. 23 oz. at 5s 10d per oz.	6	14	0
	— a tankard, - - 10 10 6 0 ——	3	3	0
	—— 20. ——	9	17	2
1	*Sir John Mosley, Dr.*			
	To a ream of fine post paper ——	1	5	0
	—— 27. ——			
2	*Mr. John Summers, schoolmaster, Dr.*			
	To 6 cyphering books, at 1s 2d each ——	0	7	0
	— 3 dozen copy books, 2 4 per dozen ——	0	7	0
	— 4 quires of fool's cap, 0 9 per quire ——	0	3	0
	— 1 quire, thin post, —— ——	0	0	8
	—— February 5. ——	0	17	8
1	*Mr. Anthony Archer, Dr.*			
	To a ledger, ruled, —— —— ——	1	0	0
	— 5 hundred quills, at 2s per hund. ——	0	10	0
	— 4 reams of thick post, 1*l* 2s 4*d* per ream, —	4	9	4
	— 8 reams, fool's cap, - 1 1 0 —— —	8	8	0
	—— 12. ——	14	7	4
	Mr. William Grove, Dr.			
1	To 4 gallons of rum, - at 12s 0d per gallon	2	8	0
	— 2 —— brandy, - 8 0 —— —	0	16	0
	— 3 —— English gin, 4 6 ——	0	13	6
		3	17	6

	February 20.			
2	*William Warner, Esq.* *Dr.*	£.	s.	d.
	To 10 oz. of nutmegs, - at 1s 2d per oz. —	0	11	8
	— 4 pounds coffee, - - 4 0 per lb. —	0	16	0
	— 5 —— almonds, - 1 2	0	5	10
	— 8 —— raisins, - 0 8	0	5	4
	—— 27. ——	1	18	2
1	*Sir John Mosley,* *Cr.*			
	By cash received of him in full —	1	5	0
	—— March 22. ——			
1	*George Birch, Esq. of Bath,* *Dr.*			
	To 8 gallons sherry, - at 6s 4d per gallon —	2	10	8
	— 12 —— rhenish, - 6 6 —	3	18	0
	— 8 —— Lisbon, - 4 2 — —	1	17	6
		8	6	2
	—— April 24. ——			
1	*Mrs. Sarah Moore,* *Cr.*			
	By cash received of her in full, —	3	18	11
	—— May 3. ——			
1	*Mr. John Holland, of York,* *Dr.*			
	To 25 yards of yard-wide cloth, at 5s 2d per yard	6	9	2
	— 8 ditto drugget, — 5 8 —	2	5	4
	— 9 —— serge, — 2 6 —	7	9	6
	— 36 —— shalloon, — 1 8 —	3	0	0
	—— 14. ——	12	17	0
2	*Mr. John Flint, of Nottingham,* *Dr.*			
	To 12 pair worsted stockings, at 4s 2d per pair -	2	10	0
	— 5 — silk ditto, — 16 4	4	1	8
	— 16 — thread dit. — 5 0 —	4	0	0
	—— June 3. ——	10	11	8
3	*Mr. James Davies,* *Dr.*			
	To 8 quarters of wheat, at 2l 8s 0d per quarter -	19	4	0
	— 4 —— rye, - 1 8 2 —	5	12	8
	— 20 —— oats, - 0 10 9 —	10	10	0
	—— 12. ——	35	6	8
2	*Sir Joseph Johnson,* *Cr.*			
	By a bank note, received of his servant,	5	0	0
	—— 17. ——			
1	*Mrs. Sarah Moore,* *Dr.*			
	To 6 pounds of hard soap, - at 6½d per lb. —	0	3	3
	— 4 —— soft ditto, — 6 —	0	2	0
	— 5 —— starch, — 5 —	0	2	1
	— 4 dozen of candles, — 6 per dozen —	0	4	0
		1	11	4

		£.	s.	d.
	June 21.			
	Mrs. Sarah Moore, *Cr.*			
1	By cash received in full	1	11	4
	July 7.			
	Mr. John Summers, schoolmaster, *Cr.*			
2	By cash received in full	0	17	8
	28.			
	Mr. James Davies, *Dr.*			
3	To 12 bushels of peas, - at 2*s* 6*d* per bushel -	1	10	0
	— 8 — beans, — 2 2 —	0	17	4
	— 10 — malt, — 5 0 —	2	10	0
		4	17	4
	August 1.			
	William Warner, Esq. *Dr.*			
2	To 9 gross of bottles, - at 33*l* 0*s* per gross -	14	17	0
	— 2 — small ditto, - 12 0	1	4	0
	— 4 decanters, — 1 4 each —	0	5	4
		16	6	4
	7.			
	Mr. Anthony Archer, *Cr.*			
1	By a note upon Mr. John Steventon, for	10	0	0
	— cash in full	4	7	4
		14	7	4
	21.			
	Mr. Charles Jones, of Shrewsbury, *Dr.*			
1	To 24*lb* of cochineal, at 1*l* 2*s* 6*d* per *lb*	27	0	0
	— 3 — opium, - 0 8 0	1	4	0
	— 12 — rose pink, 0 0 10	0	10	0
		28	14	0
	September 4.			
	Mr. John Summers, schoolmaster. *Dr.*			
2	To 12 schoolmaster's guides, at 2*s* 2*d* each	1	6	0
	— 3 dozen copy books, - 2 6 per dozen	0	7	6
	— 1 ream fool's cap,	1	0	0
		2	13	6
	9.			
	Mr. John Flint, *Cr.*			
2	By a bank note for	5	0	0
	Mr. John Summers, *Dr.*			
2	To 6 dozen of Dyche's spelling book, at 10*s* per dozen	3	0	0
	October 2.			
	Mr. Samuel Taylor, *Dr.*			
3	To 20*lb* of flax, at 1*s* per lb.	1	0	0
	15.			
	Mr. John Johnson, of Great Haywood, *Dr.*			
2	To 4½ *cwt.* iron, at 18*s* per cwt.	4	1	0
	21.			
	Mrs. Phebe Young, *Cr.*			
3	By 60 yards of Irish cloth, at 2*s* 6*d* per yard	7	10	0

		£.	s.	d.
	October 27.			
1	*George Birch, Esq. Cr.*			
	By cash in full	17	6	11
	30.			
3	*Mr. Samuel Taylor, Dr.*			
	To 12*lb.* of flax, at 1*s* 0*d* per lb.	0	12	0
	— 10 — 0 6	0	5	0
	November 13.	0	17	0
1	*Mr. John Holland, Cr.*			
	By a bill for	20	0	0
	15.			
3	*Mr. James Davies, Cr.*			
	By cash in full	40	4	0
	22.			
6	*Mr. Thomas Green, Dr.*			
	To 2 dozen knives and forks, at 12*s* per dozen,	1	4	0
	— a set of china,	3	10	6
	— a mahogany tea board,	0	12	0
	26.	5	6	6
8	*Mr. Thomas Green, Cr.*			
	By 30 ells of holland, at 5*s* 2*d* per ell	7	15	0
	28.			
2	*Sir Joseph Johnson, Cr.*			
	By cash in full	4	17	8
	December 1.			
2	*Mr. John Summers, schoolmaster, Cr.*			
	By cash in full	5	13	6
	3.			
1	*Mr. Anthony Archer, Dr.*			
	To a cask of rum	10	0	0
	6.			
2	*William Warner, Esq. Cr.*			
	By cash in full	18	4	6
	8.			
3	*Mr. John Hunter, of Friesely, Dr.*			
	To 4 tons of coals, at 7*s* 6*d* per ton	1	10	0
	10.			
1	*Mr. William Grove, Cr.*			
	By cash in full	3	17	6
	12.			
3	*Mr. Carless, Dr.*			
	To a tun of oil, containing 236 gallons, at 2*s* 6*d* per gallon	29	10	0
	13.			
	Mr. John Johnson, Cr.			
	By cash in full	4	1	0

		£.	s.	d.
	December 18.			
2	*Mrs. Hill, Dr.*			
	To a lump of sugar, wt. 26 *lb* at 12*d* per lb.	1	6	0
	23.			
3	*Mr. John Young, Dr.*			
	To 4*cwt* 2*qrs* cheese, at 32*s* per cwt. ——	7	4	0
2	*Mrs. Hill, Cr.*			
	By cash in full —— —— ——	1	6	0

L E D G E R.

The ALPHABET.

A		B		C	
Mr. Anth. Archer	1	George Birch, Esq.	1	Mr. Carless	3
D		**E**		**F**	
Mr. Jas. Davies	3			Mr. John Flint	3
G		**H**		**I**	
Mr Wm. Grove	2	Mr. John Holland	1	Sir Joseph Johnson	2
Mr. John Grove	1	Mrs. Hill	2	Mr Charles Jones	1
Mr. Thomas Green	3			Mr. John Johnson	2
K		**L**		**M**	
				Mrs. Sarah Moore	1
				Sir John Mosley	1
N		**O**		**P**	
Q		**R**		**S**	
				Mr. John Summers	2
T		**V**		**W**	
Mr. Sam. Taylor	3			Wm. Warner, Esq.	2
X		**Y**		**Z**	
		Mrs. Phebe Young	3		
		Mr. John Young	3		

		£.	s.	d.			£.	s.	d.
1782	*Dr.* *Mr. John*				1782	*Holland, of York,* *Cr.*			
January 1	To sundries - -	26	2	10	Nov. 13	By a bill for - - -	20	0	0
May 3	To sundries - - -	12	17	0		By cash remains to balance -	18	19	10
		38	19	10			38	19	10
1782	*Dr.* *George*				1782	*Birch, Esq, of Bath,* *Cr.*			
July 1	To sundries - - -	9	0	9	Oct. 27	By cash in full - -	17	6	11
March 22	To sundries - - -	8	6	2					
		17	6	11					
1782	*Dr.* *Mrs. Sarah*				1782	*Moore,* *Cr.*			
January 4	To sundries - - -	3	18	11	April 24	By cash in full - -	3	18	11
June 17	To sundries - - -	1	11	4	June 21	By cash in full - -	1	11	4
		5	10	3			5	10	3
1782	*Dr.* *Sir John*				1782	*Mosler,* *Cr.*			
Jan. 20	To a ream of paper - -	1	5	0	Feb. 27	By cash in full - - -	1	5	0
1782	*Dr.* *Mr. Anthony*				1782	*Archer,* *Cr.*			
Feb. 5	To sundries - - -	14	7	4	August 7	By sundries - - -	14	7	4
Dec. 3	To a cask of rum - -	10	0	0		By cash remains to balance -	10	0	0
		24	7	4			24	7	4
1782	*Dr.* *Mr. William*				1782	*Grove,* *Cr.*			
Feb. 12	To sundries - - -	3	17	6	Dec. 10	By cash in full -	3	17	6
1782	*Dr.* *Mr. Charles*					*Jones, of Shrewsbury,* *Cr.*			
August 21	To sundries - - -	28	14	0		By cash remains to balance -	28	14	0

		£.	s.	d.			£.	s.	d.
1782	*Dr.* *Mr. John*				1782	*Summers, schoolmaster,* *Cr.*			
Jan. 27	To sundries - - -	0	17	8	July 7	By cash in full - -	0	17	8
Sept. 4	To sundries - - - -	2	13	6	Dec. 1	By cash in full - - -	5	13	6
— 9	To 6 dozen of spelling books, at 10*s*	3	0	0			6	10	2
		6	10	2					
1782	*Dr.* *William*				1782	*Warner, Esq.* *Cr.*			
Feb. 10	To sundries - - -	1	18	2	Dec. 6	By cash in full - -	18	4	6
August 1	To sundries - - -	16	6	4					
		18	4	6					
1782	*Dr.* *Sir Joseph*				1782	*Johnson,* *Cr.*			
January 9	To sundries - - -	9	17	2	June 12	By a bank note - -	5	0	0
					Nov. 28	By cash in full - - -	4	17	2
							9	17	2
1782	*Dr.* *Mr. John*				1782	*Flint, of Nottingham,* *Cr.*			
May 14	To sundries - - -	10	11	8	Sept. 9	By a bank note - -	5	0	0
						Cash remains to balance -	5	11	8
							10	11	8
1782	*Dr.* *Mrs.*				1782	*Hill,* *Cr.*			
Dec. 18	To a lump of sugar, wt. 26*lb* at 12*d*	1	6	0	Dec. 23	By cash in full - - -	1	6	0
1782	*Dr.* *Mr. John*				1782	*Johnson,* *Cr.*			
Oct. 15	To $4\frac{1}{2}$ *cwt* of iron, at 18*s* -	4	1	0	Dec. 13	By cash in full - - -	4	1	0

Date	Dr.	£	s.	d.	Date	Cr.	£	s.	d.
3) 1782	*Dr.* *Mr. James*					*Davies,* *Cr.*			
June 3	To fundries - - -	35	6	8	1782				
July 28	To fundries - -	4	17	4	Nov. 15	By cash in full - -	40	4	0
		40	4	0					
1782	*Dr.* *Mr. Samuel*					*Taylor,* *Cr.*			
October 2	To 20 pounds of flax, at 1s -	1	0	0					
30	To fundries - - -	0	17	0		Cash remains to balance -	1	17	0
		1	17	0					
1782	*Dr.* *Mrs. Phebe*				1782	*Young,* *Cr.*			
	To balance - - -	7	10	0	Oct. 21	By 60 yards of Irish cloth, at 2s 6d -	7	10	0
1782	*Dr.* *Mr. Thomas*				1782	*Green,* *Cr.*			
Nov. 22	To fundries - - -	5	6	6	Nov. 26	By 30 ells of Holland, at 5s 2d -	7	15	0
	Cash remains to balance - -	2	8	6					
		7	15	0					
1782	*Dr.* *Mr. John*					*Hunter,* *Cr.*			
Dec. 8	To 4 tons of coals, at 7s 6d -	1	10	0		Cash remains to balance -	1	10	0
1782	*Dr.* *Mr.*					*Carless,* *Cr.*			
Dec. 12	To a tun of oil - -	29	10	0		Cash remains to balance -	29	10	0
1782	*Dr.* *Mr. John*					*Young,* *Cr.*			
Dec. 23	To 4 hundred 2 quarters cheese, at 32s	7	4	0		Cash remains to balance -	7	4	0

B A L A N C E.

1782	*Dr.* Balance	£.	s.	d.		*Cr.*	£.	s.	d.
	To Mr. John Holland, due to me,	18	19	10		By Mrs. Phebe Young - -	7	10	0
	To Mr. Anthony Archer - -	10	0	0		By Mr. Thomas Green - -	2	8	6
	To Mr. John Flint - -	5	11	8			9	18	6
	To Mr. Samuel Taylor - -	1	17	0					
	To Mr. John Hunter - -	1	10	0					
	To Mr. Carlefs - -	29	10	0					
	To Mr. John Young - -	7	4	0					
		75	12	6					

BOOK-KEEPING BY DOUBLE ENTRY,

ACCORDING TO

THE ITALIAN METHOD.

THIS method was firſt invented in Italy, for which reaſon it is called the Italian Method; and it is ſaid to be by Double Entry, becauſe every article is twice entered in the Ledger.

The Books generally uſed in this way of keeping accounts, are three, viz. the Waſte Book, the Journal, and Ledger, of all which I ſhall give a ſhort account.

1. *Of the* WASTE BOOK.

The Waſte Book contains a complete memorial of every tranſaction in buſineſs, recorded promiſcuouſly as they happen with reſpect to time.

This book opens with an inventory of the perſon's money, effects, and debts, which at his firſt ſetting out in trade, are to be gathered from the particulars that make up his real eſtate.

2. *Of the* JOURNAL.

The Journal agrees with the Waſte Book in the form or manner of ruling, dating, and order or ſucceſſion of accounts, according to their dates; and differs from it by having the Debtors and Creditors of all accounts ſpecified.

On the right hand margins of each folio, or page, of the Journal and Waſte Book, are ruled three columns for pounds, ſhillings, and pence; and on the left hand margins, a column to receive the figures expreſſing the folios, or pages, where the ſame accounts are entered in the ſucceeding book; viz. in the Waſte Book margin are ſet the correſponding Journal pages, and in the Journal margin, the Ledger pages.

3. *Of the* LEDGER.

The Ledger is a large volume, containing all the tranſactions of a man's affairs, in ſuch order, that thoſe belonging to every different ſubject, lie together in one place, making ſo many diſtinct accounts.—In this book, all the accounts diſperſed in the Journal, are drawn out and tituled at the top Debtor and Creditor.

To form each account, two pages are required, oppoſite to each other; that on the left hand ſerving for Debtor, the other for Creditor; by which means, at any time, the merchant may be ſatisfied how any particular account ſtands: And for the more readily finding any particular account, the Ledger has always an alphabet prefixed

prefixed to it: The right hand margin of each page is ruled into three columns for money, and one for the figures expressing the folio's where the same articles stand, on the other folio of some other accounts; and on the left hand margin is formed a column, for the dates of the articles.

In any entry, to know what to make Debtor, and what Creditor, observe the following RULES.

1. What money, goods, and wares you have in possession, or are owing to you, make each particular account Debtor to Stock, and Stock Creditor by each account.

2. What you owe to any person, make Stock Debtor for so much to the person, and the same person Creditor by Stock.

3. What money is owing to you, make the person owing Debtor, and Stock Creditor.

In buying and selling GOODS.

1. To enter goods bought in, for ready money, make the goods bought Dr. and cash Cr.

2. To enter goods bought upon trust, make the goods Dr. and the seller Cr.

3. To enter goods bought for part ready money and part trust, make the goods Dr. and the seller Cr.

4. To enter goods sold for ready money, make the cash Debtor, and the goods Creditor.

5. To enter goods sold upon trust, make the buyer Debtor, and the goods Creditor, the same when several payments are to be made to you, mentioning in the journal the several times of payment.

6. To enter goods sold for part ready money and part trust, make the buyer Debtor for the whole, and the goods Creditor; then make the cash received Debtor, and the buyer Debtor, for what remains unpaid.

BARTERING.

1. When you give one sort of goods for another sort of equal value, make the goods received Debtor, and those you part with Creditor.

2. When you give one sort of goods for several other sorts of equal value, make each particular sort of goods received Debtor for its respective value, and the goods delivered by sundry accounts Creditor for the whole.

BORROWING *and* LENDING.

1. Make cash Creditor for what you lend, and the person that borrows Debtor.

2. What money you borrow, make cash Debtor, and the person lending Creditor.

BILLS.

1. When you draw bills of exchange upon your factor, and receive the contents, Debtor cash, and Creditor the factor's account current.

2. When your factor draws bills of exchange upon you for goods bought by him abroad, and you pay the contents, make the drawer Debtor and cash Creditor.

3. When bills of exchange are drawn by one of my factors on another, and I receive the contents at usance, I Debtor the accepting man, and Creditor the factor drawing.

PROFIT *and* LOSS.

1. What money you gain, win, or receive gratis, make cash Debtor to profit and loss, and profit and loss Creditor by its value.

2. What money or goods you give away, lose, or is spoiled, &c. is Creditor, and profit and loss is Debtor for its value.

FOREIGN TRADE.

1. When goods are sent to your factor abroad, make the voyage Debtor and the goods Creditor.

2. When you have advice that your factor has received the goods, then he becomes Debtor to the voyage, and the voyage Creditor. If he gains by selling the goods, he becomes Debtor to profit and loss, on account of the gain.

3. If he returns the goods he is Creditor, and the goods are Debtor.

HOUSE EXPENCES.

When you pay servants wages, house expences, &c. make profit and loss Debtor, and cash Creditor for its value.

ERRORS.

If you have entered any thing in your ledger under a wrong title, or false, you need not blot it out, but make this mark (X) in the margin against it, and write on the contrary side *error per contra*, with the sum against it, and the same mark in the margin, and the account will be right.

Directions for the learner.

Having ruled your books according to the forms of the following specimens, copy into your waste book the first month's transactions as they stand in the following waste books, omitting the left hand marginal figures, which are to be inserted according to the following directions.

Enter these articles one by one in the journal, according to the journal form, and when any article is entered in the journal, turn to the same article in the waste book, and directly against it, in the margin, write the number of the folio where it stands in the journal.

THE

THE WASTE BOOK.

			£	s	d
	Birmingham, January 6, 1783.				
	An inventory of the money, goods, and debts due to or by me, A—— B——.				
1	I have in ready money ——	100*l* 0*s* 0*d*			
	— 10 bags of hops, each 1*cwt* at 3*l* per bag	30 0 0			
	— 4 pipes of wine, at 20*l* 5*s* per pipe -	81 0 0			
	— 6 pieces of broad cloth, at 25*l* 10*s* per piece —— ——	153 0 0			
	— Thomas Rigby owes me on demand	10 10 0			
			374	10	0
	I owe as follows,				
1	To John Fletcher on bond ——	16*l* 0*s* 0*d*			
	To Samuel Turner, on account —	2 0 0			
			18	0	0
	January 1, 1783.				
1	Bought of John Jackson half a ton of cheese, at 20*l* 5*s* per ton, for which I paid ready money —		10	2	6
	3.				
1	Bought of Richard Perks 4 hogsheads of cyder, at 2*l* 5*s* per hogshead				
	paid in ready money ——	5*l* 0*s* 0*d*			
	remains due ——	4 0 0			
			9	0	0
	5.				
1	Bought of Samuel Tonks 2 hogsheads of tobacco, at 10*l* 6*s* per hogshead, to pay in three months		20	12	0
	7.				
1	Sold to John Wheeler 4 bags of hops, at 4*l* 10*s* per bag, for which I received ready money		18	0	0
	10.				
1	Sold to Samuel Tonks 2 pipes of wine, at 25*l* per pipe				
	received in part ——	30*l* 0*s* 0*d*			
	remains due on demand	20 0 0			
			50	0	0
	16.				
1	Sold John Jackson a piece of broad cloth, at 28*l* 10*s* to be paid in one month ——		28	10	0
	19.				
1	Bartered 4 hogsheads of cyder, at 3*l* per hogshead, for half a tun of cheese at the same value ——		12	0	0
	22.				
1	Lent Abraham Taylor the sum of 5*l* to be paid on demand —— ——		5	0	0
	26.				
1	Drawn a bill on Thomas Rigby, to be paid at sight		5	0	0
	28.				
2	Thomas Rigby paid the bill when I drew on him		5	0	0

		£.	s.	d.
	January 30.			
2	John Fletcher has drawn a bill on me, payable at sight	10	0	0
	February 5.			
2	Paid the bill to John Fletcher	10	0	0
	7.			
2	Borrowed of James Steventon the sum of ten pounds	10	0	0
	9.			
2	Paid John Fletcher a quarter's interest, due at Christmas last	0	4	0
	10.			
2	This day dined with the Honourable B. C. Esq. and gave his servants	0	5	0
	21.			
2	Won to-day at Quadrille	2	2	0
	13.			
2	Houshold expences last month	4	15	0
	14.			
2	Paid my book-keeper a quarter's salary, and other expences	15	0	0
	16.			
2	Shipped 2 *cwt* of Gloucester cheese, in the Diligence, John Lowe, master, consigned to William Cartwright, at the Hague, the cheese valued at — 2*l* 10*s* 0*d* paid freight and custom — 0 12 0	3	2	0
	20.			
2	Received advice that my factor has received the cheese safe at the Hague sold for — 6*l* 10*s* 0*d* charges — 0 18 0 being deducted makes	5	11	6
	23.			
2	Received from my factor, William Cartwright, at the Hague, a chest of sugar, weight nt. 3*cwt* 2*qrs* valued at — 4*l* 8*s* 0*d* paid freight and custom here — 1 3 6	5	11	6
	25.			
2	Received a legacy left me by my uncle	5	0	0
	27.			
2	Paid church and poor	0	2	6

THE JOURNAL.

Birmingham, January 1, 1783.

				£.	s.	d.
Sundries Dr. to Stock -	£. 374	10	0			
Cash in ready money - -	100	0	0			
Hops, for 10 bags, at 3*l* per bag -	30	0	0			
Wine, for 4 pipes, at 20*l* 5*s* per pipe -	81	0	0			
Broad cloth, for 6 pieces, at 25*l* 10*s* per piece	153	0	0			
Thomas Rigby, per note on demand -	10	10	0			
				374	10	0
Stock Dr. to sundries - -	18	0	0			
To John Fletcher, on bond -	16	0	0			
To Samuel Turner, on account -	2	0	0			
				18	0	0
January 1, 1783.						
Cheshire cheese Dr. to Cash -	£. 10	2	6			
For half a tun - -				10	2	6
3.						
Cyder Dr. to sundry accounts - -	9	0	0			
To cash paid — —	5	0	0			
To Richard Perks, remains due at 1 month	4	0	0	9	0	0
5.						
Tobacco Dr. to Samuel Tonks -	20	12	0			
For two hogsheads, at 10*l* 6*s* per hhd. to pay in 3 months				20	12	0
7.						
Cash Dr. to Hops -	18	0	0			
Received for 4 bags, at 4*l* 10*s* per bag				18	0	0
10.						
Sundry accounts Dr. to Wine -	50	0	0			
Cash received in part for 2 pipes -	30	0	0			
Samuel Tonks, for the rest on demand	20	0	0			
				50	0	0
16.						
John Jackson Dr. to broad cloth, for 1 piece valued	28	10	0			
To be paid in one month — —				28	10	0
19.						
Cheese Dr. to Cyder - -	12	0	0			
For 4 hogsheads; received in barter half a tun of Gloucester cheese, of the same value -				12	0	0
22.						
Abraham Taylor Dr. to Cash —	5	0	0			
To pay on demand — —				5	0	0
26.						
Bills received Dr. to Thomas Rigby						
For one drawn on him, to be paid at sight —				5	0	0

				£.	s.	d.
January 28.						
Cash Dr. to bills receivable						
Received the bill of Thomas Rigby - 1 -				5	0	0
30.						
John Fletcher, Dr. to bills payable						
To one to be paid at sight - - -				10	0	0
February 5.						
Bills payable Dr. to cash -	£.10	0	0			
Paid John Fletcher the bill - -				10	0	0
7.						
Cash Dr. to James Steventon for -	10	0	0			
Borrowed of him - - -				10	0	9
9.						
Profit and loss Dr. to cash -	4	0	0			
Paid John Fletcher a quarter's interest -				0	4	0
10.						
Profit and loss Dr. to cash -	0	5	0			
For 5s given to B. C. Esq. servants - -				0	5	0
11.						
Cash Dr. to profit and loss -	2	0	0			
Won at quadrille - - -				2	2	0
13.						
Profit and loss Dr. to cash -	4	15	0			
For one month's house expences - -				4	15	0
14.						
Profit and loss Dr. to cash -	15	0	0			
For a quarter's salary and other expences -				15	0	0
16.						
Voyage to the Hague Dr. to sundry accounts	3	2	0			
To cheese, 2 *cwt* shipped on board, valued at	2	10	0			
Paid freight and custom - -	0	12	0			
				3	2	0
20.						
William Cartwright, Dr. to sundry accounts	5	11	6			
To a voyage to the Hague - -	3	2	0			
To profit and loss gained by selling the goods	2	9	6			
				5	11	6
23.						
Sugar Dr. to sundry accounts -	5	11	6			
To William Cartwright, for one chest received. wt. 3*c* 2*qrs* valued at -	4	8	0			
To cash paid freight and custom -	1	3	6			
				5	11	6
25.						
Cash Dr. to profit and loss -	5	0	0			
For a legacy left me by my uncle -				5	0	0
27.						
Profit and loss Dr. to cash -	0	2	6			
Paid to church and poor - -				0	2	6

THE LEDGER, 1783.

INDEX or ALPHABET.

A	B	C
	Broad cloth folio 1 Bills receivable 2 Bills payable 2 Balance 3	Cash - - 1 Cheese - - 2 Cyder - - 2 Cartwright Wm. 2
D	E	F
		Fletcher, John - 1
G	H	I
	Hops - - 1	Jackson, John - 2
K	L	M
N	O	P
		Perks, Richard - 2 Profit and Loss - 3
Q	R	S
	Rigby, Thomas - 1	Stock - - 1 Steventon, James 3 Sugar - - 3
T	V	W
Tonks, Samuel - 2 Tobacco - - 2 Taylor, Abraham 2	Voyage to the Hague 3	Wine - - 1
X	Y	Z

		fol.	£.	s.	d.
1) 1783	*Stock* Dr.				
Jan. 1	To sundry accounts as per journal —	1	18	0	0
	To Balance		367	7	0
			385	7	0

		fol.	£.	s.	d.
	Cash Dr.				
Jan. 1	To stock — —	1	100	0	0
7	To hops — —	1	18	0	0
10	To wine — —	1	30	0	0
28	To bills receivable —	2	5	0	0
Feb. 7	To James Steventon —	3	10	0	0
11	To profit and loss —	3	2	2	0
25	To profit and loss —	3	5	0	0
			170	2	0

		fol.	£.	s.	d.
1783	*Hops* Dr.				
Jan. 1	To stock 10 bags, at 3*l* per bag —	1	30	0	0
	To profit and loss gained —	3	6	0	0
			36	0	0

		fol.	£.	s.	d.
	Wine Dr.				
Jan. 1	To stock 4 pipes, at 20*l* 5*s* per pipe	1	81	0	0
	To profit and loss gained —	3	9	10	0
			90	10	0

		fol.	£.	s.	d.
	Broad cloth Dr.				
Jan. 1	To stock 6 pieces, at 25*l* 10*s* per piece	1	153	0	0
	To profit and loss gained —	3	3	0	0
			156	0	0

		fol.	£.	s.	d.
	Thomas Rigby, Dr.				
Jan. 1	To stock per note on demand —	1	10	10	0

		fol.	£.	s.	d.
	John Fletcher, Dr.				
Jan. 30	To bills payable for one drawn on me payable at sight — —	2	10	0	0
	To balance remains due to me —	3	6	0	0
			16	0	0

			fol.	£.	s.	d.
1783		*Per contra* Cr.				
Jan.	1	By sundry accounts as per journal	1	374	10	0
		~~By Profit and Loss~~		10	17	0
				385	7	0

			fol.	£.	s.	d.
		Per contra Cr.				
				18	2	6
Jan.	1	By cheese paid for one ton ——	2	~~20~~	~~10~~	~~0~~
	3	By cyder —— ——	2	5	0	0
	22	By Abraham Taylor ——	2	5	0	0
Feb.	5	By bills payable —— ——	2	10	0	0
	9	By profit and loss —— ——	3	0	4	0
	10	By profit and loss —— ——	3	0	5	0
	13	By do. do. ——	3	4	15	0
	14	By do. do. —— ——	3	15	0	0
	16	By voyage to the Hague paid freight and custom —— ——	3	0	12	0
	23	By freight and custom ——	3	1	3	6
	27	By profit and loss ——	3	0	2	6
		By balance remaining in hand ——	3	110	17	6
				170	2	0

			fol.	£.	s.	d.
1783		*Per contra* Cr.				
Jan.	7	By cash for 10 bags, at 4^l 10^s per bag	1	18	0	0
		By balance remaining in hand, 6 bags at 3^l per bag —— ——	3	18	0	0
				36	0	0

			fol.	£.	s.	d.
		Per contra Cr.				
Jan.	10	By sundry accounts as per journal				
		By cash received in part for 2 pipes £30 0 0	1			
		By Samuel Tonks remains due on demand —— } 20 0 0				
				50	0	0
		By balance remains 2 pipes ——	3	40	10	0
				90	10	0

			fol.	£.	s.	d.
		Per contra Cr.				
Jan.	16	By John Jackson one piece, to be paid in one month ——	1	28	10	0
		By balance remains 5 pieces ——	3	127	10	0
				156	0	0

			fol.	£.	s.	d.
		Per contra Cr.				
Jan.	26	By bill receivable for one drawn on him, to be paid at sight ——	2	5	0	0
		By balance remains due on demand	3	5	10	0
				10	10	0

			fol.	£.	s.	d.
		Per contra Cr.				
Jan.	1	By stock due on bond ——	1	16	0	0

Date	Particulars		fol.	£.	s.	d.
(2) 1783	*Samuel Tonks,* Dr.					
Jan. 10	To wine on demand		2	20	0	0
	To balance due to him		3	2	12	0
				22	12	0
	Cheese Dr.					
Jan. 1	To cash paid for half a ton of Gloucester		1	10	2	6
19	To cyder bartered 4 hogsheads, at 3*l* per hhd. for half a ton of Cheshire cheese at the same value		2	12	0	0
	To profit and loss gained		3	0	2	0
				22	4	6
	Cyder Dr.					
	To sundry accounts as per journal					
Jan. 3	To cash paid in part for 4 hogsheads	£.5 0 0	1			
	To Richard Perks, remains due	4 0 0	2			
				9	0	0
	To profit and loss gained		3	3	0	0
				12	0	0
	Richard Perks, Dr					
Jan. 3	To balance remains due to him		3	4	0	0
	Tobacco Dr.					
Jan. 5	To Samuel Tonks 2 hogsheads, to be paid in 3 months, at 10*l* 6*s* per hhd.		2	20	12	0
	John Jackson Dr.					
Jan. 16	To broad cloth for 1 piece, to be paid in one month		1	28	10	0
	Abraham Taylor, Dr.					
Jan. 22	To cash 5*l* lent him to be paid on demand		1	5	0	0
	Bills receivable Dr.					
Jan. 26	To Thomas Rigby, for one drawn on him, to be paid at sight		1	5	0	0
	Bills payable Dr.					
Feb. 5	To cash paid John Fletcher, his bill drawn on me payable at sight		1	10	0	0
	William Cartwright, Dr.					
Feb. 20	To sundry accounts as per journal					
	To a voyage to the Hague	£.3 2 0	3			
	To profit and loss gained by selling the goods	2 9 6	3			
				5	11	6

			fol.	£.	s.	d.
2) 1783		*Per contra* Cr.				
Jan.	1	By stock due on account ———	1	2	0	0
	5	By tobacco 2 hogsheads, at 10*l* 6*s* per hhd.	2	20	12	0
				22	12	0
		Per contra Cr.				
Jan.	19	By voyage to the Hague, shipped on board 2*cwt* of Cheshire cheese ———	3	2	10	0
		By balance remains 2*cwt* of Gloucester	3	10	2	6
		By balance remains 8*cwt* of Cheshire	3	9	12	0
				22	4	6
		Per contra Cr.				
Feb.	16	By Cheshire cheese half a ton received in barter for 4 hhds. at 3*l* per hhd. ———	2	12	0	0
		Per contra Cr.				
Jan.	3	By cyder remains due at 3 months, in part for 4 hhds. ———	2	4	0	0
		Per contra Cr.				
Jan.	5	By balance remains 2 hhds. at 10*l* 6*s* per hhd.	3	20	12	0
		Per contra Cr.				
		By balance remains due to me ———	3	28	10	0
		Per contra Cr.				
		By balance remains due to me	3	5	0	0
		Per contra Cr.				
Jan.	28	By cash received the bill ———	1	5	0	0
		Per contra Cr.				
Jan.	30	By John Fletcher, for one bill drawn on me, to be paid at sight ———	1	10	0	0
		Per contra Cr.				
Feb.	23	By sugar received 1 chest, net weight 3*cwt* 2*qrs* valued at ——— £.4 8 0	3			
		By balance remains due to me 1 3 6	3			
				5	11	6

3) 1783		*Profit and loss* Dr.	fol.	£.	s.	d.
Feb.	9	To cash paid John Fletcher —	1	0	4	0
	10	To cash given to B. C. Esq. servants	1	0	5	0
	13	To cash for one month's houshold expences	1	4	15	0
	14	To cash for a quarter's salary to my book-keeper and other expences —	1	15	0	0
	27	To cash paid to church and poor —	1	0	2	6
		To stock gained by trade —	1	10	17	0
				31	3	6
		James Stevenson Dr.				
		To balance remains due on demand	3	10	0	0
		Voyage to the Hague Dr.				
Feb.	16	To sundry accounts as per journal —	3	3	2	0
		Sugar Dr.				
Feb.	23	To sundry accounts as per journal —				
		To William Cartwright for one chest, received net weight 3c 2qrs valued at	3	4	8	0
		To cash paid freight and custom	1	1	3	6
				5	11	6

B A L A N C E.

1783	*Balance* Dr.	fol.	£.	s.	d.
	To cash remaining in my hands - -	1	117	17	6
	To hops, 6 bags remains, at 3*l* per bag -	1	18	0	0
	To wine, remains 2 pipes, at 20*l* 5*s* per pipe	1	40	10	0
	To broad cloth, remains 5 pieces, at 25*l* 10*s* per piece - - - -	1	127	10	0
	To Thomas Rigby, remains due on demand	1	5	10	0
	To Gloucester cheese, remains 10 cwt. at 20*l* 5*s* per ton - - - - -	2	10	2	6
	To Cheshire cheese, remains 8 cwt. at 24*l* per ton - - - - - -	2	9	12	0
	To tobacco, remains 2 hhds. at 10*l* 6*s* per hhd.	2	20	12	0
	To John Jackson, remains due to me -	2	28	10	0
	To Abraham Taylor, remains due to me -	2	5	0	0
	To William Cartwright, remains due to me	2	1	3	6
	To sugar, remains in my hands - -	3	5	11	6
			389	19	0

1783	Per contra Cr.	fol.	£.	s.	d.
Feb. 11	By cash won at quadrille	1	2	2	0
20	By William Cartwright	3	2	9	6
25	By profit and loss for a legacy left by my uncle	1	5	0	0
	By hops gained	1	6	0	0
	By wine gained	1	9	10	0
	By broad cloth	1	3	0	0
	By cheese	2	0	2	0
	By cyder	2	3	0	0
			31	3	6

	Per contra Cr.	fol.	£.	s.	d.
Feb. 7	By cash borrowed	1	10	0	0

	Per contra Cr.	fol.	£.	s.	d.
Feb. 20	By William Cartwright, who has received the goods	2	3	2	0

	Per contra Cr.	fol.	£.	s.	d.
23	By balance remains one chest, net weight 3*cwt* 2*qrs* valued with charges	3	5	11	6

1783	Per contra Cr.	fol.	£.	s.	d.
	By John Fletcher, remains due to him -	1	6	0	0
	By Samuel Tonks, remains due to him -	2	2	12	0
	By Richard Perks, remains due to him -	2	4	0	0
	By James Steventon, remains due to him -	3	10	0	0
	By Stock, for my neat estate -	1	367	7	0
			389	19	0

Forms of Acquittances, Notes, Bills of Exchange, &c.

ACQUITTANCES UPON RECEIPT OF MONEY.

A general Receipt.

RECEIVED March 12, 1782, of Mr. John Smith, the sum of thirty pounds, in full of all demands.

£. 30 0 0 By me, W. T.

An Acquittance, proper to be given by a servant, when he receives money for the use of his master

RECEIVED March 21, 1782, of Mr. Thomas Brown, nine pounds, six shillings, for the use of my master, Daniel Young, on account.

£. 9 6 0 By me, A. B.

A Receipt or Acquittance for Rent paid.

RECEIVED Nov. 16, 1782, of Mr John Simpson, twenty pounds. for a quarter's rent, due at Michaelmas last.

£. 20 0 0 By me, B. C.

An Acquittance for Debt, received of a third hand.

RECEIVED the 1st day of March, 1782, of Mr. A. C. by the hands of Mr. C. D. the sum of eight pounds, ten shillings, in full, for certain goods, &c. bought by the said A. C. of me, in full of all demands.

£. 8 10 0 By me, W. P.

A Receipt for Interest due on Bond.

RECEIVED this, &c. of Mr. A. B. the sum of ten pounds, in full for one year's interest of 200*l*. due to me at Christmas last, on bond, from the said A. B.

£. 10 0 0 By me, C. D.

An Acquittance for a Legacy.

RECEIVED this, &c. of A. B. executor of the last will and testament of C. D. late of ——, &c. deceased, the sum of fifty pounds, in full of a legacy bequeathed to me, in and by the last will and testament of the said C. D. in full of all demands.

£. 50 0 0 By me, Y. Z.

A Receipt

A Receipt for Writings entrusted in a Person's Hands.

RECEIVED this, &c. of A. B, of ——, &c. two several deeds or conveyances, one thereof being a lease, and the other a release, made between ——, &c. for which several deeds or writings I hereby promise to be accountable, and to re-deliver the same to the said A. B. on demand.

Witness *Hugh Whatley.*

PROMISSORY NOTES.

The form of one payable on Demand.

I promise to pay to A. B. or order, the sum of thirty pounds, on demand, for value received. Witness my hand this first day of March, 1783.

£.30 0 0 — B. C.

Note. All promissory notes, bills of exchange, or drafts, being negotiable or transferable, for the payment of twenty shillings, or any sum of money above that sum and less than five pounds, must specify the names and places of abode of the persons respectively, to whom, or to whose order, the same shall be made payable, and shall bear date before or at the time of drawing or issuing thereof, and shall be made payable within the space of twenty-one days next after the day of the date thereof, and shall not be transferable or negociable after the time thereby limited for the payment thereof.

The form of one payable at a certain time.

Birmingham, 1st of March, 1783.

TWENTY-ONE days after date, I promise to pay to A. B. or his order, the sum of four pounds, for value received.

Witness, J. K. — J. T.

The Indorsements

12th of March, 1783.

PAY the contents to E. F. of Birmingham, or his order.

Witness, T. B. — A. B.

Note. Promissory notes and book debts, if not legally demanded in the space of six years, cannot be recovered by law.

INLAND BILLS OF EXCHANGE.

Form of one payable at sight.

£.100 0 0 — *Birmingham, Jan.* 1, 1783.

AT sight pay Mr. R. B. or order, the sum of one hundred pounds, the value received of B. C. and place it to account, as per advice from

To Mr. A. B. merchant,
Bath-street, Bristol. — E. D.

Form of one payable after Sight.

£.96 12 6 — *Birmingham, June* 1, 1782.

AT ten days sight pay Mr. R. S. or order, the sum of ninety-six pounds, twelve shillings and six-pence, for value received of T. L. and place it to account, as per advice from

To Mr. James Cox, jeweller,
High-street, Liverpool. — W. T.

Form of one payable after date.

£.300 0 0 *Birmingham, June 3, 1782.*

TWO months after date pay Mr. A. Y. or order, three hundred pounds, value received of G. B. Esq. and place it to account, as per advice from

To Mr. O. M. at the Angel, High Green, Wolverhampton

F. T.

Another at Sight.

£.2 18 6 *Birmingham, April 8, 1782.*

At sight hereof pay Thomas Hurd, or order, two pounds eighteen shillings and six-pence, for value received, as advised.

To Mr. William Shepard, Leeds.

Per *John Gray.*

FOREIGN BILLS OF EXCHANGE.

For crowns 600, at usance. *London, March 10, 1782.*

London on Paris. First bill. } AT usance pay this first bill of exchange to C. D. or order, six hundred crowns, for the value here received of Sir A. B. and place it to account, as per advice from

To Mr. X. Y. merchant, at Paris.

C. F.

For crowns 600, at usance. *London, Feb. 4. 1783.*

Second bill. } AT usance pay this my second of exchange (my first not paid) to C. D. or order, six hundred crowns, for value received of Sir A. B. and place it to account, as per advice

To Mr. X. Y. merchant, at Paris.

C. F.

For crowns 400, at 34*d.* per crown. *Paris, June 4, 1782.*

Paris on London. The bill. } AT double usance pay this my only bill of exchange to myself, the sum of four hundred crowns, exchange at thirty-four pence sterling per crown, ~~the~~ value received of Monf. E. B. and place it, as per advice, to the accompt of

To Mr. P. L. merchant, in London.

F. F.

The Correspondent's Letter of Advice.

Mr. P. L.

SIR, *Paris, June 4, 1782.*

By this post I have drawn on you for four hundred crowns, at 34*d.* payable to yourself, value of Monsieur E. B. which, with my other bills depending, please to honour, and the timely remittance shall be punctually made you by

To Mr. P. L. merchant, London.

Sir, your humble servant,

F. F.

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A

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